THE IMPACT OF DIFFERENT MIMO STRATEGIES ON THE NETWORK-OUTAGE PERFORMANCE

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ABSTRACT

In this paper, we analyze the performance of different multiple antenna transmission techniques in wireless networks with interference treated as noise. Our focus is on the impact of simple orthogonal space time codes (STC) on the so-called network-outage probability. Analytical results are given for some simple networks. These results show insufficiency of many traditional space-time coding designs under interference conditions. Simulations suggest that the main statements of this paper may also hold for general wireless networks, provided that the interference is sufficiently strong.

1. INTRODUCTION AND MOTIVATION

A wireless sensor network (WSN) consists of a number of sensor nodes spread across a geographical area to perform various sensing tasks and/or act on the environment. Some of the most existing applications for WSNs require that sensor nodes are powered by batteries. Their capabilities are strictly limited so that they can only perform simple operations. In particular, in many cases of practical interest, interference is treated as noise.

To ensure some quality-of- service (QoS), we assume that each connection has to achieve a certain signal-to-interference-and-noise-ratio (SIR). Due to channel variations and interference, it might be impossible to maintain the desired SIR on each link permanently. Given some established network topology and channel statistics, one of the most important objectives is then to guarantee certain outage probability performance of the network. In this paper, the network is said to be in outage if there exists at least one link, for which the SIR target cannot be satisfied. This event is called network-outage. The network-outage probability P_{NO} is the probability for this event. An exact definition is given later in the paper.

Most studies on multiple antenna techniques focuses either on point-to-point communication or on multiuser communication scenarios such as broadcast or multiple access channels. There is little literature on the performance analysis of multiple antenna systems that are exposed to (unknown) interference from other connections. The work of Blum et al. [1, 2] shows that in scenarios with large interference, standard multiple antenna techniques could fail to achieve the desired performance objectives. In addition, for some systems, it was shown that transmitting with only one antenna is optimal.

In this paper, we analyze the impact of different multiple antenna transmission strategies on the network-outage probability. Our main focus is on simple orthogonal space time codes (STC). Due to the complexity of the problem, analytical results are presented only for some simple networks. These results show insufficiency of many traditional space-time coding designs under interference conditions. Simulations suggest that the main statements of this paper may also hold for general wireless networks, provided that the interference is sufficiently strong. To the best of our knowledge, the impact of multiple antenna techniques on the network-outage probability under an optimal power control has not been considered before.

The paper is organized as follows: In Section 2, we introduce the system model and some important definitions. In that section, we also describe the main problem addressed in this paper. Section 3 presents the probability density functions of signal and interference attenuation for line-of-sight and non-line-of-sight channels. In doing so, we consider several widely studied transceiver strategies such as SISO, beamforming with two transmit and two receive antennas as well as the Alamouti STC with one receive antenna, with two receive antennas and with antenna selection where the receiver is equipped with two antennas. Section 4 shows an inherent drawback of orthogonal STCs in networks with interference if the SIR is used as a performance measure. In this case, general orthogonal STCs lead to different SIRs for the symbols transmitted in one STC symbol. The conclusion is that in contrast to the usual point-to-point communication, some channel knowledge may improve the performance of STCs. Section 5 analyzes the probability density function of the spectral radius for a simple network with only two users. We give analytical results for the density function of the spectral radius and the network outage probability for all considered transceiver strategies. The conclusions are validated by simulations for a larger network in Section 6.

2. SYSTEM SETUP AND DEFINITIONS

We consider a power-controlled wireless network with $K > 1$ transmitter-receiver pairs that, for simplicity, are referred to as users. We assume that a stream of independent information-bearing symbols is generated at each transmitter and this data stream is transmitted to the corresponding receiver over a wireless channel. We have no special requirements on the placement of the transmitters and receivers except that the wireless channels between them are assumed to have certain statistical properties. Each transmitter is equipped with one or more antennas, depending on the transceiver strategy. The users are fully synchronized and use the same transceiver strategy. There is no scheduling in time and frequency domain so that the signal of every user occupies the entire (available) frequency band at the same time. The interference is treated as noise.

The main figure of merit is the minimum SIR over data symbols transmitted simultaneously. To be more precise, assume for a moment that for each user, only one symbol is sent simultaneously at any particular time. Then, the SIR of user k is given by

$$\mathrm{SIR}_k = \frac{D_k p_k}{\sum_{i \neq k} G_{k,i} p_i + \sigma^2},\ 1 \leq k \leq K \tag{1}$$

where σ^2 denotes the noise variance per antenna, which is fixed for all transmission strategies, $D_k > 0$ represents the effective attenuation of the desired signal at the k^{th} receiver, and $G_{k,i} \geq 0$ is the effective attenuation of the interference signal caused by the i^{th} user. These quantities are collected in two matrices $\boldsymbol{D} = \mathrm{diag}(D_1, D_2, \ldots, D_K)$ and $\boldsymbol{G} = (G_{k,i})$. In words, the signal attenuation $D_k > 0$ is the k^{th} diagonal entry of the diagonal matrix $\boldsymbol{D} \geq 0$ and the interference attenuation $G_{k,i}$ is the $(k,i)^{th}$ entry of the de-traced matrix $\boldsymbol{G}$. p_k is the total power used by the k^{th} transmitter for one channel use. If a tranmission strategy (such as STC) serves several data symbols simultaneously, then we define the SIR of a user to be the minimum SIR over these data symbols. In what follows, assume that the SIR in (1) is the minimum SIR over the simultaneously transmitted symbols of user k

