OPTIMUM NUMBER OF BEAMS IN MULTIUSER OPPORTUNISTIC SCHEMES UNDER QOS CONSTRAINTS

Nizar Zorba

Centre Tecnològic de Telecomunicacions de Catalunya
Av. Canal Olímpic, 08860-Castelldefels, Barcelona, Spain
`nizar.zorba@cttc.es`

Ana I. Pérez-Neira

Universitat Politècnica de Catalunya (UPC)
C/ Jordi Girona, Mòdul D5, 08034-Barcelona, Spain
`anuska@gps.tsc.upc.edu`

ABSTRACT

Scheduling in a Broadcast (BC) channel based on partial Channel State Information at the Transmitter (CSIT) is carried out through an opportunistic random beamforming technique. Within a more practical perspective, this paper first presents a transmission strategy where a minimum rate per user is required. This minimum rate is demanded by each scheduled user to properly decode and manage its received signal, which stands as a possible Quality of Service (QoS) indicator for the system behaviour. Then, the paper considers an optimization of the number of generated beams to satisfy the QoS constraints while maximizing the number of served users. The derived optimum number of beams is compared via simulations to available schemes in literature.

1. INTRODUCTION

The Opportunistic multiantenna schemes are attractive due to their high performance, and at the same time, low complexity design. But while maximal sum rate has typically been considered as the objective of multibeam opportunistic schemes [1], an alternative approach that focuses on the QoS of the served users is required for a system implementation [2] [3]. A potential measure of the system QoS is through the minimum rate per user, so that each served user is ensured a minimum Signal-to-Noise-Interference-Ratio (SNIR), allowing it to properly decode its intended data with a predefined Bit Error Rate (BER).

Previous studies have shown [4] that the user satisfaction is insignificantly increased by a service rate higher than the user demands, while on the other hand, if the provided rate fails below its requirement, the satisfaction drastically decreases [5]. Thus, a good scheduling scheme is achieved through delivering service to the highest possible number of users restricted to a minimum rate per user (similar to the approach presented in [6] for CDMA systems).

This work was partially supported by the Catalan Government under grant SGR2005-00996; by the Spanish Government under project TEC2005-08122-C03; and by the European MEDEA+ A121 project FIT-330220-2005-111 (PlaNets).

Several proposals appear in the literature for the Opportunistic random beamforming, where [7] proposes a single transmitting beam while [1] suggests the generation of a number of beams equal to the available number of transmitting antennas, and even a lot of studies have compared both approaches via simulations [8], but the optimum number of beams have been only characterized for the asymptotic conditions.

Even some QoS studies have been presented [5] for the single beam opportunistic transmission [7], but no previous proposals (up to the authors' knowledge) for the multibeam opportunistic beamforming are suggested in literature. This is mainly due to the cross interference terms that the multibeam opportunistic beamforming originates in the multiuser scenario as each user receives an interference component from each one of the generated beams, while only partial CSIT is available.

Furthermore, the wireless operators realize that some users can provide deficient channel conditions for communication, and delivering service to such users can be very expensive in term of system resources, driving down the whole system performance; so that if these users are dropped, the operator can offer better service to all the remaining users in the system. Based on this practical point of view, operators are more interested in probability of outage measures rather than absolute QoS fulfillment, making all the commercial systems to fix a target probability of outage in the users QoS.

This work considers a scheduling scheme that allows for some predefined outage to match the wireless operator business model, and derives the optimal number of transmitting beams under QoS constraints for the served users. The obtained theoretical result is later compared to the available opportunistic schemes in the literature. The remainder of this paper is organized as follows: while section II deals with the system model, in section III a review of the multibeam opportunistic procedure is presented. Section IV presents the optimal number of beams under QoS restrictions followed by section V with the numerical results and simulations. The paper finally draws the conclusions in section VI.

