A COMPARISON OF HIGH RATE ALGEBRAIC AND NON-ORTHOGONAL STBCS

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ABSTRACT

In this paper we unify and compare two high rate space-time coding constructions and layering techniques for MIMO systems. Algebraic space-time coding constructions are revisited and their relation to non-orthogonal codes (with quasi-orthogonal layers) is established. We discuss the class of perfect and golden space-time block codes, consisting of the version used in IEEE 802.16e specification for a system with 2 tx and 2 rx antennas. The main contribution of the paper is to consider both algebraic and non-orthogonal space-time codes for the 4 tx and 2 rx antenna setup.

1. INTRODUCTION

MIMO modulation methods that simultaneously improve diversity and transmission rate have attracted significant interest in recent years. The development of these modulation methods progressed along two separate paths. In crude terms, on one path, one begins from a full rate MIMO scheme, such as spatial multiplexing, and optimizes the symbol constellations for each layer jointly, see [6,9] and references therein. On the other main path, one begins from a full diversity transmission scheme, such as an orthogonal space-time block code [3], and adds new layers or new symbol matrices to improve the symbol rate [16, 18]. In terms of matrix modulation terminology, these two approaches differ from each other in the way the basis matrices are selected for the modulation matrix. For both methods, it is important to optimize symbol constellations so that the diversity benefits are fully exploited.

MIMO modulation methods that transmit two symbols per channel use have recently been adopted to OFDM-based wireless standards, such as IEEE 802.16e (WiMAX) [21]. The 802.16e specification includes a variant of a symbol-rate-two space-time code, known today as the *Golden code* [9]. This transmission method requires only two transmit antennas and obtains the highest coding gain know today. It was independently discovered in [7, 9, 18] under different names. Their algebraic properties were further analyzed and the codes were generalized in [9–11]. Incidentally, the code proposed in [15] is similar, but obtains lower coding gain. Extensions of the symbol-rate-two transmission methods for use with four transmit antennas are also attractive to wireless standards. Such transmission schemes can be constructed e.g. by puncturing two layers from 4-layer 4×4 MIMO code, or by puncturing a set of basis matrices from symbol-rate-four non-orthogonal space-time code [17, 18].

In this paper, we compare different symbol-rate-two MIMO modulators (or codes) with either two or four transmit antennas, and with two receive antennas. In particular, we explore the similarities and differences between algebraic MIMO modulators from [9, 10] and the non-orthogonal MIMO modulators from [17, 18], and evaluate their performance using a realistic coding chain that models WiMax [20] channel encoding.

The following notations are used in the paper: T denotes transpose and $\dagger$ denotes transpose conjugate. Let $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{C}$ and $\mathbb{Z}[j]$ denote the ring of rational integers, the field of rational numbers, the field of complex numbers, and the ring of Gaussian integers, where $j^2 = -1$. Let $\mathcal{S}$ and $\mathcal{S}^2$ denote PAM and QAM constellation sets, respectively. Let $\mathbb{Q}(\theta)$ denote an algebraic number field generated by the primitive element θ. The $m \times m$ dimensional identity matrix is denoted by $\mathbf{I}_m$. The matrix $\mathbf{1}_m$ is defined as an all ones $m \times m$ matrix. Given an m dimensional vector $\mathbf{v}$, $\mathbf{V} = diag(\mathbf{v})$ is the $m \times m$ diagonal matrix with $\mathbf{V}_{i,i} = v_i$ and $\mathbf{V}_{i,k} = 0$ for all $i, k = 1, \ldots, m$, and $i \neq k$.

