ON ENERGY COST OF BIT AND BIT/S IN MULTI-ANTENNA WIRELESS NETWORKS UNDER HARDWARE CONSTRAINTS

Marcin Wiczanowski, Angela Feistel

* Heinrich-Hertz Chair, EECS,
University of Technology Berlin,
Einsteinufer 25, 10587 Berlin, Germany
Tel/Fax: +49(0)30-314-28462/-28320
Email: Marcin.Wiczanowski@TU-Berlin.de

*Slawomir Stanczak, Holger Boche**

Fraunhofer German-Sino Lab
for Mobile Communications
Einsteinufer 37,
10587 Berlin, Germany,
Email: {feistel,stanczak,boche}@hhi.fhg.de

ABSTRACT

The paper addresses the problem of minimizing the overall energy consumption per frame in wireless networks with multiple antennas at all transmitters and receivers. In doing so, we take into account the energy expenditure for signal processing using a realistic hardware model. We provide a characterization of different regimes of transmit covariance matrices, in which the total energy exhibits different behaviors. This allows us to obtain valuable insight into the design of energy-optimal policies and their dependence on some hardware parameters. Furthermore, we introduce and interpret the metric of energy per bit/s and characterize its behavior under the energy-optimal policy.

1. INTRODUCTION AND PROBLEM STATEMENT

In recent years, the quality-of-service (QoS) in wireless communications networks has significantly improved due to the development of multi-antenna transmission techniques. The multiple-input multiple-output (MIMO) communication has been extensively studied and is relatively well understood in the context of cellular networks (see [1], [2], [3], [4] and references therein) and ad-hoc networks (see [5] and references therein). Together with the saturation of the research on MIMO communication, we are now in the phase of incorporating MIMO techniques into the existing standards and the corresponding adaption of existing hardware.

However, due to the increase of complexity and energy consumption, the benefits of multiple antenna transmission are not obvious when transmitters and receivers are implemented using low-cost hardware and powered by (energy-constrained) batteries. In such cases, it is desired to combine the energy efficient operation with sufficient service quality, measured e.g. by the data-rate or delay, perceived by each user. This leads us to the problem of minimizing total energy consumption in a network subject to link-specific constraints on quality-of-service, which is of the main interest in this paper.

A number of recent results on energy consumption deals with the minimum energy cost for transmission of one bit. In particular, the regime of low-energy per bit for a separate MIMO link is studied in [6]. In [7], the authors extend some of these results to a multi-user case.

Some insights into the energy-efficiency of multi-user multi-antenna communication are provided in [8]. Clearly, the problem of minimizing energy consumption is strongly related (but, in general, not equivalent) to the problem of minimizing transmit powers. The latter problem is well understood and is addressed frequently for different types of networks and physical layers. Usually, the problem is to minimize the total transmit power in a network under the condition that some signal-to-interference ratios are achieved on the links. This problem, is addressed in [4] under the assumption of a single antenna element on one side of the links (see also [9], [10]).

In this work, our objective is to minimize the total energy (also called sum-energy) consumption per frame in a multi-antenna network when a fixed backlog per frame is transmitted on each link. The minimization is subject to constraints on the data-rate on each link under the assumption of the maximum-likelihood receiver. In our analysis, we include the real-world constraints on transceiver hardware, such as the dissipation power of integrated transceiver circuits and the power consumption of microcontroller units. We study the solvability of the sum-energy minimization problem in the sense of the existence of local and global minimizers. We characterize two regimes of transmit covariance matrices, the power-efficiency and power-inefficiency regime, in which the sum-energy function exhibits different behavior. The impact of hardware parameters on the regimes of the sum-energy is analyzed. The understanding of the different regimes and the influence of hardware provides insight into the design of energy-optimal policies. In particular, the impact of the con-

This work was partly supported by the German Ministry for Education and Research (BMBF) under Grant 01BU0680

straints on the properties of the energy-optimal policy is addressed. Finally, we introduce the notion of link energy consumption per bit/s as a metric of energy-efficiency of a bit-transfer per unit time. We characterize the behavior of the energy consumption per bit/s under the energy-optimal policy.

Model and preliminaries are provided in Section 2. The solvability of the sum-energy optimization is addressed in Section 3. In Sections 4 ad 5, we analyze the power-efficiency and power-inefficiency regime of the sum-energy, respectively, and study the consequences for the energy-optimal policy and the influence of hardware. In Section 6 we study the link energy consumption per bit/s when the sum-energy is minimized.

2. PRELIMINARIES

2.1. Network model

We consider a single-hop wireless network with the set of $K \in \mathbb{N}_+$ node-to-node links $\mathcal{K} = \{1, \ldots, K\}$. This includes the particular cases of multiple access channel and broadcast channel, if all links have a common receiver or transmitter, respectively. Each node is equipped with an array of $N \in \mathbb{N}_+$ antennas. The node-to-node multiple-input multiple-output (MIMO) channels are assumed to be time-invariant and frequency flat. Let $\boldsymbol{H}_{ij} \in \mathbb{C}^{N \times N}$ denote the channel from transmitter of link $j \in \mathcal{K}$ to the receiver of link $i \in \mathcal{K}$. We assume the noise on link $i \in \mathcal{K}$ to be white Gaussian and spatially uncorrelated with variance n_i. The transmit covariance matrices $\boldsymbol{Q}_i \in \mathbb{S}_+^N$ of links $i \in \mathcal{K}$ are grouped in the matrix $\boldsymbol{Q} = (\boldsymbol{Q}_1', \ldots, \boldsymbol{Q}_K')' \in (\mathbb{S}_+^N)^K$. We extend the notation of the semi-order of positive semidefinite matrices by writing $\boldsymbol{Q}' \succeq \boldsymbol{Q}''$ if and only if $\boldsymbol{Q}' - \boldsymbol{Q}'' \in (\mathbb{S}_+^N)^K$.

