

# TESTBED MEASUREMENTS OF OPTIMIZED LINEAR DISPERSION CODES

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## ABSTRACT

High-rate optimized linear dispersion (LD) codes have been investigated extensively in theory and by simulation throughout the last years. Regardless of the findings of the research so far, little work has been performed to investigate the performance of such codes in a realistic transmission system. We implemented an optimized full-rate full-diversity space-time block code, derived by Damen, on the Vienna MIMO testbed for a  $2 \times 2$  MIMO channel, and measured the uncoded bit error rate (BER) performance of the space-time block code, if decoded with a sphere decoder. Moreover, we compare the obtained results with the uncoded BER of the well-known Alamouti STBC, decoded with a ZF receiver, at various SNR values. Our results show that in a realistic environment high-rate optimized LD codes offer no advantage over the conventional Alamouti STBC.

## 1. INTRODUCTION

Currently, there is a high level of interest in multiple-input-multiple-output (MIMO) antenna systems for enhancing the data rates of modern communication systems, e.g. WiMax or UMTS LTE. Many of these systems are orthogonal frequency-division multiplexing (OFDM) based, thus effectively separating the frequency selective MIMO channel into subcarriers each with flat fading channels, respectively. Based on the initial theoretical studies by Foschini [1] and Telatar [2], the performance and practical feasibility of different space-time block coding (STBC) techniques was investigated, e.g. [3–5], to put the predicted MIMO gains in flat fading channels into practice.

However, most of the research has been conducted under simplified conditions, and results have been generated mainly by means of simulations. Bearing that in mind, it is of great importance to investigate the promised gains and usability of such STBC techniques in a realistic environment in order

to evaluate their performance in modern communication systems.

The literature contains little work about the realistic performance of STBCs for high-rate communication [4]. In this paper, we assess the performance of an optimized linear dispersion code [6] in a realistic environment by a measurement campaign, carried out on the “Vienna MIMO Testbed” [7]. Our results are compared with the performance of the well-known Alamouti STBC [8] in order to investigate whether the predicted coding gain of the LD code holds in a practical setting. Throughout this work, we restrict our evaluations to the flat fading MIMO channel (by choosing the bandwidth of the measurement system narrow enough), which is - as mentioned above - of interest for OFDM based communication systems. This means that we examine the performance of the space-time block codes when applied to a single OFDM sub-carrier, thus not considering any possible coding across sub-carriers.

Through measurements, we are able to address the following important aspects not covered in a simulation:

- **Imperfections of a real system.** In simulation it is required to model complex behavior by simpler mathematical descriptions. Accordingly, essential parts of a realistic communication system, like synchronization and linear power amplification are mostly not modeled at all, which often shows a too optimistic performance in simulation.
- **True physical behavior of the wireless channel.** Most theoretical analyses are based on a simplified modeling of the wireless channel by means of an i.i.d. flat fading channel matrix  $\mathbf{H}$ . In practice however, one faces antenna correlation, as well as time-variant channels and frequency selectivity, even in OFDM.

The paper is organized as follows: Section 2 describes the investigated STBCs and provides the theoretical performance of them. Section 3 explains the measurement setup and gives some details on the “Vienna MIMO Testbed”. Measurement

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results are shown in Section 4 and the paper ends with conclusions in Section 5.

## 2. INVESTIGATED STBCS

Before we go into the details of our measurement setup, we define the MIMO transmission model and specify the investigated STBCs. Suppose, there are  $n_T$  transmit antennas,  $n_R$  receive antennas, and an interval of  $T$  symbols available to us during which the propagation channel is (nearly) constant. Then, in a narrow-band, flat-fading, multiple-antenna communication system, the transmitted and received signals are related by

$$\mathbf{Y} = \sqrt{\frac{\rho}{n_T}} \mathbf{H} \mathbf{S} + \mathbf{N}, \quad (1)$$

where  $\mathbf{Y} \in \mathbb{C}^{n_R \times T}$  denotes the matrix of complex received signals,  $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$  denotes the channel matrix,  $\mathbf{S} \in \mathbb{C}^{n_T \times T}$  denotes the matrix of complex transmitted signals, and the additive noise is denoted by  $\mathbf{N} \in \mathbb{C}^{n_R \times T}$ . The transmit block matrix  $\mathbf{S}$  contains the transmitted complex data symbols  $s_n$ , which are chosen from an arbitrary constellation,  $\mathcal{A}$ , say  $r$ -QAM or  $r$ -PSK. By normalization of  $\mathbf{S}$ , i.e.  $E\{\|\mathbf{S}\|_F^2\} = n_T \cdot T$ , and assuming that the channel matrix  $\mathbf{H}$  and the noise  $\mathbf{N}$  have unit variance entries, the normalization  $\sqrt{\rho/n_T}$  in (1) ensures that  $\rho$  is the signal-to-noise ratio (SNR) at each receive antenna, independently of  $n_T$ .