2. SYSTEM MODEL

We focus on the BC channel where N receivers, each one of them equipped with a single receiving antenna, are being served by a unique transmitter at the Base Station (BS) provided with n_t transmitting antennas, and supposing that N is greater than n_t. A multiantenna channel $\mathbf{h}_{[1\times n_t]}$ is considered between each of the users and the BS where a quasi static block fading model is assumed, which keeps constant through the coherence time, and independently changes between consecutive time intervals with independent and identically distributed (i.i.d.) complex Gaussian entries $\sim \mathcal{N}(0,1)$. Let $\mathbf{x}$ be the $n_t \times 1$ transmitted vector, while denote y_i as the i^{th} user received signal given by

$$y_i = \sqrt{\beta}\mathbf{h}_i\mathbf{x} + z_i \tag{1}$$

where z_i is an additive complex noise component with zero mean entries and unit variance and β stands for the channel Signal-to-Noise-Ratio (SNR) value.

The transmitter delivers service up to $W \leq n_t$ simultaneous users, where W denotes the number of generated beams. The transmitted signal $\mathbf{x}$ incloses the uncorrelated data symbols s_i to each one of the selected users with $E\{|s_i|^2\} = 1$, where a total transmitted power constraint of $P = 1$ is considered. For ease of notation, time index is dropped throughout the paper.

3. MULTIBEAM OPPORTUNISTIC TRANSMISSION

One of the main transmission techniques in multiuser scenarios is the multiple user opportunistic beamforming [1], where up to $W \leq n_t$ random orthogonal beams are generated at the BS to simultaneously serve more than one user. Within the acquisition step, each one of the users sequentially calculates the SNIR that it receives from each beam, and feeds back the best value to the BS together with an integer indicating the beam index. The BS scheduler chooses the user with the largest SNIR value for each one of the beams, enters the transmission stage and forwards every one of the selected users with its intended data.

This multibeam strategy achieves high system sum rate by serving several users at the same time, making the transmitted signal to enclose the data symbols for the n_t selected users as

$$\mathbf{x} = \sqrt{\frac{1}{W}}\sum_{m=1}^{W}\mathbf{b}_m\, s_m \tag{2}$$

with $\mathbf{b}_m$ as the unit power beam assigned to the m^{th} user.

This transmission scheme is characterized by its SNIR term due to the interference that each beam generates to its non-intended users, representing a major drawback of this system. The SNIR formulation for the i^{th} user through the m^{th} beam, with several transmitting orthogonal beams, is as

$$SNIR_{i,m} = \frac{\frac{1}{W}\left|\mathbf{h}_i\mathbf{b}_m\right|^2}{\frac{1}{\beta} + \sum\limits_{u\neq m}^{W}\frac{1}{W}\left|\mathbf{h}_i\mathbf{b}_u\right|^2} \tag{3}$$

where no receiver processing is present and assuming uniform power allocation among all the users. This formulation stands for any number of transmitting beams $W \leq n_t$ as long as the power P is redistributed among the transmitting beams. Obviously if a single beam is generated as in [7], then no interference terms are present.

The system sum rate capacity of this multiple user opportunistic beamforming, when $W = n_t$ can be written [1] as

$$SR \simeq E\left\{\sum_{m=1}^{n_t}\log(1+\max_{1\leq i\leq N} SNIR_{i,m})\right\} \tag{4}$$

which has been shown to be asymptotically optimal with the number of users, so that the optimum number of beams W is equal to the maximum allowed orthogonal beams (i.e, $W = n_t$) when the number of users approaches infinity. But for practical number of active users in the system, no previous works, up to the authors' knowledge, have characterised the optimum number of beams. This paper considers an outage scenario and provides a closed form expression for the optimal number of beams under QoS restrictions.

4. OPTIMUM NUMBER OF BEAMS

The requirement of a certain system QoS is presented in terms of a minimum rate value per user, where each scheduled user needs for this minimum rate to detect and manage its received signal. This is easily accomplished when a single user is scheduled at a time, so that through the power control, the delivered rate is regulated to the user requirements, but when several simultaneous users are scheduled through a multibeam opportunistic transmission, with cross interference terms among them as shown in (3), then this task is not a trivial one.

Driving opportunistic schemes towards a practical point of view to match the commercial operators requirements, an alternative service policy can be adapted, where a predefined probability of outage ξ_{out} in the service rate is tolerated. Through this policy, an important parameter to characterize the behaviour of a multiantenna system from the operator point of view, is the maximum number of served users (W) that the system supports under a restriction of minimum rate value per user.