Let $\sigma(k)$ denote the set of links which provide interference at the receiver of link $k \in \mathcal{K}$. The sets $\sigma(k)$, $k \in \mathcal{K}$, are determined by the kind of multi-user processing, that is, precoding and postprocessing. In general we have

$$\sigma(k) \subseteq \{j \in \mathcal{K} : j \neq k\}, \quad k \in \mathcal{K}. \tag{1}$$

For instance, if no two links in the network have a common transmitter or receiver, then it is likely that multi-user processing reduces to per-link single-user processing, so that equality in (1) is possible [5]. Under postprocessing by Successive Interference Cancellation (SIC) in the multiple access channel we have

$$\sigma(k) = \{j \in \mathcal{K} : \pi(j) < \pi(k)\}, \quad k \in \mathcal{K}, \tag{2}$$

where the permutation $k \mapsto \pi(k)$, $k \in \mathcal{K}$, denotes the SIC order in the sense that link $\pi(j)$ is decoded after link $\pi(k)$ if $\pi(j) < \pi(k)$, $j, k \in \mathcal{K}$, [11], [12]. Under superposition precoding with perfect Channel Side Information (CSI) in the broadcast channel (2) is satisfied as well, with π as the precoding order in the sense that link $\pi(j)$ is precoded with perfect knowledge of (the information on) link $\pi(k)$ if $\pi(j) < \pi(k)$, $j, k \in \mathcal{K}$ (see [1], [4] and references therein). It is known that multi-user processing by SIC and superposition precoding with CSI is optimal in terms of achievable rates in the Gaussian multiple access channel and broadcast channel, respectively [1], [3].

Each link receiver uses the Maximum Likelihood receiver algorithm. The achievable link rate (function) $\boldsymbol{Q} \mapsto R_i(\boldsymbol{Q})$, $\boldsymbol{Q} \succeq 0$, can be then expressed as [3]

$$R_i(\boldsymbol{Q}) = \log \frac{|\boldsymbol{I}n_i + \sum_{k \in \sigma(i) \cup i} \boldsymbol{H}_{ik} \boldsymbol{Q}_k \boldsymbol{H}_{ik}'|}{|\boldsymbol{I}n_i + \sum_{k \in \sigma(i)} \boldsymbol{H}_{ik} \boldsymbol{Q}_k \boldsymbol{H}_{ik}'|}, \quad i \in \mathcal{K},$$

where, for simplicity, unit bandwidth is assumed.

2.2. Hardware model and problem statement

The network operation time is partitioned into frames of duration T and each link is assumed to transfer a backlog of c bits per frame. Each transceiver hardware is assumed to consist of a single microcontroller unit and N integrated transceiver circuits, each with a logic part and high-frequency part including a single antenna. We denote by P_a the hardware-related power consumption on each link $k \in \mathcal{K}$, excluding the transmit power itself, during the backlog transmission on the link (active mode). Similarly, P_s denotes the hardware power consumption on each link $k \in \mathcal{K}$ when no transmission takes place on the link (passive mode). Precisely, the hardware power consumptions are functions of the antenna number $N \mapsto P_a(N)$, $N \mapsto P_s(N)$ of the form

$$\begin{aligned} P_a(N) &= P_a^c(N) + P_e(N) + P_a^m \\ P_s(N) &= P_s^c(N) + P_s^m, \end{aligned} \qquad N \in \mathbb{N}_+. \tag{3}$$

Thereby, $N \mapsto P_a^c(N)$ and $N \mapsto P_s^c(N)$, $N \in \mathbb{N}_+$, denote the power consumptions of integrated circuits on link transmitter and link receiver in the active and passive mode, respectively, and include dissipation power and direct current supply power. Further, P_a^m and P_s^m denote the (antenna number invariant) powers consumed by the microcontrollers on the transmitter and the receiver in the active and passive mode, respectively, and $N \mapsto P_e(N)$, $N \in \mathbb{N}_+$, is the power needed for channel estimation and signaling.

In our focus is the energy-optimal policy, which minimizes the sum-energy consumption in the network per frame while some minimum rate γ_k is ensured for the backlog transmission on each link $k \in \mathcal{K}$. The minimum rates are adjusted to link-specific levels of expected service quality. When the antenna transmit power and the symbol rate is adaptable (in particular, is not fixed by standard), the energy-optimal policy is understood as a set of transmit covariance matrices which solves the problem

$$\min_{\boldsymbol{Q} \in \mathcal{Q}_\gamma} \sum_{k \in \mathcal{K}} \Big(\frac{c}{R_k(\boldsymbol{Q})} (\mathrm{tr}(\boldsymbol{Q}_k) + P_a) + (T - \frac{c}{R_k(\boldsymbol{Q})}) P_s \Big) \tag{4}$$

with $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_K)$, with

$$\mathcal{Q}_{\boldsymbol{\gamma}} = \{-\boldsymbol{Q} \preceq 0 : \gamma_k - R_k(\boldsymbol{Q}) \leq 0, \quad k \in \mathcal{K}\},$$

and with $P_a = P_a(N)$, $P_s = P_s(N)$, for some fixed $N \in \mathbb{N}_+$.