Based on this MIMO transmission model, we now want to explain the investigated STBCs and show their performance in simulations.

### 2.1. Alamouti STBC

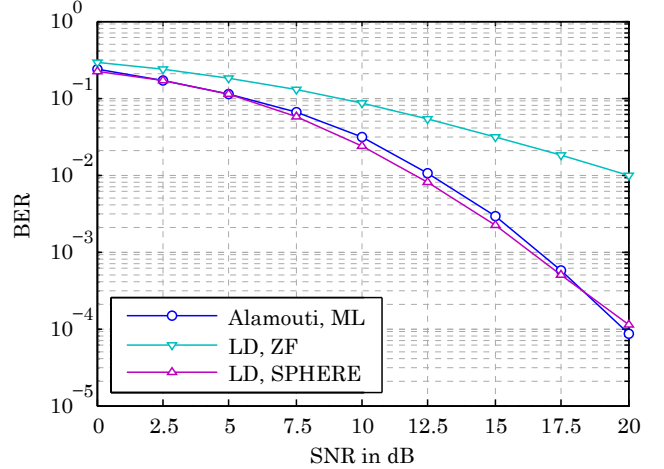
Nearly eight years ago, Alamouti [8] proposed a simple transmit diversity scheme which improves the signal quality at the receiver by simple processing across two antennas at the transmitter side. The proposed block coding technique has been theoretically extended by Tarokh [5] in the framework of orthogonal STBCs, so that codes with comparable properties for more than two transmit antennas are available.

Orthogonal STBCs generally have to fulfil the unitary property of the encoded transmission matrix  $\mathbf{S}$ , i.e.

$$\mathbf{S} \mathbf{S}^H = \mathbf{I} \sum_{n=1}^{n_S} |s_n|^2, \quad (2)$$

where  $(\cdot)^H$  denotes the Hermitian transpose,  $n_S$  equals the number of symbols mapped into the transmit block matrix and  $\mathbf{I}$  is the identity matrix. Accordingly, the Alamouti STBC (as a special case of the class of orthogonal STBCs for two transmit antennas) is specified as

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2^* \\ s_2 & -s_1^* \end{bmatrix}, \quad (3)$$



**Fig. 1.** BER Simulation results of Alamouti orthogonal STBC and optimized linear dispersion STBC in a  $2 \times 2$  i.i.d. flat fading MIMO channel.

with  $(\cdot)^*$  denoting the complex conjugate, which obviously fulfills Equation (2). The Alamouti scheme (3) can be used in a transmission system with an arbitrary number of receive antennas. In our setup we use two receive antennas only.

The Alamouti STBC offers two important advantages for a practical implementation, namely: (1) It effectively orthogonalizes the flat fading channel, thus decreasing the effect of fading (diversity) and (2) as for all orthogonal STBCs, the optimum ML receiver reduces to a symbol-by-symbol decision after a ZF receiver. However, from a capacity perspective, the orthogonal STBCs are far from optimum. In [9] it has been proven that orthogonal STBCs effectively limit the capacity of the MIMO communication system compared to the ergodic channel capacity offered by the MIMO channel. Accordingly, for high-rate communications, orthogonal STBCs do not seem the best choice.

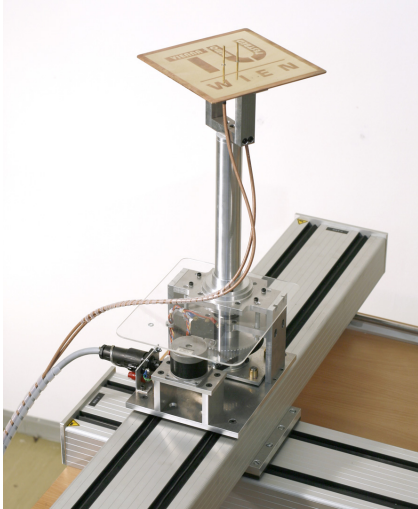
### 2.2. Optimized linear dispersion STBC

The universal class of linear dispersion STBC, proposed in [4] formally also includes the class of orthogonal STBC, but in the work performed by Hassibi *et al.*, the abstract coding structure was optimized constraint to a capacity criterion. The obtained code matrices showed an improved system capacity compared to that of the orthogonal STBCs, but suffered in terms of BER performance (diversity).