A common SIR-target γ is required to be satisfied for all users. It is well known [3], that, given $\boldsymbol{D}$ and $\boldsymbol{G}$, there exists a valid power allocation, iff the spectral radius $\rho(\boldsymbol{D}^{-1}\boldsymbol{G})$ of the matrix $\boldsymbol{D}^{-1}\boldsymbol{G}$ satisfies

$$\rho(\boldsymbol{D}^{-1}\boldsymbol{G}) < 1/\gamma. \tag{2}$$

Note that a power allocation is said to be valid, if the SIR requirements are satisfied for every user.

The wireless channels is inherently stochastic in nature so that $\boldsymbol{D}$ and $\boldsymbol{G}$ are random matrices. Furthermore, instant realisations as well as statisticaql properties of the matrices $\boldsymbol{D}$ and $\boldsymbol{G}$ depend on a transmission strategy. Now we say that the network is in outage or that network outage[1] occurs if there exists no power allocation such that the SIR target is satisfied for all the users, or equivalently, if $\rho(\boldsymbol{D}^{-1}\boldsymbol{G}) \geq 1/\gamma$. Furthermore, we define the network-outage probability to be

$$P_{NO}(\gamma) = P(\rho(\boldsymbol{D}^{-1}\boldsymbol{G}) \geq 1/\gamma)\,. \tag{3}$$

This probability depends on stochastic properties of the wireless channel as well as on the transmission strategy. In what follows, the channel between transmitter k and receiver i is called the desired channel if $k = i$. All other channels are called interference channels.

3. THE STATISTICS OF SIGNAL AND INTERFERENCE

In this paper we consider several strategies for the transmission. The observation of user k is given by

$$\boldsymbol{Y}_k = \boldsymbol{H}_{k,k}\boldsymbol{X}_k + \sum_{i \neq k} \boldsymbol{H}_{k,i}\boldsymbol{X}_i + \boldsymbol{N}_k.$$

Here and hereafter, $\boldsymbol{Y}_k$, $\boldsymbol{X}_k$ and $\boldsymbol{N}_k$ are matrices, whose dimension depend on the transmission strategy. For instance, in SISO systems, they become scalars, while in a beamforming scenario they are vectors whose length depend on the number of transmit and receive antennas. In general, the dimension of the matrices is $M \times N$ for $\boldsymbol{H}_{k,i}$, $M \times T$ for $\boldsymbol{X}_k$, $N \times T$ for $\boldsymbol{Y}_k$, $\boldsymbol{X}_k$ and $\boldsymbol{N}_k$ where $M(N)$ is the number of transmit (receive) antennas and T indicates how often the channel is used for the transmission.

The channel matrix $\boldsymbol{H}_{k,i}$ is a stochastic variable, which captures the properties of the channel between transmitter and receiver. To analyze the effects of transmission strategies in different scenarios, we use two extreme channel models for the analysis: In the first model the coefficients of the matrix are i.i.d. circular symmetric complex Gaussian variables of variance $\sigma_{k,i}^2$. This model is used to account for channels that offer full diversity, i.e. all variables are independent and the channel offers the maximum degrees of freedom in a statistical meaning. In the second model the matrix $\boldsymbol{H}_{k,i}$ is created by

$$\boldsymbol{H}_{k,i} = h_{k,i}\boldsymbol{C} \tag{4}$$

where $h_{k,i}$ is a circular symmetric complex Gaussian variable with variance $\sigma_{k,i}^2$ and $\boldsymbol{C}$ is any constant matrix, such that its entries satisfy $|c_{n,m}| = 1$. This model is used to to account for channels, that have only one degree of freedom, as e.g. pure line-of-sight channels.

[1] The SIR-target may be achieved for some of the users while it is not by other users. In this case, some users may be disabled. As this changes, the network structure and therefore can be seen as a new problem of the same structure. The question which users should be shout down is beyond the scope of this paper.

In our analysis, we primarily focus on the first model, and use the second model to determine, how much multiple-antenna strategies suffer if the channel between transmitter and receiver provide only limited diversity. This is interesting from practical point of view, as network nodes are often placed such that interfering channels are non-line-of-sight channels, while desired channels have often line-of-sight components. Therefore, we apply the second model only to the desired channels and not to the interfering channels.

For the analysis, we first focus on two transmit antenna systems $M = 2$ and apply Alamouti STC [4] with one receive antenna $N = 1$, with antenna selection at the receiver and with two receive antennas $N = 2$. As a reference, we also provide some results for SISO and beamforming with $M = N = 2$.

To keep the notation simple, we assume that $\sigma_{k,i}^2 = 1$ for all the channels. Furthermore, for the analysis, the matrix $\boldsymbol{C}$ in the second model is assumed to be the same for all the desired channels. This means that a somehow symmetric setup is used in the analysis. We use these constraints to obtain some insight in what happens if we use different transmission strategies. The specific results for the calculated statistics will vary, if other parameters or less symmetric scenarios are considered, but the analysis can be done in a similar way. Nevertheless, the choice of parameters will not change the implications of the results for the use of multiple-antenna techniques in interference networks. In simulations, we consider systems that are not subject to these constraints.