Besides the energy-optimal policy itself, our interest is also in the characterization of energy consumption per bit and energy consumption per bit/s of each link. By (4), the link energy (consumption) per frame is a function $\boldsymbol{Q} \mapsto e_k(\boldsymbol{Q})$, $\boldsymbol{Q} \succeq 0$, which can be written as

$$e_k(\boldsymbol{Q}) = \frac{c}{R_k(\boldsymbol{Q})}(\text{tr}(\boldsymbol{Q}_k) + P_a - P_s) + TP_s, \quad k \in \mathcal{K}. \quad (5)$$

The link energy per bit $\boldsymbol{Q} \mapsto e_k^b(\boldsymbol{Q})$, $\boldsymbol{Q} \succeq 0$, follows then simply as $e_k^b(\boldsymbol{Q}) = e_k(\boldsymbol{Q})/c$, $k \in \mathcal{K}$, and is an established metric of energy efficiency of the transfer of a single backlog unit (bit) [6], [7]. The link energy per bit/s $\boldsymbol{Q} \mapsto e_k^{bs}(\boldsymbol{Q})$, $\boldsymbol{Q} \succeq 0$, follows further as

$$e_k^{bs}(\boldsymbol{Q}) = \frac{e_k(\boldsymbol{Q})}{cR_k(\boldsymbol{Q})}, \quad k \in \mathcal{K},$$

and can be seen as a metric of energy efficiency of the transfer of a single bit per unit time (equivalently, at certain rate). Thus, link energy per bit/s suits good for the consideration with constraints on the link rate in (4).

3. SOLVABILITY OF THE ENERGY OPTIMIZATION

In this section we address the basic issue of the energy-optimal policy - the computational tractability of problem (4).

We say that an optimization problem is locally solvable, if it has some local optimizers. Local solvability represents the basic requirement on an optimization problem and ensures that at least a local optimum is attained. More restrictively, an optimization problem is said to be globally solvable, if any its local optimizer is a global optimizer as well (in particular, the global optimizer might be unique). Global solvability property is highly desired in terms of algorithmic tractability of the problem. A global optimizer of a globally solvable optimization problem can be found by means of locally convergent iterations, such as the descent iteration, the gradient iteration or the Newton iteration [13].

Regarding the solvability of problem (4), we can prove the following.

Lemma 1 *Problem (4) is locally solvable and there exist values of K, $\boldsymbol{H}_{ij}$, $\sigma(i)$, n_i, γ_i, with $i, j \in \mathcal{K}$, for which problem (4) is not globally solvable.*

The lack of general global solvability is a negative result in terms of algorithmic computation of the energy-optimal policy. Lemma 1 implies that locally convergent iterations fail at finding the globally energy-optimal policy under some values of link number, channels, noise variances and rate requirements. This means that one either resorts to finding any locally energy-optimal policy by locally convergent iterations or one utilizes more intricate globally convergent optimization methods.

4. THE REGIME OF POWER-EFFICIENCY

According to Lemma 1, the considered optimization of sum-energy is merely a locally solvable problem, so that, in general, multiple locally energy-optimal policies exist. In the current and the next section we deepen the insight into the energy optimization problem (4). We provide sufficient characterizations of regimes of different behavior of the sum-energy. This allows for conclusions on the energy-optimal policy in terms of rate requirements and transceiver hardware.

Consider first the following property of the sum-energy.

Lemma 2 *Let a set of transmit covariance matrices*

$$\mathcal{Q}_e = \{\boldsymbol{Q} \succeq 0 : (e_k^b(\boldsymbol{Q}) - \tfrac{T}{c}P_s)\nabla_{\boldsymbol{Q}_k} R_k(\boldsymbol{Q}) \preceq \boldsymbol{I}, \quad k \in \mathcal{K}\}$$

be defined. Then, we have

$$\nabla \sum_{k \in \mathcal{K}} e_k(\boldsymbol{Q}) \succeq 0, \quad \boldsymbol{Q} \in \mathcal{Q}_e.$$

The Lemma say that the sum-energy is an operator-monotone function on the set $\mathcal{Q}_e$ [14]. This implies that whenever the transmit covariance matrices remain in the set $\mathcal{Q}_e$, no decrement of the sum-energy is achieved when the transmit covariance matrices increase jointly according to the semi-order of positive semidefinite matrices. Analogously, the sum-energy is nonincreasing under joint decrement of the transmit covariance matrices according to the semi-order of positive semidefinite matrices. In particular, an increment and decrement in such semi-order is implied by an increment and decrement of the eigenvalues of transmit covariance matrices, respectively. The spectrum of a transmit covariance matrix $\boldsymbol{Q}_k$ is known to represent the spatial allocation of power among the transmit directions (beams) of link $k \in \mathcal{K}$ [2]. Thus, Lemma 2 implies in particular that when no spatial power allocation of a link is decreased, the sum-energy can not be decreased as well whenever the transmit covariance matrices remain in the set $\mathcal{Q}_e$. Equivalently, given transmit covariance matrices from set $\mathcal{Q}_e$, the sum-energy does not grow when the spatial power allocations of all links are jointly decreased.