2. SYSTEM MODEL

We consider a $n_T \times n_R$ MIMO system. We review symmetric MIMO systems with $n_T = n_R$, but our main focus is on asymmetric systems with $n_T > n_R$. We assume that the channel matrix $\mathbf{H}$ remains constant for the duration T of a codeword $\mathbf{X}$

$$\mathbf{Y}_{n_R \times T} = \mathbf{H}_{n_R \times n_T} \mathbf{X}_{n_T \times T} + \mathbf{Z}_{n_R \times T} \tag{1}$$

The work is in part supported by the STREP project No. IST-026905 (MASCOT) within the sixth framework programme of the European Commission, as well as the Christian Doppler Laboratory for Design Methodology of Signal Processing Algorithms and ARC Seibersdorf Research GmbH.

In (1), $\mathbf{Z}$ is the complex i.i.d. Gaussian noise matrix with entries $\mathcal{CN}(0, N_0)$ and $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the independent Rayleigh fading channel matrix with complex i.i.d. entries $\mathcal{CN}(0, 1)$, which is given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_{n_R} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,n_T} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,n_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R,1} & h_{n_R,2} & \cdots & h_{n_R,n_T} \end{bmatrix}.$$

The space-time codes are designed under the assumption that the elements of the channel matrix are Rayleigh distributed and they vary independently from one block to another.

3. HIGH-RATE SPACE-TIME CODES

Consider first square ($n_T = T$) linear dispersion ST block coding schemes that achieve the diversity/multiplexing gain tradeoff. For each codeword we can transmit $T \times n_R$ QAM information symbols arranged in the matrix

$$\begin{aligned} \mathbf{B} &= \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_{n_R} \end{bmatrix} \\ &= \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,n_R} \\ \vdots & \vdots & \ddots & \vdots \\ b_{T,1} & b_{T,2} & \cdots & b_{T,n_R} \end{bmatrix} \end{aligned} \tag{2}$$

where $b_{i,l} \in \mathbb{Z}[j], i = 1, \ldots, n_T, l = 1, \ldots, n_R$. We say that such codes have *full rate* of n_R symbols per channel use. Let $\mathcal{E}$ denote the average energy of the QAM symbols $b_{i,l}$.

3.1. TAST Codes

We recall threaded-algebraic space-time (TAST) codes codes from [6]. The TAST codes are constructed by transmitting a scaled DAST code in each layer (or thread) l, where $l = 1, \ldots, n_R$, i.e.,

$$\mathbf{x}_l = \phi_l \mathbf{M} \mathbf{b}_l, \tag{3}$$

where $\mathbf{x}_l$ are the encoded symbols, $\mathbf{b}_l$ are the complex QAM information symbol vectors, and ϕ_l is chosen to ensure full diversity and maximize the coding gain of the component codes. In [6], ϕ_l is given by

$$\phi_l = \phi^{(l-1)/n_T}, \tag{4}$$

where $\phi = e^{i\lambda}$ ($\lambda \neq 0$) is either an algebraic number or transcendental number [6].

In (3), $\mathbf{M} \in \mathbb{C}^{n_T \times n_T}$ is a rotation matrix defining a DAST code, which is constructed from an algebraic number field $\mathbb{Q}(\theta)$ of degree n_T [6, 13]. Let $\mathbf{s} = [s_1, \ldots, s_{n_T}]^T = \mathbf{M}\mathbf{b}$ and $\hat{\mathbf{s}} = [\hat{s}_1, \ldots, \hat{s}_{n_T}]^T = \mathbf{M}\hat{\mathbf{b}}$ be two different DAST codewords, where $\mathbf{b}$ and $\hat{\mathbf{b}}$ are two different information symbol vectors. The rotation matrix $\mathbf{M}$ is chosen to maximize the associated minimum product distance $d_p(\mathbf{s}, \hat{\mathbf{s}})$. One can easily verify that DAST codes achieve full diversity, and their coding gains are proportional to the minimum product distance associated with the rotations used.