3.1. SISO

For SISO systems the statistics of the entries in $\boldsymbol{D}$ and $\boldsymbol{G}$ are immediately obvious. From

$$y_k = h_{k,k}x_k + \sum_{i \neq k} h_{k,i}x_i + n_k.$$

with $E\{|x_k|^2\} = p_k$ it follows that $D_k = |h_{k,k}|^2$ and $G_{k,i} = |h_{k,i}|^2$. All the non-zero coefficients of $\boldsymbol{D}$ and $\boldsymbol{G}$ are distributed according to an exponential distribution, whose parameter depends on the variance of the channel $\sigma_{k,i}^2$. Furthermore all the entries are independent of each other. With $\sigma_{k,i}^2 = 1$, we have

$$p_G(x) = p_D(x) = e^{-x}, x \geq 0 \tag{5}$$

where $p_G(x)$ and $p_D(x)$ are the probability density functions of the coefficients $G_{k,i}, k \neq i$ and D_k of the matrices $\boldsymbol{G}$ and $\boldsymbol{D}$ respectively. This formula holds for both models under consideration.

3.2. Beamforming matched to the desired channel

The beamforming scenario refers to a situation where transmitter and receiver beamformers are matched to the desired channel. With the channel $\boldsymbol{H}_{k,k} = \boldsymbol{V}_{k,k}\boldsymbol{\lambda}_{k,k}\boldsymbol{U}_{k,k}^H$ where $\boldsymbol{\lambda}_{k,k} = \mathrm{diag}(\lambda_1^{(k,k)}, \lambda_2^{(k,k)}, \ldots, \lambda_{\min\{M,N\}}^{(k,k)})$ is a diagonal matrix whose diagonal elements are ordered in descending order according to their absolute values, the received symbol is

$$\boldsymbol{y}_k = \boldsymbol{H}_{k,k}\boldsymbol{u}_{k,k}x_k + \sum_{i \neq k} \boldsymbol{H}_{k,i}\boldsymbol{u}_{i,i}x_i + \boldsymbol{N}_k.$$

where $\boldsymbol{u}_{k,k}$ with $\|\boldsymbol{u}_{k,k}\| = 1$ is the first colum of $\boldsymbol{U}_{k,k}$ and $E\{|x_k|^2\} = p_k$. The receiver computes $\hat{x}_{k,k} = \boldsymbol{v}_{k,k}^H\boldsymbol{y}_k$ where $\boldsymbol{v}_{k,k}$ with $\|\boldsymbol{v}_{k,k}\| = 1$ is the first colum of $\boldsymbol{V}_{k,k}$. Therefore, $D_k = \|\boldsymbol{v}_{k,k}^H\boldsymbol{H}_{k,k}\boldsymbol{u}_{k,k}\|^2 = \lambda_1(\boldsymbol{H}_{k,k}\boldsymbol{H}_{k,k}^H)$ and $G_{k,i} = \|\boldsymbol{v}_{k,k}^H\boldsymbol{H}_{k,i}\boldsymbol{u}_{i,i}\|^2$.

As $p(\boldsymbol{V}^H\boldsymbol{H}\boldsymbol{U}) = p(\boldsymbol{H})$ for unitary matrices $\boldsymbol{U}, \boldsymbol{V}$ independent from $\boldsymbol{H}$ and a matrix $\boldsymbol{H}$ with i.i.d. circular symmetric complex Gaussian distributed coefficients, the density function of $G_{i,j}$ is an exponential distribution

$$p_G(x) = e^{-x}, \ x \geq 0$$

as in the SISO scenario. The density function of D_k for the first model where the entries of $\boldsymbol{H}$ are i.i.d. circular symmetric complex Gaussian variables can be found in the literature [5] and is given for the case of $M = N = 2$ by

$$p_D(x) = e^{-x}(-2 + e^x(2 - 2x + x^2)), x \geq 0.$$

In the second model, the same transceiver strategy leads again to $D_k = \lambda_1(\boldsymbol{H}_{k,k}\boldsymbol{H}_{k,k}^H)$. Now since $\lambda_1(\boldsymbol{H}_{k,k}\boldsymbol{H}_{k,k}^H) = \lambda_1(\boldsymbol{C}_{k,k}\boldsymbol{C}_{k,k}^H)|h_{k,i}|^2 = c|h_{k,i}|^2$ with $\max\{M, N\} \leq c \leq MN$, the density function of D_k is a scaled exponential distribution. Compared to the SISO system, from a statistical perspective, the beamforming performance is always at least as good as the SISO performance as $c \geq 1$; it follows that the outage performance is at least as good as the SISO outage performance.

3.3. Alamouti

For the Alamouti scenario, we consider the following formula for a system with only one interfering channel. As all the channels are independent, an extension to the general setup is straightforward. To keep the notation simple, in this section, we use $\boldsymbol{H}$ for the desired channel and $\boldsymbol{G}$ for the interfering channel. Furthermore, d is used for the desired data symbols while s represent data symbols of the interfering user.