This is actually a very practical approach for commercial systems as the wireless operator is always concerned about maximizing the number of served customers within the available resources, while at the same time, QoS guarantees are required for the served users in terms of a minimum SNIR value ($snir_{th}$), to allow a proper data decoding with predefined BER measures. Therefore, an attractive formulation for the QoS problem states as

$$\begin{aligned} &\max \ W \\ &s.t. \ Prob\big(SNIR_{W(i)} < snir_{th}\big) \leq \xi_{out} \quad \forall \ i \end{aligned} \tag{5}$$

where the calculation of the value of W is restricted by the value of $snir_{th}$ that is required for each user and the ξ_{out} value required by the system operator. To characterize the probability of outage, the serving SNIR cumulative distribution function (cdf) is needed, where a higher number of simultaneously serviced users causes more interference terms in the SNIR formulation (see (3)), which results in a lower serving SNIR per user.

When using any opportunistic scheme (even n_t beams, single beam or any value between the two extremes) to deliver service to the users, the serving SNIR value corresponds to the maximum SNIR over the active users in the system, so that the cumulative distribution function (cdf) of the maximum SNIR is required for the calculation of the maximum number of users W in equation (5). Using the SNIR equation in (3) with W transmitted beams, note that the numerator follows a Chi-square $\chi^2(2)$ distribution while the interference terms in the denominator are modeled as $\chi^2(2(W-1))$, which allows to obtain the probability distribution function (pdf) as

$$f(x) = \frac{e^{-(x*W/\beta)}}{(1+x)^W}\left(\frac{W}{\beta}(1+x) + W - 1\right) \tag{6}$$

and the cdf is then formulated as

$$F(x) = 1 - \frac{e^{-(x*W/\beta)}}{(1+x)^{W-1}} \tag{7}$$

and since the serving SNIR is the maximization over all the users' SNIR values, then the serving SNIR cdf is stated as

$$FF(x) = (F(x))^N = \left[1 - \frac{e^{-(x*W/\beta)}}{(1+x)^{W-1}}\right]^N \tag{8}$$

Enabling to reformulate the restriction of outage probability in (5) through the $cdf(snir_{th})$ as follows

$$\left[1 - \frac{e^{-(snir_{th}*W/\beta)}}{(1+snir_{th})^{W-1}}\right]^N \leq \xi_{out} \tag{9}$$

where the relation between the number of served users and both the ξ_{out} and $snir_{th}$ restrictions is established, thus making possible to calculate the maximum number of served users under the QoS restrictions as

$$W \leq \frac{\ln(1+snir^{th}) - \ln(1-\sqrt[N]{\xi_{out}})}{\ln(1+snir^{th}) + \frac{snir^{th}}{\beta}} \tag{10}$$

providing a closed form solution for W. Remind that a high W value results into smaller SNIR, so that W can not be increased a lot for practical SNIR magnitudes, and left for operator requirements and restrictions to set the values of ξ_{out} and $snir_{th}$.

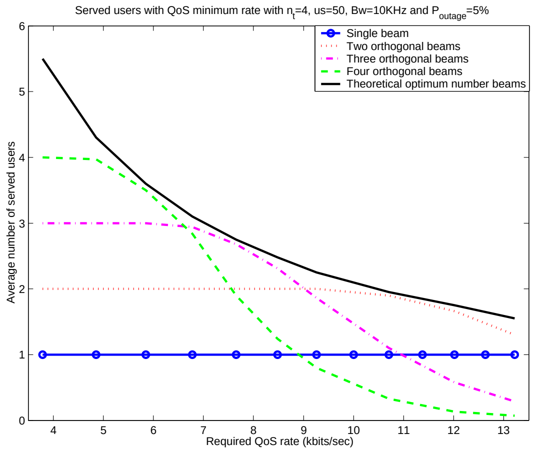

Fig. 1. Average number of served users under QoS restrictions.

5. SIMULATIONS

The performance of the proposed scheme is presented by Monte Carlo simulations, where the objective is to determine the optimum number of simultaneously serviced users. The serving policy ensures the QoS requirements in terms of minimum SNIR per served user, while a maximum outage restriction is imposed on the system performance. We consider a wireless scenario with $n_t = 4$ transmitting antennas in a cell with a variable number of active users, each one equipped with a single receiving antenna. The transmitter runs a multibeam opportunistic beamforming technique where a maximum of 4 orthogonal beams are set up. A total system bandwidth of 10KHz is considered together with a noise variance of $\sigma^2 = 1$ and $\beta = 0dB$.