For L layers, where $L = n_R$ for the system in this paper, we can write the TAST codeword matrix as

$$\mathbf{X} = \sum_{l=1}^{n_R} \left(\phi_l \mathbf{e}^{l-1}\right) \mathrm{diag}\left(\mathbf{M}\mathbf{b}_l\right), \tag{5}$$

where

$$\mathbf{e} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & & 0 \\ 0 & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{6}$$

3.2. Perfect STBCs

Perfect codes [10, 11] are full rate and full diversity $n_T n_R$. Furthermore they posses the non-vanishing determinant property that guarantees that they achieve the DMG tradeoff. The QAM information symbols are linearly encoded by such STBCs into an $n_T \times n_T$ codeword matrix $\mathbf{X} = \{x_{i,l}\} \in \mathbb{C}, i, l = 1, \ldots, n_T$. For the special cases of $n_T = 3, 4, 6$, perfect STBCs were proposed in [10, 11]. The perfect STBCs are constructed based on cyclic division algebras, where the codeword with L layers is given by [10, 11],

$$\mathbf{X} = \sqrt{\frac{n_T}{L}} \sum_{l=1}^{L} \mathbf{e}^{l-1} \mathrm{diag}\left(\mathbf{M}\mathbf{b}_l\right), \tag{7}$$

where

$$\mathbf{e} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & & 0 \\ 0 & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & & 1 \\ \gamma & 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{8}$$

and γ is chosen from $\mathbb{Z}[j]$ in order to achieve the full diversity and non-vanishing determinant [10]. The factor $\sqrt{\frac{n_T}{L}}$ is a power normalization and assures transmit power the same total power is transmitted when not all layers are encoded. Comparing to TAST codes, we have a different $\mathbf{e}$ matrix and $\phi = 1, \gamma = j$.

In this paper, we only consider subcodes of the perfect STBCs with a reduced number of layers, i.e., $L = n_R = 2$ and the transmission matrix is

$$\mathbf{X} = \sqrt{\frac{n_T}{2}} \sum_{l=1}^{2} \mathbf{e}^{l-1} \mathrm{diag}\left(\mathbf{M}\mathbf{b}_l\right).$$

Note that with the Perfect code given above only two transmit antennas (out of four) are used at any time instant.

The unitary generator matrix $\mathbf{M}$ for 4×4 Perfect STBC is given in [11].

3.3. Quasi-orthogonal codes

Quasi-orthogonal or non-orthogonal codes use a Clifford basis when constructing the linear dispersion code. The basis matrices are explicitly given for several cases in [17] and therefore they are not repeated here. In this way, they induce quasi-orthogonal layers where only some symbols interfere with each other while others remain orthogonal.

In a variant of Double ABBA [17], coined in what follows as DjABBA, with $\mathbf{X}_A, \mathbf{X}_B, \mathbf{X}_C$ and $\mathbf{X}_D$ STTD blocks encoding the symbol pairs (x_1, x_2), (x_3, x_4), (x_5, x_6), (x_7, x_8) [18]

$$\mathbf{X}^T = \begin{bmatrix} \cos\rho\,\mathbf{X}_A + \sin\rho\,\mathbf{X}_C & \cos\rho\,\mathbf{X}_B + \sin\rho\,\mathbf{X}_D \\ \jmath\,(\sin\rho\,\mathbf{X}_B - \cos\rho\,\mathbf{X}_D) & \sin\rho\,\mathbf{X}_A - \cos\rho\,\mathbf{X}_C \end{bmatrix} \tag{9}$$

Thus, the matrix transmits eight symbols using a modulation matrix of size 4×4, which is identical to that of TAST or Perfect codes with $L = 2$, given above. Due to STTD structure, by puncturing antennas 2 and 4 from $\mathbf{X}$, the result is

$$\begin{bmatrix} \cos\rho\,x_1 + \sin\rho\,x_5 & \cos\rho\,x_3 + \sin\rho\,x_7 \\ \cos\rho\,x_2 + \sin\rho\,x_6 & \cos\rho\,x_4 + \sin\rho\,x_8 \\ \jmath\,(\sin\rho\,x_3 - \cos\rho\,x_7) & \sin\rho\,x_1 - \cos\rho\,x_5 \\ \jmath\,(\sin\rho\,x_4 - \cos\rho\,x_8) & \sin\rho\,x_2 - \cos\rho\,x_6 \end{bmatrix}, \tag{10}$$