The received signal in the Alamouti system with two receive antennas is given by

$$\boldsymbol{Y} = \underbrace{\begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix}}_{\boldsymbol{H}} \begin{bmatrix} d_1 & -d_2^* \\ d_2 & d_1^* \end{bmatrix} + \underbrace{\begin{bmatrix} g_{1,1} & g_{2,1} \\ g_{1,2} & g_{2,2} \end{bmatrix}}_{\boldsymbol{G}} \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + \boldsymbol{N} \tag{6}$$

where $\boldsymbol{N}$ is a matrix containing the Gaussian noise. The data symbols satisfy $E\{|d_1|^2\} = E\{|d_2|^2\} = \frac{p_k}{2}$ and $E\{|s_1|^2\} = E\{|s_2|^2\} = \frac{p_i}{2}$ so that the transmit power per channel use is given by p_k for the k^{th} transmitter.

The received signal can be also written as follows

$$\tilde{\boldsymbol{Y}} = \underbrace{\begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \\ h_{2,1}^* & -h_{1,1}^* \\ h_{2,2}^* & -h_{1,2}^* \end{bmatrix}}_{\tilde{\boldsymbol{H}}} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \underbrace{\begin{bmatrix} g_{1,1} & g_{2,1} \\ g_{1,2} & g_{2,2} \\ g_{2,1}^* & -g_{1,1}^* \\ g_{2,2}^* & -g_{1,2}^* \end{bmatrix}}_{\tilde{\boldsymbol{G}}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} \quad (7)$$

with

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H} \\ \boldsymbol{H}^*\boldsymbol{J} \end{bmatrix}, \tilde{\boldsymbol{G}} = \begin{bmatrix} \boldsymbol{G} \\ \boldsymbol{G}^*\boldsymbol{J} \end{bmatrix} \quad (8)$$

and

$$\boldsymbol{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \quad (9)$$

Similar equations follow for a system with one receive antenna. Using these definitions, the receiver output yields

$$\begin{aligned} \tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{Y}} &= \tilde{\boldsymbol{H}}^H \left(\tilde{\boldsymbol{H}} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + \boldsymbol{G} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{n}_{12} \\ \boldsymbol{n}_{34} \end{bmatrix} \right) \\ &= \begin{bmatrix} \sum_{i,j} |h_{i,j}|^2 & 0 \\ 0 & \sum_{i,j} |h_{i,j}|^2 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} \boldsymbol{H}^H & \boldsymbol{J}^T\boldsymbol{H}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{G} \\ \boldsymbol{G}^*\boldsymbol{J} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} \boldsymbol{H}^H & \boldsymbol{J}^T\boldsymbol{H}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{n}_{12} \\ \boldsymbol{n}_{34} \end{bmatrix} \\ &= \begin{bmatrix} \sum_{i,j} |h_{i,j}|^2 & 0 \\ 0 & \sum_{i,j} |h_{i,j}|^2 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \\ &\quad + \left(\boldsymbol{H}^H\boldsymbol{G} + \boldsymbol{J}^T \left(\boldsymbol{H}^H\boldsymbol{G} \right)^* \boldsymbol{J} \right) \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \\ &\quad + \boldsymbol{H}^H\boldsymbol{n}_{12} + \boldsymbol{J}^T\boldsymbol{H}^T\boldsymbol{n}_{34} \quad (10) \end{aligned}$$

To analyze the statistics of $G_{k,l}$ and D_k, note that the SIR of both the transmitted symbols is the same. Therefore, without loss of generality, we can focus on the first symbol d_1. As we have normalized the SIR such that the noise power is $\sigma^2 = E\{|n_i|^2\}$, we arrive at

$$D_k = \frac{1}{2} \sum_{i,j} |h_{i,j}|^2,$$

where the factor $1/2$ accounts for the fact that we use only half the power for the transmission of each of the symbols. Therefore, the probability density function of D_k is a scaled $\chi^2_{4N}, N \in \{1, 2\}$, distribution for the first model of a channel matrix with i.i.d. entries. So, in case of one receive antenna, we have

$$p_D(x) = 4xe^{-2x}$$

and, in case of two receive antennas,

$$p_D(x) = \frac{8}{3}x^3e^{-2x}. \quad (11)$$

Furthermore, in case of antenna selection, if we always chose the receive antenna j, such that $D_k = \max_j D_k(j) = \frac{1}{2}\sum_i |h_{i,j}|^2$, we have

$$p_D(x) = 8x(e^{-2x} - e^{-4x} - 2xe^{-4x})$$

which can be easily obtained from (11) by noting that the signal attenuation for both antennas is independent. For the second model the signal attenuation is

$$D_k = N|h_{k,k}|^2,$$

which has a χ^2_2 distribution (as in the SISO case), scaled by the number of receive antennas. For all these systems, the coefficients of the matrix $\boldsymbol{D}$ are i.i.d.