Summarizing, Lemma 2 implies that in the regime $\mathcal{Q}_e$ of transmit covariance matrices a growing power-inefficiency causes suboptimality in terms of the sum-energy. Thus, in some sense, the set $\mathcal{Q}_e$ can be interpreted as the regime of power-efficiency of the sum-energy.

The following property improves the understanding of the power-efficiency regime.

Lemma 3 *Given $P_a - P_s \geq 0$, $\mathcal{Q}_e$ is nonempty and there exists some $\boldsymbol{Q}' \succeq 0$, such that*

$$e_k^b(\boldsymbol{Q})\nabla_{\boldsymbol{Q}_k} R_k(\boldsymbol{Q}) \preceq e_k^b(\boldsymbol{Q}'')\nabla_{\boldsymbol{Q}_k''} R_k(\boldsymbol{Q}''),\ \ \boldsymbol{Q} \succeq \boldsymbol{Q}'' \succeq \boldsymbol{Q}'.$$

By Lemma 3 follows that the set $\mathcal{Q}_e$ is nonempty, whenever the hardware power consumption in the active mode is no smaller than the one in the passive mode. This implies that the power-efficiency regime of the sum-energy exists under real-world hardware, when on each link no additional power consuming tasks are conducted during the passive mode.

Further, Lemma 3 implies that when a set of transmit covariance matrices from the power-efficiency regime $\mathcal{Q}_e$ is increased according to the semi-order of positive semidefinite matrices, then a set of matrices from the regime $\mathcal{Q}_e$ is obtained as well. In particular, the transmit covariance matrices remain within the power-efficiency regime when no spatial power allocation of a link is decreased.

Summarizing, one can say that the power-efficiency regime of the sum-energy is the regime of sufficiently large transmit powers. Largeness is thereby measured by the semi-order of positive semidefinite matrices.

4.1. Consequences for the energy-optimal policy

The properties of the power-efficiency regime allow for some conclusions on the energy-optimal policy.

Lemma 4 *Let $\boldsymbol{Q}' = \arg\min_{\boldsymbol{Q}\in\mathcal{Q}_{\boldsymbol{\gamma}}} \sum_{k\in\mathcal{K}} e_k(\boldsymbol{Q})$ and $P_a - P_s \geq 0$. If $\boldsymbol{\gamma}$ is such that $\mathcal{Q}_{\boldsymbol{\gamma}} \subseteq \mathcal{Q}_e$ or, equivalently, if*

$$\frac{\gamma_k}{R_k(\boldsymbol{Q})} \leq 1 \Rightarrow (e_k^b(\boldsymbol{Q}) - \tfrac{T}{c}P_s)\nabla_{\boldsymbol{Q}_k} R_k(\boldsymbol{Q}) \preceq \boldsymbol{I}, \quad k \in \mathcal{K}, \tag{6}$$

$\boldsymbol{Q} \succeq 0$, then we have

$$\boldsymbol{Q}' = \arg\min_{\boldsymbol{Q}\in\mathcal{Q}_{\boldsymbol{\gamma}}} \sum_{k\in\mathcal{K}} tr(\boldsymbol{Q}_k) \quad \textit{and} \quad \gamma_k = R_k(\boldsymbol{Q}'), \quad k \in \mathcal{K}.$$

Lemma 4 characterizes the energy-optimal policy when $\mathcal{Q}_{\boldsymbol{\gamma}}$ is included in the power-efficiency regime of the sum-energy, which case is characterized by condition (6). For the case of multi-user processing satisfying (2), consider first the property

$$R_k(\boldsymbol{Q}) \succeq R_k(\boldsymbol{Q}'), \quad \boldsymbol{Q} \succeq \boldsymbol{Q}' \succeq 0, \tag{7}$$

which follows from the theory of operator-monotone functions [15], [14], and let the rate requirements jointly increase. Then, it follows by (7) that the hypothesis in (6) is satisfied only when the transmit covariance matrices are also correspondingly increased in the semi-order of positive semidefinite matrices. But by Lemma 3 we have that under sufficient increment of the transmit covariance matrices, the implication in (6) is satisfied as well. Thus, a simple conclusion is that $\mathcal{Q}_{\boldsymbol{\gamma}}$ is within the power-efficiency regime of the sum-energy, when the rate requirements are sufficiently large and the multi-user processing satisfies (2). This includes the case of multiple-access channel with SIC and broadcast channel with superposition precoding with CSI.