which is a redundant but equivalent representation of a particular 2×2 Golden code, provided that ρ is appropriately selected. However, in the presence of four transmit antennas, the optimal precoder given in [18] differs slightly from that of the Golden code. Nevertheless, though the layering structure is different, we see that TAST, Perfect and Quasi-orthogonal codes are linked to each other. DABBA, in contrast to TAST, is shown in [17] to reach second order capacity of the 4×2 MIMO channel.

4. PERFORMANCE

4.1. Uncoded

We evaluate the performance of selected designs using 4 transmit and 2 receive antennas. The comparison in what follows assuming QPSK modulation, with 4 bps/Hz spectral efficiency. In particular, we compare DjABBA (with $\rho = \pi/4$) to a two layer Perfect code. Fig. 1 shows the result with Sphere and LMMSE detection in an iid Rayleigh fading channel. It is seen that DjABBA improves in perfect code by about 0.5 dB at high SNR. Although TAST result are not depicted, the simulations have shown that the Perfect code improves on TAST by about 0.5 dB at high SNR.

4.2. Coded

The performance evaluation for coded systems is carried out in an IEEE 802.16-2004 [20] compliant WiMAX simulator. For the simulations we use the OFDM physical layer with an FFT size of 256. The standard-conform coding consists of a concatenation of an outer Reed-Solomon code and an inner convolutional code. A two stage interleaver after the encoder avoids error bursts caused by subcarriers with low SNR. The relevant WiMAX system parameters are summarized in Table 1.

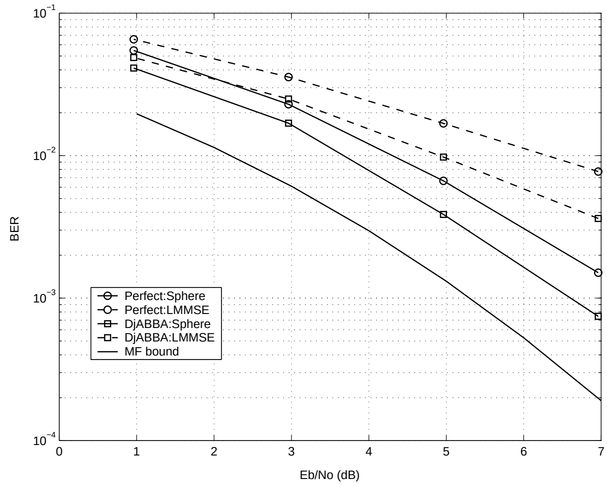

Fig. 1. Uncoded BER of a two layer Perfect code and DjABBA in a 4 tx - 2 rx configuration and iid Rayleigh channel.

Number of OFDM carriers	256
Modulation	16-QAM (Alamouti) 4-QAM (all other schemes)
Bandwidth	10 MHz
Cyclic prefix	1/4 = 5.56 us
Channel estimation	perfect
Code block size	35328 bits
Overall code rate	1/2

Table 1. WiMAX system simulation parameters.

We evaluate the performance for spatially uncorrelated flat and frequency selective block fading channels. The frequency selective channel is generated according to the ITU Pedestrian B power delay profile.

The receivers for the different space-time codes are maximum likelihood receivers with hard demapping. For the 4×2 systems the ML receiver is implemented as sphere decoder.