To see the statistics of the interference, note that given $\boldsymbol{H}$, the entries in the first row of $\boldsymbol{H}^H\boldsymbol{G}$ are independent Gaussian random variables with variance $\sum_i |h_{1,i}|^2$, while the entries in the second row are independent Gaussian random variables with variance $\sum_i |h_{2,i}|^2$. Furthermore, the dependence between the entries is only columnwise. The entries of $\boldsymbol{H}^H\boldsymbol{G} + \boldsymbol{J}^T \left(\boldsymbol{H}^H\boldsymbol{G}\right)^* \boldsymbol{J}$ are statistical dependent, but each entry has a Gaussian distribution with variance $\sum_{i,j} |h_{i,j}|^2$. Furthermore, the dependence of the variables does not matter for the statistics of $G_{k,l}$, as the interfering data symbols s_1 and s_2 are assumed to be independent. From this, it follows that the distribution of $G_{k,l}$ is a χ^2_4-distribution independent of the number of receive antennas N and independent of the model used for the desired channel $\boldsymbol{H}$. The distribution is given by

$$p_G(x) = xe^{-x}$$

An interpretation of this result is that not only the desired signal gains from diversity, but also the interference. In fact, from a statistical point of view and compared to the SISO setup, each interferer counts twice, if the Alamouti scheme is used for transmission. This is significantly different from the beamforming setup, where the interference is similar to the SISO case although two antennas are used for the transmission.

This gives rise to the question whether STCs are of any use in channels with interference. The answer is not obvious, and one has to take a look at the statistics of the spectral radius, which is the key performance measure for the outage of a network. We will analyze this statistics for a network with a simple structure in the next section and compare it to those of beamforming and SISO.

4. A NOTE ON GENERAL ORTHOGONAL SPACE-TIME-CODES AND INTERFERENCE

The analysis of the Alamouti setup rises the question about the performance of other orthogonal STCs. The following analysis shows that general orthogonal STCs suffer from interference since the SIR for the different symbols is in general not the same anymore. Indeed, if the minimum SIR is of interest, an equal distribution of the signal-to-interference ratios over data symbols is necessary to achieve the optimal performance. Therefore, for these STCs, some performance gains may be possible if the channel is known at the transmitter. In this case, different powers could be allocated to different data symbols of the STC in order to make the corresponding SIRs as equal as possible.

For any orthogonal STC, the received signal in a two user scenario can be written as

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{G}\boldsymbol{S} + \boldsymbol{N} \tag{12}$$

where the desired term $\boldsymbol{X}$ is

$$\boldsymbol{X} = \sum_{k=1}^{K} \boldsymbol{A}_k x_k^R + i\boldsymbol{B}_k x_k^I \tag{13}$$

and the interference term

$$\boldsymbol{S} = \sum_{k=1}^{K} \boldsymbol{A}_k s_k^R + i\boldsymbol{B}_k s_k^I \,. \tag{14}$$

For the code, it holds that with $k \neq l$

$$\boldsymbol{A}_k\boldsymbol{A}_k^H = \boldsymbol{I} \tag{15}$$

$$\boldsymbol{B}_k\boldsymbol{B}_k^H = \boldsymbol{I} \tag{16}$$

$$\boldsymbol{A}_k\boldsymbol{A}_l^H = -\boldsymbol{A}_l\boldsymbol{A}_k^H \tag{17}$$

$$\boldsymbol{B}_k\boldsymbol{B}_l^H = -\boldsymbol{B}_l\boldsymbol{B}_k^H \tag{18}$$

$$\boldsymbol{A}_k\boldsymbol{B}_l^H = \boldsymbol{B}_l\boldsymbol{A}_k^H \tag{19}$$

The detection of the real part of symbol x_m is then

$$\begin{aligned}
&\mathrm{ReTr}\left(\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{Y}\right)\\
&=\mathrm{ReTr}\left(\boldsymbol{A}_m^H\boldsymbol{H}^H\left[\boldsymbol{H}\boldsymbol{X}+\boldsymbol{G}\boldsymbol{S}+\boldsymbol{N}\right]\right)\\
&=\mathrm{ReTr}\Bigg(\boldsymbol{A}_m^H\boldsymbol{H}^H\Bigg(\boldsymbol{H}\left(\sum_{k=1}^{K}\boldsymbol{A}_k x_k^R + i\boldsymbol{B}_k x_k^I\right)\\
&\quad+\boldsymbol{G}\left(\sum_{k=1}^{K}\boldsymbol{A}_k s_k^R + i\boldsymbol{B}_k s_k^I\right)+\boldsymbol{N}\Bigg)\Bigg)\\
&=\mathrm{ReTr}\left(\boldsymbol{A}_m\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{H}x_m^R\right)\\
&\quad+\mathrm{ReTr}\left(\boldsymbol{A}_m\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{G}s_m^R\right)\\
&\quad+\mathrm{ReTr}\left(\sum_{k\neq m}^{K}\boldsymbol{A}_k\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{H}x_k^R+\sum_{k=1}^{K}i\boldsymbol{B}_k\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{H}x_k^I\right)\\
&\quad+\mathrm{ReTr}\left(\sum_{k\neq m}^{K}\boldsymbol{A}_k\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{G}s_k^R+\sum_{k=1}^{K}i\boldsymbol{B}_k\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{G}s_k^I\right)\\
&\quad+\mathrm{ReTr}\left(\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{N}\right)\\
&=||\boldsymbol{H}||_F^2 x_m^R+\mathrm{ReTr}\left(\boldsymbol{H}^H\boldsymbol{G}\right)s_m^R\\
&\quad+\mathrm{ReTr}\left(\sum_{k\neq m}^{K}\boldsymbol{A}_k\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{G}s_k^R+\sum_{k=1}^{K}i\boldsymbol{B}_k\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{G}s_k^I\right)\\
&\quad+\mathrm{ReTr}\left(\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{N}\right)
\end{aligned}$$