Figure (1) shows results for the proposed optimization, where the theoretical optimum number of beams value derived in (10) is calculated for a scenario with 50 active users with $\xi_{out} = 5\%$, and compared to the performance of transmitting with either 4, 3, 2 or 1 generated beams. The result displays the spectral efficiency in terms of the number of served users, and how it changes as the required minimum rate per served user increases, where the rate is mapped from the $snir^{th}$ value. For a QoS rate of 6kbps, generating a single transmitting beam shows that its system behaviour is deficient, as the system resources can be used with a larger number of users without affecting the single user QoS satisfaction, while with 4 beams, the QoS requirement will be satisfied for all the 4 users. This highlights how the number of beams can be exploited to achieve a dynamic resource management scheme based on the scenario characteristics.

Also notice that the orthogonal beamforming techniques show a gap in satisfying higher number of users than n_t when the QoS restrictions are not so high, where if the minimum rate per user equals 4kbps, then from the figure (1), it seems that an extra user can be serviced. The presented orthogo-

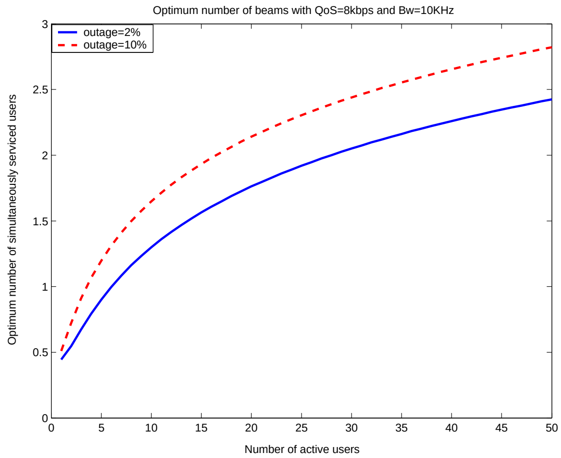

Fig. 2. Optimum number of simultaneously serviced users under QoS restrictions.

nal beamforming can not generate a number of beams higher than n_t, so that the solution for small QoS restrictions can be solved through non orthogonal Grassmannian beamforming as proposed in [9].

Together with the QoS and outage system requirements, an important system driving factor is the available number of users N in the cell, as all opportunistic schemes strongly rely on this variable. Considering the calculation of the optimum number of beams in (10), the effect of the N variable can not be neglected, as a higher number of users will enable the transmitter to select a subset of users that simultaneously show high channel rate and low interference among themselves, so that their SNIR increase and consequently, the QoS can be satisfied even a higher number of beams is generated.

Figure (2) presents the optimum number of users in a scenario with a variable number of users asking for service, and it shows how the number of simultaneously serviced users increases with higher available users in the scenario. Two outage probabilities are considered to also expose the effect of a higher ξ_{out}, as it allows the scheduler to have a higher freedom in the allocation process. Obviously, as the number of serviced users can only be reflected by integer numbers, then the BS scheduler chooses to allocate a number of users that shows the largest integer below the optimum calculated value in figure (2).

Another benefit from the presented scheme is its high robustness to feedback uncertainty, as the optimum number of beams is calculated over statistical information rather than instantaneous ones (as done in [8] through simulations), so that the feeded back values from the users are used for the user selection process and not as an indicator of their channel qualities and separability. Further comment on the robustness of the transmission schemes based on the statistical distribution of SNIR in an outage scenario are found in [2].

6. CONCLUSIONS

For a system implementation of the opportunistic schemes, a QoS system objective is required to match the commercial operators requirements, and this work showed the benefits of the opportunistic beamforming schemes in terms of a potential measure of QoS, through the minimum rate per scheduled user. To further adapt the study to realistic scenarios, a rate outage is considered in the scheduling process.

Under the QoS restrictions, the paper developed the optimum number of generated beams (i.e, simultaneously serviced users) in a closed form solution, so that a decision over the available opportunistic schemes in literature is done under the developed scenario.

To test the theoretical results, several simulations were performed to show the effect of the several design parameters in the system, and the impact they have on the QoS compliance.

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