EFFECT OF MULTIPLE CARRIER FREQUENCY OFFSETS IN MIMO SC-FDMA SYSTEMS

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ABSTRACT

SC-FDMA with dynamic bandwidth assignment was chosen as uplink transmission scheme for the Long Term Evolution (LTE) of the 3GPP UMTS standard [1]. In this contribution we focus on the extension of the Single-Carrier Frequency Division Multiple Access (SC-FDMA) scheme to a Multiple-Input Multiple-Output (MIMO) SC-FDMA scheme employing spatial multiplexing and dynamic bandwidth assignment.

In the first part of this contribution, we define a flexible mathematical Multi-User(MU) MIMO SC-FDMA framework. This gives us a tool to study the two different sub-carrier assignment strategies and different MIMO system setups. The second part extents the framework towards user-specific Carrier Frequency Offsets (CFOs). In addition we study the resulting MU interference, when a CFO Correction (CFOC) for the desired user is deployed . Finally, we compare the sub-carrier assignment strategies Localized FDMA (LFDMA) and Interleaved FDMA (IFDMA).

1. INTRODUCTION

Single-Carrier (SC) transmission is well-known for its low Peak-to-Average-Power-Ratio (PAPR) property and robustness to carrier frequency offsets, but a multi-path channel can severely impact the performance of a high data-rate SC signaling by causing Inter-Symbol-Interference (ISI). In such a traditional SC scheme the compensation of ISI requires adaptive equalizers of high complexity. Orthogonal Frequency Division Multiplexing (OFDM) is well-known to overcome the detrimental effect of multi-path channels and to offer a solution with low processing complexity due to the usage of the Fast Fourier Transform (FFT). Nevertheless, such multi-carrier systems are known for being sensitive to frequency offsets and phase noise. The high Peak-to-Average-Power-Ratio of multi-carrier systems require additional back-off for the Power Amplifiers (PAs) of the transmitters, which is a high burden for low-cost User Equipments (UEs). In addition OFDM requires strong channel coding or adaptivity in order to make use of the frequency diversity offered by the channel.

Another attractive signaling scheme, which is called SC Frequency Domain Equalization (SC-FDE), allows as good as OFDM the effective signaling over frequency selective channels, but it combines the benefits of OFDM with the advantages of SC transmission. This is done by employing a block-based transmission with cyclic extensions, quite similar to OFDM. Since single carrier transmission uses the frequency diversity directly, no strong channel coding is required. A single carrier scheme features low PAPR, in particular by using Phase Shift Keying (PSK) for a constant envelope. The low PAPR property allows cheap PAs in the UEs, which makes SC-FDE a strong candidate for uplink transmission in wireless communication systems [2], [3].

The quite similar SC-FDMA scheme with dynamic bandwidth assignment was elected for the uplink of 3GPP LTE, while in the downlink adaptive OFDMA with dynamic bandwidth assignment is deployed. SC-FDMA can be seen as an SC scheme, where an OFDM system with larger bandwidth is employed for modulating and demodulating the SC signals to/from different bands. Another interpretation sees SC-FDMA as OFDM system, where the Discrete Fourier Transform (DFT) is employed as coding matrix before sub-carrier assignment and corresponding to this the Inverse DFT (IDFT) as de-coding matrix.

Beside other choices the sub-carrier assignment as shown in Fig. 1 can follow two main strategies. The first one is Localized FDMA (LFDMA), where each user band is occupying a dedicated part of the usable spectrum. The second one is Interleaved FDMA (IFDMA), where the user bands are interleaved to each other. IFDMA combines the advantages of a spread spectrum technique with “multi-carrier”-like transmission, but comes on the cost of sensitivity to frequency offsets and phase noise, which is comparable to that of OFDMA [4]. Such frequency offsets occur in a multi-user system due to the different local oscillators deployed in the User Equipments

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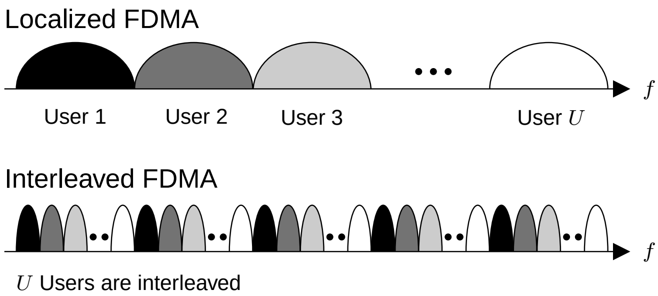

Fig. 1. Localized and Interleaved FDMA.

(UEs) and the Base Station (BS) or due to the movements of the UEs relative to the Base-Station (BS) - the so called Doppler effect.

In section 2 we provide a system model and the general mathematical framework for MU MIMO SC-FDMA under the assumption of perfect synchronization in time and frequency. The section 3 extents the framework further to study the effects of multiple Carrier Frequency Offsets (CFOs) in MIMO SC-FDMA, when LFDMA or IFDMA is employed.

2. SYSTEM MODEL

We assume the following scenario: U UEs are communicating at the same time with a fixed BS, which employs N_{R} receive antennas. Each user u has $N_{\mathrm{T}}^{(u)}$ transmit antennas. The data block length $N_{\mathrm{DFT}}^{(u)}$ can be chosen differently for each user, but can not differ for transmit branches. The channel is at least quasi static for a single block transmission and a block-wise transmission is assumed. In the following we consider the transmission and reception of $N_{\mathrm{T}}^{(u)}$ blocks, which are multiplexed to the $N_{\mathrm{T}}^{(u)}$ transmit branches and are transmitted simultaneously.