where the last equation follows from

$$\begin{aligned}
&2\mathrm{ReTr}\left(\boldsymbol{A}_k\boldsymbol{A}_l^H\boldsymbol{H}^H\boldsymbol{H}\right)\\
&\quad=\mathrm{Tr}\left(\boldsymbol{A}_k\boldsymbol{A}_l^H\boldsymbol{H}^H\boldsymbol{H}\right)+\mathrm{Tr}\left(\left(\boldsymbol{A}_k\boldsymbol{A}_l^H\boldsymbol{H}^H\boldsymbol{H}\right)^H\right)\\
&\quad=\mathrm{Tr}\left(\boldsymbol{A}_k\boldsymbol{A}_l^H\boldsymbol{H}^H\boldsymbol{H}\right)+\mathrm{Tr}\left(\boldsymbol{A}_l\boldsymbol{A}_k^H\boldsymbol{H}^H\boldsymbol{H}\right)\\
&\quad=\mathrm{Tr}\left(\left(\boldsymbol{A}_k\boldsymbol{A}_l^H+\boldsymbol{A}_l\boldsymbol{A}_k^H\right)\boldsymbol{H}^H\boldsymbol{H}\right)\overset{(17)}{=}0
\end{aligned}$$

and similar

$$\mathrm{ReTr}\left(i\boldsymbol{B}_k\boldsymbol{A}_l^H\boldsymbol{H}^H\boldsymbol{H}\right)=0$$

Similar steps has to be performed for all k as well as for the imaginary part. An important part here is the term

$$\mathrm{ReTr}\left(\sum_{k\neq m}^{K}\boldsymbol{A}_k\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{G}s_k^R+\sum_{k=1}^{K}i\boldsymbol{B}_k\boldsymbol{A}_m^H\boldsymbol{H}^H\boldsymbol{G}s_k^I\right).$$

As the matrix $\boldsymbol{H}^H\boldsymbol{G}$ is in general random, this contribution to the interference will not vanish. Furthermore, the result of this may be quite different for the decoding of different data symbols x_m.

The Alamouti code is a special case in the sense that the sets

$$\mathcal{A}_m = \{\boldsymbol{A}_k\boldsymbol{A}_m^H | k = 1..K\}$$

and

$$\mathcal{B}_m = \{\boldsymbol{B}_k\boldsymbol{A}_m^H | k = 1..K\}$$

are — up to a possible change of the sign of the matrices in the set due to the requierment (17) — independent of m. So, all the symbols suffer the same interference.[2] Codes for more than two antennas do not have this property in general. Of course, the property can always be fulfilled with low rate STCs by setting $\mathcal{A}_m = \{\boldsymbol{X}_k | \forall_{k \neq m} \boldsymbol{X}_k = \boldsymbol{0}, \boldsymbol{X}_m = \boldsymbol{I}\}$ and $\mathcal{B}_m = \{\boldsymbol{0} | \boldsymbol{X}_k = \boldsymbol{0}, k = 1..K\}$. An interesting questions for future research is, if there exist high rate codes with this property of higher dimension and what is the maximum rate achievable with such codes. This discussion is not in the scope of this paper.

If such symmetry conditions are not fulfilled, the SIRs for different symbols may be different. Furthermore, as mentioned above, since the SIR depends on the channels, some channel knowledge at the transmitter can be used to enhance the system performance. However, this would deprive the STCs of their main advantage, namely, the fact that they do not require channel state information at the transmitter in case of point-to-point communication.

5. STATISTICS OF THE SPECTRAL RADIUS FOR SIMPLE NETWORKS

In order to figure out whether the Alamouti STC is able to improve the performance of networks with interference, we consider the density function of the spectral radius of the matrix $\boldsymbol{D}^{-1}\boldsymbol{G}$. In general, the problem is intractable. Therefore, we confine our attention to a simple network structure with only two users. From the statistics of the spectral radius, we can calculate the outage probability, which can be used to assess the suitability of a transmission scheme for the use in networks with interference. In our analysis, all involved channel matrices have complex Gaussian coefficients with variance 1. The coefiicients of the desired channel in the first scenario and those of the interfering channels for all scenarios are independent distributed; in the second considered scenario which models a line-of-sight channel, the coefficients have a deterministic relation as in (4). Furthermore all channels are independent from each other. This setup allows some intuition about the performance of the different transmission schemes. The conclusions of this analysis are verified by simulations for some larger networks in the next section.

The equations that appear in the analysis are quite involved for more complicated transmission schemes and do not provide further insight. Therefore, we only sketch the analysis for the SISO setup with some more details and confine ourself to visualize the results for the other schemes.