2×2 systems

The 2 tx and 2 rx antenna system is compared for Alamouti coded and Golden coded transmit signals. For Alamouti coding we employ 16-QAM modulation, and for Golden code 4-QAM to allow for a fair comparison. For both space-time codes the channel coding is exactly the same. The results in Fig. 2 show that the Golden Code enjoys approx. 0.8 dB gain over Alamouti in a Pedestrian B environment when combined

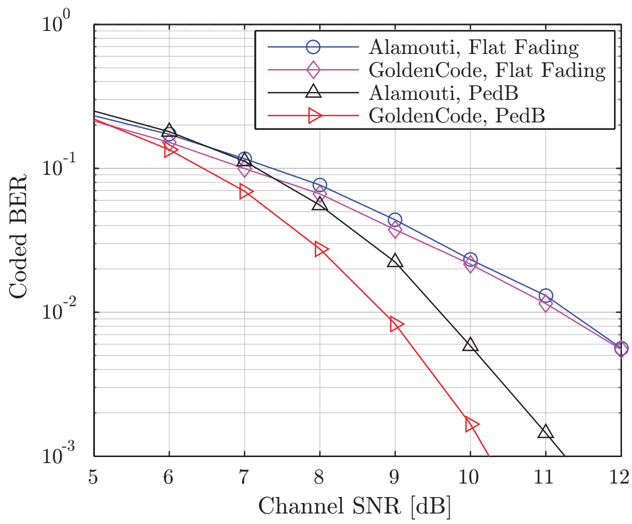

Fig. 2. Comparison of Golden Code and Alamouti for a coded 2x2 WiMAX transmission.

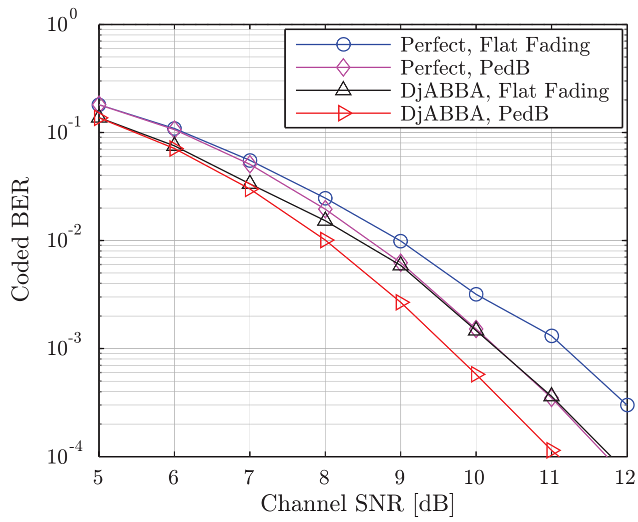

Fig. 3. Comparison of DjABBA and Perfect code for a coded 4x2 WiMAX transmission.

with the WiMAX conformant concatenated Reed-Solomon-Convolutional code. In a flat fading environment however, the gain is only about 0.3 dB.

4×2 systems

A comparison of DjABBA (with optimal $\rho = 0.8881$ taken from [18]) and the Perfect code with two encoded layers is shown in Fig. 2. Here, we averaged over 11000 channel realizations for the flat fading case and 6000 channel realizations for the Pedestrian B channel model. Channel coding and 4-QAM modulation is the same as for the 2×2 system employing the Golden space-time coding to allow for comparisons between the 4×2 and 2×2 systems. In both scenarios, flat fading and Pedestrian B, the DjABBA outperforms the Perfect code by 0.6 dB.

The weaker performance of the two-layer Perfect code is due to the reduced number of diversity branches available in one channel use. On the contrary, the DjABBA uses all independent branches between the 4 tx and the 2 rx antennas in each channel use.

5. CONCLUSION

In this paper we have reviewed and evaluated two different high rate space-time coding concepts, an algebraic (Perfect) code and a quasi-orthogonal code (DjABBA). The paper compares the two code constructions by stating their design principles and by extensive simulations using a WiMAX compatible simulation chain. The comparisons are done for a medium rate service, where the modulation symbol alphabet is QPSK and the coding rate is 1/2. In this setup, DjABBA outperforms Perfect code by a fraction of a decibel. On the other hand, the Perfect code may have some implementation advantages in that at each time instant only two transmit antennas (out of four) are used.

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