Lemma 4 implies that, given $\mathcal{Q}_{\boldsymbol{\gamma}}$ within the power-efficiency regime, the energy-optimal policy is optimal in terms of sum-power minimization as well. In other words, the task of sum-energy minimization can be replaced by the task of sum-power minimization, when the set $\mathcal{Q}_{\boldsymbol{\gamma}}$ is a subset of $\mathcal{Q}_e$. Obviously, in such case the obtained link rates coincide with the corresponding rate requirements. By (7), we have for the case (2) that the energy-optimal policy and the power-optimal policy are equivalent when the link rate requirements are sufficiently large (in the semi-order of positive semidefinite matrices). Thus, when sufficiently large minimum rates per link are required, the sum-energy minimization is equivalent to sum-power minimization in the multiple-access channel with SIC and in the broadcast channel with superposition precoding with CSI.

4.2. Influence of hardware

It is intuitive that the hardware has influence on the properties of the power-efficiency regime $\mathcal{Q}_e$. Precisely, let P_s be fixed and the link energy per frame (5) be additionally a function of the hardware power consumption in the sense $(\boldsymbol{Q}, P_a) \mapsto e_k(\boldsymbol{Q}, P_a)$, $\boldsymbol{Q} \succeq 0, P_a \geq 0$. It is obvious that the link energy per bit is a linear function of the hardware power consumption with slope

$$\tfrac{\partial}{\partial P_a} e_k^b(\boldsymbol{Q}, P_a, P_s) = \tfrac{1}{R_k(\boldsymbol{Q})}, \quad k \in \mathcal{K}, \boldsymbol{Q} \succeq 0. \tag{8}$$

Thus, with Lemmas 2 and 3 we recognize that an increase of the hardware power consumption in the active mode shifts the power-efficiency regime $\mathcal{Q}_e$ linearly towards larger transmit covariance matrices (in the corresponding semi-order). With (3) it follows in particular that the power-efficiency regime is shifted towards larger transmit covariance matrices when the power consumption of the channel estimation algorithm grows.

The hardware power consumption influences also the energy-optimal policy. Precisely, consider multi-user processing satisfying (2), as in the multiple-access channel with SIC and in the broadcast channel with superposition precoding with CSI. By Lemma 4 and (7) follows that under increase of the hardware power consumption in the active mode, the link rate requirements for which the set $\mathcal{Q}_{\boldsymbol{\gamma}}$ is included in the power-efficiency regime $\mathcal{Q}_e$ are shifted towards larger values as well. Thus, under an increase of P_a in the multiple-access channel with SIC (and broadcast channel with superposition precoding with CSI), the rate requirements for which the energy-optimal policy and the power-optimal policy become equivalent increase as well.

By (8) can be however seen, that the slope of the shift of the power-efficiency regime due to an increment of P_a diminishes with the shift itself. In other words, the influence of the hardware power consumption in the active mode on the properties of the sum-energy is asymptotically vanishing.

5. THE REGIME OF POWER-INEFFICIENCY

While the existence of the power-efficiency regime of the sum-energy is intuitive, the existence of an opposite regime is no more obvious.

The corresponding property is the following.

Lemma 5 *Let a set of transmit covariance matrices*

$$\mathcal{Q}_i = \{\boldsymbol{Q} \succeq 0 : R_k(\boldsymbol{Q}) \times \sum_{j\in\sigma(k)\cup k} (e_j^b(\boldsymbol{Q}) - \tfrac{T}{c}P_s)\nabla_{\boldsymbol{Q}_k} \log R_j(\boldsymbol{Q}) \succeq \boldsymbol{I}, k \in \mathcal{K}\}$$

be defined. Then, we have

$$\nabla \sum_{k\in\mathcal{K}} e_k(\boldsymbol{Q}) \preceq 0, \quad \boldsymbol{Q} \in \mathcal{Q}_i.$$

By Lemma 5, the sum-energy is an operator-convex function on the set $\mathcal{Q}_i$ [14]. This implies that whenever the transmit covariance matrices remain within the set $\mathcal{Q}_i$, then the sum-energy is nonincreasing when the transmit covariance matrices increase according to the semi-order of positive semi-definite matrices. Obviously, in such case the sum-energy can not be decreased under a joint decrement of the transmit covariance matrices according to the same semi-order.

We conclude from Lemma 5 that the regime $\mathcal{Q}_i$ of transmit covariance matrices has properties opposite to the regime $\mathcal{Q}_e$. An improvement of the power-efficiency, in the sense of a decrement of transmit covariance matrices in the corresponding semi-order, provides no decrement in the sum-energy on $\mathcal{Q}_i$. Equivalently, the sum-energy is nonincreasing when the transmit covariance matrices are increased in the corresponding semi-order, that is, when the inefficiency in power use grows. Consequently, in analogy to the power-efficiency regime $\mathcal{Q}_e$, the set $\mathcal{Q}_i$ can be regarded as the power-inefficiency regime of the sum-energy.

From Lemma 3 can already be recognized that the power-inefficiency regime is not within the regime of large transmit powers. The structure of the power-inefficiency regime is more intricate than the one of power efficiency regime. In particular, no monotony property analogous to the one from Lemma 3 can be proven for the power-inefficiency regime. Nevertheless, we show that the regime $\mathcal{Q}_i$ is nonempty by the occasion of studying the influence of hardware in the remainder (Lemma 8).

5.1. Consequences for the energy-optimal policy

The following conclusion on the energy-optimal policy results from the properties of the power-inefficiency regime.