In the considered setup, the matrix

$$\boldsymbol{A} = \boldsymbol{D}^{-1}\boldsymbol{G} = \begin{bmatrix} 0 & a = \frac{G_{1,2}}{D_1} \\ b = \frac{G_{2,1}}{D_2} & 0 \end{bmatrix}$$

is a 2×2 de-traced matrix with two i.i.d. entries a and b. The spectral radius of such a matrix is given by

$$\rho = \sqrt{ab}.$$

In order to obtain the probability density function (pdf) of the spectral radius, we start with the pdf of the entry a given D_1, which is given by

$$p_a(a|D_1) = p_G(aD_1)\frac{\delta G_{1,2}}{\delta a} = D_1 e^{-aD_1}\,.$$

Therefore,

$$p_a(a) = \int_0^\infty p_a(a|D)p_D(D)dD = \frac{1}{(1+a)^2}$$

where we skipped the index of D for convenience and used the result for $p_D(x)$ and $p_G(x)$ in a SISO system (5). As the entries of $\boldsymbol{D}$ and $\boldsymbol{G}$ are i.i.d. so are a and b and we have

$$p_a(x) = p_b(x).$$

For the pdf of the spectral radius given the coefficient a it follows that

$$p_\rho(\rho|a) = p_b(\frac{\rho^2}{a})\frac{\delta b}{\delta \rho} = \frac{1}{(1+\frac{\rho^2}{a})^2}\frac{2\rho}{a}$$

and[3]

$$p_\rho(\rho) = \int_0^\infty p_\rho(\rho|a)p_a(a)da = 2\rho\frac{2\rho^2 - 2 - (1+\rho^2)\log(\rho^2)}{(1-\rho^2)^3}.$$

For all considered transmission schemes this integral can be solved analytically using

$$\int_0^\infty f(x)dx = -\sum_{z \neq 0} res_z(f(\xi)\log(\xi)).$$

The resulting pdf of the spectral radius for the different considered transmission schemes is plotted in Figure 1.

With this pdf of the spectral radius the network outage probability defined in (3) is now given by

$$P_{NO}(\gamma) = \int_{\frac{1}{\gamma}}^\infty p_\rho(\rho)d\rho.$$

The analytical results for the network-outage probability for the different considered transmission schemes are plotted in Figure 2 over the SIR target in dB.

The plot shows that the Alamouti scheme with one receive antenna has a very limited advantage compared to SISO. Furthermore, if the SIR requirements are not small compared to what the network offers, SISO has less outage than the Alamouti scheme with one receive antenna. Alamouti with a second receive antenna has some more advantage compared to

[2]Similar arguments hold for the detection of the imaginary part. In this case, the sets based on $\boldsymbol{B}_k\boldsymbol{B}_m^H$ and $\boldsymbol{A}_k\boldsymbol{B}_m^H$ need to be considered.

[3]The logarithms in this paper are base e logarithms unless otherwise stated.

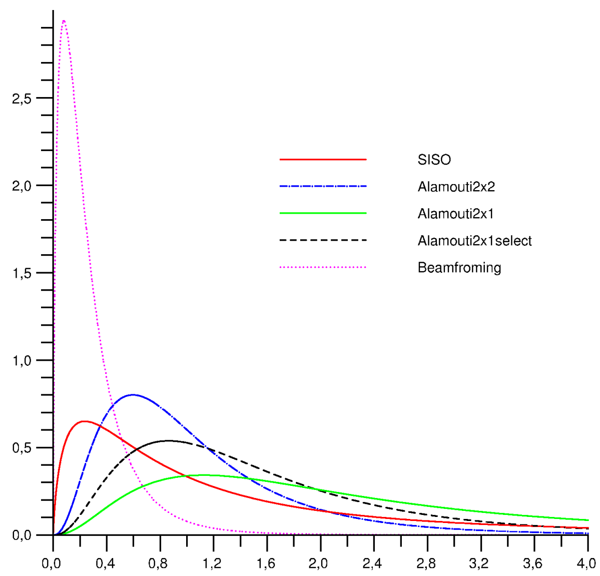

Fig. 1. The pdf of the spectral radius for a two user setup. Transmission strategies are SISO, two antenna beamforming, Alamouti, with one and two receive antennas as well as Alamouti with antenna selection. All channel coefficients are i.i.d. complex Gaussian with $\sigma = 1$.

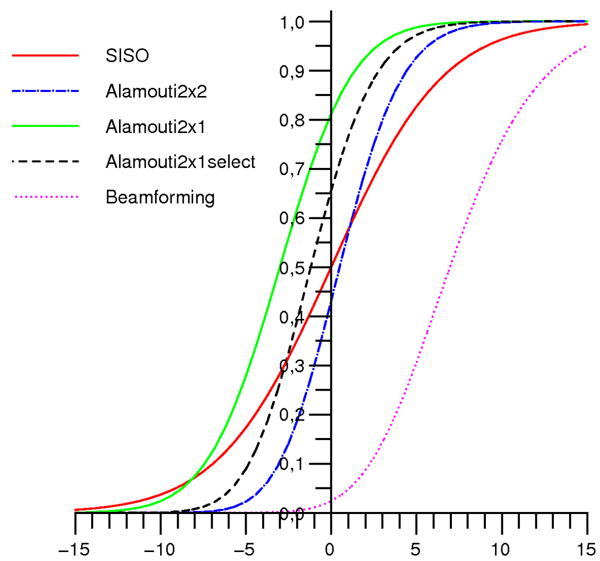

Fig. 2. Outage probability $P_{NO}(\gamma)$ (SIR Target γ in dB) for a two user setup. Transmission strategies are SISO, two antenna beamforming, Alamouti with one and two receive antennas as well as Alamouti with antenna selection. All channel coefficients are i.i.d. complex Gaussian with $\sigma = 1$.