To overcome this problem, different approaches have been taken. A promising one was proposed by Damen, *et al.* [6], where the following code matrix was suggested

$$\mathbf{S} = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 + \phi s_2 & \sqrt{\phi}(s_3 + \phi s_4) \\ \sqrt{\phi}(s_3 - \phi s_4) & s_1 - \phi s_2 \end{bmatrix}, \quad (4)$$

where  $\phi = e^{j\lambda}$ , such that  $\lambda$  is a real-valued parameter that allows for optimization. As a result of their work,  $\phi$  is derived by a construction based on number theory to ensure that  $\mathbf{S}$  has



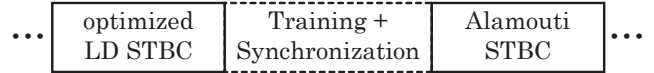
**Fig. 2.** Vienna MIMO Testbed receiver. Two carriages are able to move the antennas to obtain different channel realizations. For the measurements described in this paper, we used monopole antennas mounted on a groundplane.

a maximum transmit diversity. As they showed, the optimal choice of  $\lambda$  is dependent on the chosen symbol alphabet, for example, for a 4-QAM:  $\lambda = 0.5$ , or for a 16-QAM:  $\lambda = 0.521$ .

The so obtained optimized linear dispersion STBC does not limit the system capacity to be strictly below the ergodic capacity of the channel, and additionally offers full diversity like the orthogonal STBCs. Unfortunately, this capacity improvement has to be paid in terms of enhanced receiver complexity, since the ZF receiver is not optimal anymore, and thus the BER performance of the code would suffer greatly if this (or any comparable) suboptimum linear receiver is used.

The BER performances is shown in Figure 1, where one observes that the linear ZF receiver is not able to utilize the spatial diversity offered by the MIMO channel. However, modern receivers are able to achieve a near-ML performance with moderate complexity. For comparison, we plotted the BER performance of the optimized linear dispersion code using a sphere-decoder on the receiver side. The suboptimal sphere-decoder achieves the ML performance with much less computational effort. Accordingly, high-rate optimized LD codes promise high performance in high data-rate systems.

Furthermore, it can be seen that the optimized LD STBC outperforms the Alamouti code by approximately 0.5 dB in the BER region from  $2 \cdot 10^{-2}$  to  $2 \cdot 10^{-3}$ . Besides this SNR gain in terms of the BER performance, the LD code offers the same data rate with a smaller symbol alphabet (because of the spatial multiplexing). Accordingly, the SNR gain of the LD code (compared to the Alamouti code) grows when the rate of the transmission is increased (see also [4]).



**Fig. 3.** Block transmission / training scheme for the measurements of the two investigated STBCs.

### 3. MEASUREMENT SETUP

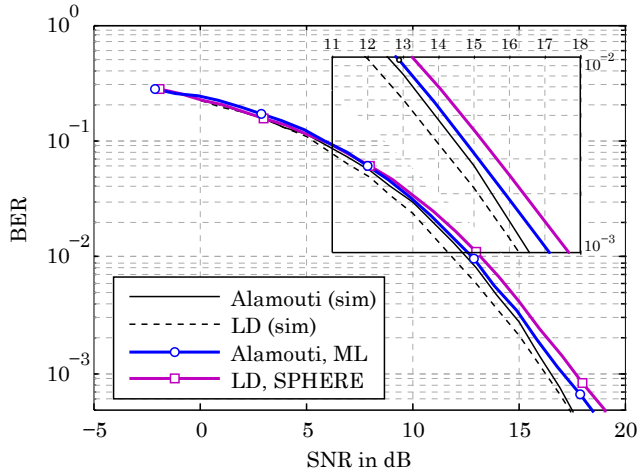
The measurements to investigate the performance of the two STBCs were conducted at the Vienna University of Technology, in the rooms of the Institute of Communications and Radio-Frequency Engineering. We used the “Vienna MIMO Testbed” [7] as a flexible measurement platform which we easily adopted to the different STBC encoding and decoding schemes.