Lemma 6 *Let $\boldsymbol{\gamma}' = \gamma'\boldsymbol{1}$, with $\gamma' > 0$, and let $\boldsymbol{Q}' = \boldsymbol{Q}'(\gamma') \in \mathcal{Q}_{\boldsymbol{\gamma}'}$ be such that $R_k(\boldsymbol{Q}') = \gamma'_k$, $k \in \mathcal{K}$. If $\boldsymbol{Q}' \in \mathcal{Q}_i$ or, equivalently, if*

$$\gamma'_k \sum_{j\in\sigma(k)\cup k} (e_j^b(\boldsymbol{Q}') - \tfrac{T}{c}P_s)\nabla_{\boldsymbol{Q}'_k} \log R_j(\boldsymbol{Q}') \succeq \boldsymbol{I}, \quad k \in \mathcal{K},$$

then $\boldsymbol{Q}'' = \arg\min_{\boldsymbol{Q}\in\mathcal{Q}_{\boldsymbol{\gamma}'}} \sum_{k\in\mathcal{K}} e_k(\boldsymbol{Q})$ is such that

$$\sum_{k\in\mathcal{K}} tr(\boldsymbol{Q}''_k) > \min_{\boldsymbol{Q}\in\mathcal{Q}_{\boldsymbol{\gamma}'}} \sum_{k\in\mathcal{K}} tr(\boldsymbol{Q}_k)$$

and $\gamma'_k < R_k(\boldsymbol{Q}'')$, for some $k \in \mathcal{K}$.

Lemma 6 characterizes the energy-optimal policy in the case when the rate requirements are symmetric and ensure the inclusion of some extremal set of transmit covariance matrices from $\mathcal{Q}_{\boldsymbol{\gamma}}$ in the power-inefficiency regime. The considered extremal set of matrices is precisely the one which obtains minimum link rates within $\mathcal{Q}_{\boldsymbol{\gamma}}$. For such case Lemma 6 implies that the sum-energy is minimized by a policy which achieves suboptimal power consumption. Consequently, the link rates obtained by the energy-optimal policy exceed for some links the corresponding rate requirements.

Recall that, by Lemma 4, the inclusion of the set $\mathcal{Q}_{\boldsymbol{\gamma}}$ in the power-efficiency regime implies the equivalence between the power- and energy-optimal policies. In contrast to this, Lemma 6 implies that if the set $\mathcal{Q}_{\boldsymbol{\gamma}}$ intersects the power-inefficiency regime in a certain way, then the energy-optimal policy and the power-optimal policy are different policies.

The rate requirements for which, by Lemma 6, the energy-optimal policy does not minimize the sum-power are characterized by the following result.

Lemma 7 *Let $\boldsymbol{\gamma} = \gamma\boldsymbol{1}$, with $\gamma > 0$, and let $\boldsymbol{Q} = \boldsymbol{Q}(\gamma) \in \mathcal{Q}_{\boldsymbol{\gamma}}$ be such that $R_k(\boldsymbol{Q}) = \gamma_k$, $k \in \mathcal{K}$. Then, there exists some $\gamma' > 0$, such that $\boldsymbol{Q} \in \mathcal{Q}_i$ for $\gamma < \gamma'$.*

Thus, the set of transmit covariance matrices which obtains minimum link rates in $\mathcal{Q}_{\boldsymbol{\gamma}}$ is also included in the power-inefficiency regime, if the rate requirements are symmetric and sufficiently small. This implies with Lemma 6 that the energy-optimal policy is power-suboptimal under small and symmetric link rate requirements. In other words, energy minimization and power minimization are different goals when small and symmetric minimum rates per link are required. This stands in contrast to the case of equivalence of energy optimality and power optimality. The latter property was shown by Lemma 4 to hold under sufficiently large (but not necessarily symmetric) link rate requirements. Summarizing Lemmas 4 and 6, one can conclude that the equivalence of the energy-optimal policy and power-optimal policy disappears under decrement and symmetrization of the link rate requirements.

5.2. Influence of hardware

The hardware power consumption has an influence on the properties of the power-inefficiency regime. Such influence is characterized by the following lemma.

Lemma 8 *Given (2), let P_s be fixed. Then, there exists some $P'_a = P'_a(P_s) > 0$, such that $\mathcal{Q}_i$ is nonempty for $P_a > P'_a$. Moreover, for any $\boldsymbol{Q}$ with $R_k(\boldsymbol{Q}) = R_l(\boldsymbol{Q})$, $k, l \in \mathcal{K}$, there exists some $P''_a = P''_a(\boldsymbol{Q}, P_s) > 0$, such that $\boldsymbol{Q} \in \mathcal{Q}_i$ for $P_a > P''_a$.*

The result addresses the case of multi-user link processing satisfying (2). This includes the multiple access channel under postprocessing by SIC and the broadcast channel under superposition precoding with CSI. For such case, Lemma 8 implies that the power-inefficiency regime of the sum-energy exists, if the hardware-related power consumption P_a in the active mode is sufficiently larger than the power consumption P_s in the passive mode.