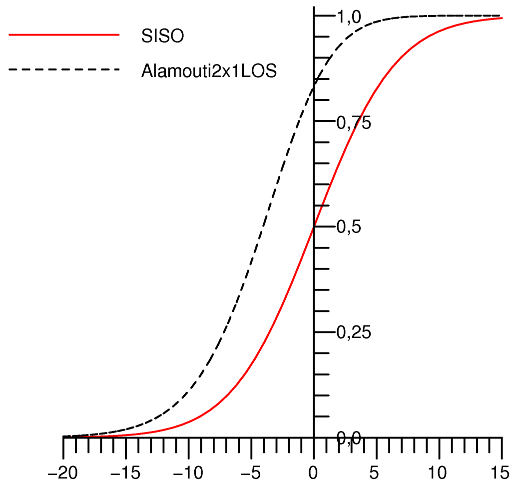

Fig. 3. Outage probability $P_{NO}(\gamma)$ (SIR Target γ in dB) for a two user setup. The two channels of the desired channels offer no diversity while the channel to interfering transmitters have full diversity. The channel coefficients are complex Gaussian with $\sigma = 1$, where the interfering channel coefficients are i.i.d. while the coefficients of the desired channels have a deterministic dependency on each other.

SISO, but even with two receive antennas Alamouti is not better for all SIR targets, even though SISO uses only one receive antenna. This shows that the interference enhancement at the receiver output due to diversity gains on the interference channels may prevail the performance improvement that results from the diversity gains achieved on the desired channel — especially if interference is dominant.

Figure 3 compares the network outage of a SISO system to that of an Alamouti system with one receive antenna in an extreme line-of-sight scenario as considered in (4). As the analysis showed, every interferer counts twice in the Alamouti setup. Therefore the network outage probability is always larger than that of the SISO setup.

6. SIMULATION OF MORE COMPLEX NETWORKS

For the simulation of more complex networks, we consider a network consisting of $K = 18$ active users where all the transmitter-receiver pairs are distributed according to a uniform distribution in an area of about $10000m^2$. Transmitter and receiver nodes of a link are close to each other (approximately $5m$) when compared to interfering nodes. In the simulated idealized non-line-of-sight (NLOS) setup, all channel coefficients are i.i.d. circular symmetric complex Gaussian distributed. Furthermore, we use a path loss exponent $n = 3.5$ and log-normal distributed shadowing with variance $\sigma^2_{sh} = 6dB$. For comparison reasons, we also show (in addi-

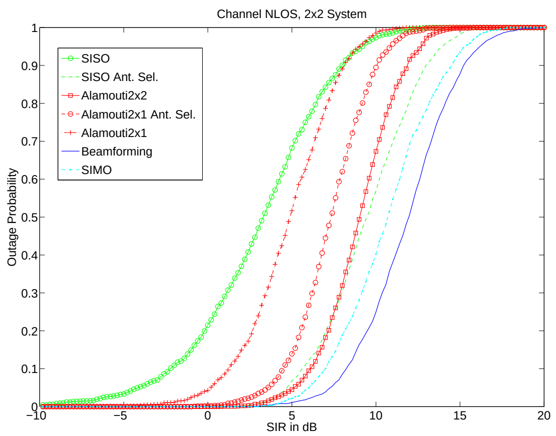

Fig. 4. Outage probability $P_{NO}(\gamma)$ (SIR Target γ in dB) for complex network with 18 active users and NLOS channels.

tion to the analyzed transceiver schemes) the performance of SISO with antenna selection and with maximum ratio combining at the receiver (SIMO) — both with two receive antennas respectively.

Figure 4 shows the network outage for the NLOS setup. We see that although the interference in this scenario is relatively low, the advantage of Alamouti with one receive antenna over SISO is small when compared to that of beamforming, SIMO or SISO with antenna selection. Compared to SISO with antenna selection, even Alamouti 2×2 and Alamouti with antenna selection have a poor performance. The reason for this is the diversity gain on the interference channels for the Alamouti scheme. Similar effects can be observed in simulations with more realistic channel models.

7. CONCLUSION

The Alamouti STC is inappropriate for many symbol synchronous networks in which interference is treated as noise since the scheme induces a diversity gain to the interference. Similar results hold for other orthogonal STCs. In many scenarios, transmitting with only one antenna is superior if one considers the increase of complexity due to the STC. In general, receive diversity proves to be give more benefit than transmit diversity by using STCs. Furthermore general orthogonal STCs lead to unequal SIR performance for the different symbols transmitted in one STC symbol. As a consequence, a channel knowledge may increase the performance of the code.

The results indicate that traditional point-to-point designs might be not suitable in distributed networks with strong interference. Is there a STC, that does not suffer in networks? Is transmitter diversity inadequate for networks in general, as long as no channel knowledge is available? Answers to these questions would be helpful when designing transmission strategies for distributed wireless networks. Distributed STCs and relaying are often located in distributed networks as e.g. sensor networks, which treat interference as noise. Most of the current work does not consider nearby nodes performing similar operations and inducing interference which — although Gaussian distributed for every time instance — may have significantly different impact on the performance than Gaussian noise, if channel statistics are taken into account.

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