We placed the transmitter and the receiver of the testbed in different rooms on the same floor to ensure that the measurement results reflect a typical indoor scenario. The linear distance between the transmitter and the receiver was approximately 16 m. As described in the introduction, only a single carrier of the testbed was activated, whereas we located the center frequency at 2.5 GHz. The available bandwidth for the transmission was limited to 1 MHz. We used root raised cosine (RRC) filters as transmit and receive filters. Furthermore, we specified the maximum transmit power by 20 dBm.

To obtain different channel realizations, we moved the transmit and receive antennas by an xy-positioning table to 256 different positions within a grid of size  $2\lambda \times 2\lambda$  respectively. Thus, we were able to generate 65.535 different channel realizations for each adjusted transmission power. According to the presented STBCs, we utilized two antennas at both ends of the system. Quarter wavelength monopole antennas, mounted on a groundplane have been used. The antennas have been spaced  $0.4\lambda$ , respectively. Figure 2 shows the receiver of the Vienna MIMO Testbed with the two carriages which carry the two monopoles mounted on the groundplane.

For a fair comparison of the two STBCs, we fixed the rate of the two STBC encoded transmissions  $R = n_S/T \log_2 |\mathcal{A}|$  ( $|\cdot|$  denoting the cardinality of the symbol alphabet) to 2 bits per channel use. Accordingly, we chose 4-QAM as symbol alphabet for the optimized LD code and 16-QAM as symbol alphabet for the Alamouti code. To minimize time-variant effects of the channel, we chose a specific block transmission /training scheme that is illustrated in Figure 3. This specific transmission procedure ensures that the optimized LD Code and the Alamouti Code use the same MIMO channel estimation as well as the same sampling time synchronization. Thus, the uncoded BER curves reflect the possible advantage (or disadvantage) of the STBC scheme solely.

The complex data symbols for transmission as well as the encoding of the STBCs were performed offline in MATLAB and were passed to the transmission chain of the testbed. After transmission of the data and training according to Figure 3, we stored the received symbols. Sampling and receive filter-



**Fig. 4.** Uncoded BER performance of the Alamouti STBC and the optimized LD code obtained from the measured data transmissions. The black lines represent the simulated curves from Figure 1.

ing were performed on the testbed, thus remaining channel estimation and decoding to be conducted offline. We accomplished channel estimation and decoding again in MATLAB, whereas we implemented the ZF receiver for the Alamouti STBC (equals the ML receiver), and the sphere decoder [10] for the optimized LD STBC. In total, we measured 7 different signal-to-noise ratios (SNR). The SNR at the receiver was set by attenuating the transmit signal power. For each of the 7 SNR values we averaged over the bit errors of the 65.535 channel realizations to obtain comparable uncoded BER performance curves.

#### 4. RESULTS

Figure 4 shows the obtained uncoded BER curves for the Alamouti STBC and the optimized LD code. It can be seen that both codes lose in terms of their performance – and even more, the Alamouti STBC outperforms the optimized LD code in the BER range where the simulation predicted a better performance. For a direct comparison, we plotted the simulated curves in Figure 4 as well.

Although the measurement was conducted in a scenario that behaves as a rich scattering and flat fading environment, the optimized LD code is not able to achieve the promised gains. The main reasons for the loss in performance are the non-ideal channel estimation and possible sampling errors, but – in our context more important – it seems that the optimized LD code is much more susceptible to correlation at either the transmit or the receive side. Although the antennas have been spaced  $0.4 \lambda$ , an influence of the remaining correlation (either on the transmit as well as on the receive side) is not impossible. Further research is planned to investigate the effect of correlation on different STBC coding structures.

#### 5. CONCLUSION

The performance of space-time block codes other than the Alamouti STBC in a real transmission scenario with the imperfections and restrictions of such systems is of great interest to rate the possible advantage of these encoding schemes. In our work, we investigated the performance of a high-rate optimized full-rate, full-diversity STBC and compared its measured uncoded BER with the uncoded BER performance of the well-known Alamouti STBC. The Vienna MIMO testbed has been used as a realistic transmission system. Encoding and decoding have been processed offline in MATLAB. Our measurements show that the predicted coding gain of the optimized LD space-time block code vanishes, although the LD code was designed to allow for a performance gain in terms of the uncoded BER. Additionally recalling that the computational complexity of the sphere-decoder is much higher than that of the ZF receiver which can be used for the Alamouti STBC, an appliance of an optimized LD code as a high-rate STBC in real transmission systems seems very questionable.

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