Further, it follows that any set of transmit covariance matrices obtaining symmetric rates falls into the power-inefficiency regime up from some value of the difference $P_a - P_s$. Together with Lemma 7 this implies that with increasing difference between the hardware power consumption in the active and in the passive mode, the power-inefficiency regime spreads towards sets of transmit covariance matrices yielding larger symmetric rates. Thus, according to Lemma 6, if the difference $P_a - P_s$ is increased, then the range of link rate requirements for which the energy-optimal policy is power-suboptimal increases towards larger symmetric requirements. Equivalently, the increment of the difference $P_a - P_s$ causes that the sum-energy minimization and sum-power minimization remain different goals up to a larger value of the rate requirement, the same for all links.

By (3), a large difference between power consumption in the active and in the passive mode can be implied by high power consumption of channel estimation and signaling. Thus, given a power-consuming channel estimation algorithm, when the symmetric rate requirements are not large enough then the energy-optimal policy is different from the power-optimal one.

6. LINK ENERGY PER BIT/S

In this section we are concerned with the behavior of the metric of link energy per bit/s under the energy-optimal policy.

The joint behavior of the metric e^b_k, $k \in \mathcal{K}$, under the energy-optimal policy can not be concluded directly from (4). The main result is the following.

Lemma 9 *Let $\boldsymbol{Q} \mapsto \boldsymbol{\Delta}(\boldsymbol{Q}) = (\boldsymbol{\Delta}_{kj}(\boldsymbol{Q}))$ be a block-matrix function with $\boldsymbol{\Delta}_{kj}(\boldsymbol{Q}) = \nabla_{\boldsymbol{Q}_k} R_j(\boldsymbol{Q})$, $k, j \in \mathcal{K}$, $\boldsymbol{Q} \succeq 0$. If $\boldsymbol{Q}' = \arg\min_{\boldsymbol{Q} \in \mathcal{Q}_{\boldsymbol{\gamma}}} \sum_{k \in \mathcal{K}} e_k(\boldsymbol{Q})$ is such that*

$$\nabla \sum_{k \in \mathcal{K}} e_k(\boldsymbol{Q}') = 0$$

and $\boldsymbol{\Delta}(\boldsymbol{Q}')$ is invertible, then

$$e^{bs}_k(\boldsymbol{Q}') = \tfrac{T}{c} P_s T_k(\boldsymbol{Q}') + \lambda_{max}\Big(\sum_{j \in \mathcal{K}} (\boldsymbol{\Delta}^{-1}(\boldsymbol{Q}'))_{kj} T_j(\boldsymbol{Q}')\Big),$$

$k \in \mathcal{K}$, with $(\cdot)_{kj}$ denoting the kj-th block and λ_{max} denoting the maximum eigenvalue.

Lemma 9 characterizes the metric of link energy per bit/s when the energy-optimal policy is a local minimizer of the sum-energy. By Lemma 4, is very unlikely to occur when set $\mathcal{Q}_{\boldsymbol{\gamma}}$ is included in the power-efficiency regime of the sum-energy. That is, when the link rate requirements are sufficiently large, so that there is equivalence between energy-optimal and power-optimal policy. According to Lemma 6 however, the energy-optimal policy can happen to be a local minimizer of the sum-energy when the set $\mathcal{Q}_{\boldsymbol{\gamma}}$ intersects the power-inefficiency regime in a certain way. By Lemma 7, this is the case when the link rate requirements are sufficiently small and symmetric. For such case, Lemma 9 says that the energy consumption per bit/s is, for any link, a jointly linear function of bit durations of all links, with coefficients depending on the transmit covariance matrices. This is interesting, since, under general policy, the energy consumption per bit/s of each link appears to be quadratically proportional to the corresponding bit duration by definition. Further, it can be seen that the linear dependence of link energy per bit/s on the bit duration on the same link becomes sharper (the slope increases) with an increase of hardware power consumption in the passive mode. In the limit, when the passive mode on link $k \in \mathcal{K}$ is highly power-consuming, e.g. due to some additional computational tasks, the dependence of e^b_k on bit durations on the other links $j \in \mathcal{K}$, $j \neq k$, becomes negligible in the relation. The same effect is obtained under large values of the ratio of frame duration and backlog, which corresponds to the case of sparse traffic.

Lemma 9 also mirrors the coupling of the link energy consumptions per bit/s by interference. For instance, let for a single link $k \in \mathcal{K}$ the corresponding rate requirement be infinitely increasing, so that the link bit duration $T_k(\boldsymbol{Q}')$ under energy-optimal policy tends to zero. Clearly, in the case of no interference this enforces a diminishing metric of link energy per bit/s as well. However, under interference, it follows by Lemma 9 that the corresponding metric e^{bs}_k is bounded above zero by some linear function of bit durations on the remaining links.

6.1. Special case of multi-user processing

For the case of particular multi-user processing we have an additional result.

Lemma 10 *Given (2), let the assumptions of Lemma 9 be satisfied. Then, we have*

$$e^{bs}_k(\boldsymbol{Q}') \leq \tfrac{T}{c} P_s T_k(\boldsymbol{Q}') + \lambda_{max}\Big(\sum_{j \in \mathcal{K}} (\boldsymbol{\Delta}^{-1}(\boldsymbol{Q}'))_{kj} T_j(\boldsymbol{Q}')\Big),$$

$k \in \mathcal{K}$, *which implies further*

$$e_k^{bs}(\boldsymbol{Q}') \leq \tfrac{T}{c} P_s \gamma_k^{-1} + \lambda_{max}(\sum_{j \in \mathcal{K}} (\boldsymbol{\Delta}^{-1}(\boldsymbol{Q}'))_{kj} \gamma_j^{-1}), \quad k \in \mathcal{K},$$

with $(\cdot)_{kj}$ *denoting the* kj*-th block and with* λ_{max} *as the maximum eigenvalue.*

Compared to Lemma 9, Lemma 10 is weaker and says that the link energy per bit/s of each link is a sublinear function of bit durations of all links (on the other side, note that the implication of Lemma 10 is not restricted to the energy-optimal policy being a local minimizer of the sum-energy). The result is applicable to multi-user processing satisfying (2), i.e. in particular to the multiple access channel with SIC and to the broadcast channel with superposition coding with CSI. We conclude from Lemmas 9 and 10, that the link energy per bit/s is in general a sublinear function of bit durations and becomes a linear one, with coefficients dependent on transmit covariance matrices, if the energy-optimal policy is a local minimizer of the sum-energy.

7. SUMMARY

In this work we investigated the problem of sum-energy minimization per frame in a multi-antenna network with real-world hardware. We showed that (at least) two different regimes of transmit covariance matrices can be specified, in which the sum-energy behaves similarly to the sum-power and contrary to the sum-power, respectively. This results in different properties of the energy-optimal policy under different values of link rate requirements and hardware parameters. Precisely, we showed that the energy-optimal policy is equivalent to the power-optimal policy, when the link rate requirements are sufficiently large. Moreover, the higher is the hardware power consumption during the transmission, the larger are the requirements for which such equivalence of energy and power optimality occurs. Further, we showed that if the minimum rates required for each link are sufficiently small and symmetric, then the energy-optimal policy becomes power-suboptimal. Thereby, the range of such suboptimality increases towards larger requirements when the difference between the hardware power consumption during the transmission and during the passive mode increases. Finally, we analyzed the behavior of link energy consumption per bit/s as a metric of energy-efficiency of bit transfer per time-unit. Under certain assumptions we showed a jointly linear behavior of the energy consumption per bit/s as a function of bit durations on all links. For the case of certain multi-user processing in the multiple-access and broadcast channel, a general sublinear behavior of energy consumption per bit/s was shown.

8. REFERENCES

[1] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz), "The capacity region of the gaussian multiple-input multiple-output broadcast channel," *IEEE Transactions on Information Theory*, vol. 52(9), pp. 3936–3964, Sep 2006.

[2] M. Bengtsson and B. Ottersten, *Handbook of Antennas in Wireless Communications*, chapter 18, Optimal and Suboptimal Transmit Beamforming, CRC press, New York, 2001.

[3] N. Jindal, S. Vishwanath, and A. Goldsmith, "On the duality of gaussian multiple-access and broadcast channels," *IEEE Transactions on Information Theory*, vol. 50(5), pp. 768–783, May 2004.

[4] H. Boche and M. Schubert, *Duality Theory for Uplink Downlink Multiuser Beamforming*, EURASIP Book Series on Signal Processing and Communications. Hindawi Publishing Corporation, smart antennas - state-of-the-art edition, 2005.

[5] R. Blum, "Mimo capacity with interference," *IEEE Journal on Selected Areas in Communications*, vol. 21(5), pp. 793–801, June 2003.

[6] A. Lozano, A. M. Tulino, and S. Verdu, "Multiple-antenna capacity in the low-power regime," *IEEE Transactions on Information Theory*, vol. 49(10), pp. 2527–2544, Oct 2003.

[7] G. Caire, A. M. Tulino, and S. Verdu, "Suboptimality of tdma in the low-power regime," *IEEE Transactions on Information Theory*, vol. 50(4), pp. 608–620, Apr 2004.

[8] S. Cui, A. Goldsmith, and A. Bahai, "Energy-efficiency of mimo and cooperative mimo techniques in sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 22(6), pp. 1089–1098, June 2004.

[9] F. Rashid-Farrokhi, L. Tassiulas, and K. J. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Transactions on Communications*, vol. 46(10), pp. 1313–1323, Oct 1998.

[10] F. Rashid-Farrokhi, K. J. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE Journal on Selected Areas in Communications*, vol. 16(8), pp. 1437–1449, 1998.

[11] H. Boche and M. Wiczanowski, "Stability-optimal transmission policy for the multiple antenna multiple access channel in the geometric view," *EURASIP Signal Processing Journal, Special Issue on Advances in Signal Processing-assisted Cross-layer Designs*, Jun 2006, invited.

[12] H. Boche and M. Wiczanowski, “Optimization-theoretic analysis of the stability-optimal transmission policy for multiple antenna multiple access channel,” *IEEE Transactions on Signal Processing*, 2007, to appear.

[13] A. V. Fiacco and G. P. McCormick, *Nonlinear Programming: Sequential Unconstrained Minimization Techniques*, Wiley & Sons, New York, 1968.

[14] R. Bhatia, *Matrix Analysis*, Springer-Verlag, New York, 1997.

[15] H. Boche and E. Jorswieck, “Optimization of matrix monotone functions: Saddle-point, worst case noise analysis, and applications,” in *Proc. IEEE Intern. Symp. on Information Theory (ISIT 2004)*, Chicago, USA, Jun 2004.