The transmitter for a single user u is shown in Fig. 2. The base-band transmit signal of a single user u can be formulated as

$$\begin{aligned}\mathbf{s}^{(u)} &= \left[\mathbf{I}_{N_{\mathrm{T}}^{(u)}} \otimes \left(\mathbf{P}_{\mathrm{add}}\left(\mathbf{F}_{N_{\mathrm{FFT}}}\right)^{-1}\right)\right] \\ &\quad \mathbf{M}_{\mathrm{T}}^{(u)}\left(\mathbf{I}_{N_{\mathrm{T}}^{(u)}} \otimes \mathbf{F}_{N_{\mathrm{DFT}}}\right)\mathbf{d}^{(u)},\end{aligned}$$

where $\mathbf{s}^{(u)} = \left[\mathbf{s}_1^{(u)}, \ldots, \mathbf{s}_{N_{\mathrm{T}}^{(u)}}^{(u)}\right]^{\mathrm{T}}$ contains the transmit signals of the $N_{\mathrm{T}}^{(u)}$ transmit antennas of the u-th user. Here, the matrix $\mathbf{I}_{N_{\mathrm{T}}^{(u)}}$ is the identity matrix of size $N_{\mathrm{T}}^{(u)} \times N_{\mathrm{T}}^{(u)}$, '$\otimes$' denotes the Kronecker product, $\mathbf{P}_{\mathrm{add}}$ is a matrix which adds a Cyclic Prefix (CP) of length L to a vector of FFT size N_{FFT}, $\mathbf{M}_{\mathrm{T}}^{(u)}$ describes the M_u-to-N sub-carrier mapping on each transmit branch for the u-th user, N_{DFT} is the DFT size used by the u-th user, and $\mathbf{d}^{(u)} = \left[\mathbf{d}_1^{(u)}, \ldots, \mathbf{d}_{N_{\mathrm{T}}^{(u)}}^{(u)}\right]^{\mathrm{T}}$ contains the $N_{\mathrm{T}}^{(u)}$ data vectors of the u-th user. $\mathbf{F}_N$ is the $N \times N$ Fourier matrix, which has the elements $[\mathbf{F}]_{i,j} = \frac{1}{\sqrt{N}} \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}(i-1)(j-1)}$. The inverse of a matrix is denoted by $(.)^{-1}$.

The vector $\mathbf{s}^{(u)}$ is always of size $(N_{\mathrm{FFT}} + L)N_{\mathrm{T}}^{(u)} \times 1$, which means that the size is only depending on the number of transmit antennas $N_{\mathrm{T}}^{(u)}$ and not on the used block length N_{DFT} (occupied resources) of the user u. Thereby, it is guaranteed, that the Transmission Time Interval (TTI) for all users is equal.

A single BS is receiving the signals. We assume in the following, that the delay in transmission and the length of the channel impulse of each user must fit into the time interval covered by the CP of length L. This condition can be maintained by a foregoing synchronization procedure involving all users. In addition we assume perfect synchronization in time and frequency.

The base-band received signals $\mathbf{r} = [\mathbf{r}_1, \ldots, \mathbf{r}_{N_{\mathrm{R}}}]^{\mathrm{T}}$ of the N_{R} receive antennas can then be written as

$$\mathbf{r} = \sum_{u=1}^{U} \mathbf{H}^{(u)}\mathbf{s}^{(u)} + \boldsymbol{\nu}, \tag{1}$$

where $\mathbf{H}^{(u)}$ is the $(N_{\mathrm{FFT}}+L)N_{\mathrm{R}} \times (N_{\mathrm{FFT}}+L)N_{\mathrm{T}}^{(u)}$ MIMO channel matrix of the u-th user, and $\boldsymbol{\nu} = [\boldsymbol{\nu}_1, \ldots, \boldsymbol{\nu}_{N_{\mathrm{R}}}]^{\mathrm{T}}$ contains the noise at the N_{R} receive antennas.

Fig. 3 depicts the MIMO SC-FDMA receiver for the U users. One can see that for each User a separate FDE is performed.

Corresponding to this, the frequency domain equalizing approach for the first user ($u = 1$) can be written as

$$\begin{aligned}\tilde{\mathbf{d}}^{(1)} &= \left(\mathbf{I}_{N_{\mathrm{T}}^{(1)}} \otimes \mathbf{F}_{N_{DFT}^{(1)}}^{-1}\right)\mathbf{W}^{(1)}\mathbf{M}_{\mathrm{R}}^{(1)} \\ &\quad \cdot \left[\mathbf{I}_{N_{\mathrm{R}}} \otimes \left(\mathbf{F}_{N_{\mathrm{FFT}}}\mathbf{P}_{\mathrm{rem}}\right)\right]\mathbf{r},\end{aligned}$$

where $\mathbf{M}_{\mathrm{R}}^{(1)}$ denotes the N_{FFT}-to-N_{DFT} sub-carrier de-mapping on all receive antennas for the first user and $\mathbf{W}^{(1)}$ is the MIMO equalization matrix, which aims to compensate the channel of the first user.

Without lose of generality the first user was chosen as the desired one.

With help of the short cuts

$$\begin{aligned}\mathbf{D}_{\mathrm{DFT}}^{(u)} &= \mathbf{I}_{N_{\mathrm{T}}^{(u)}} \otimes \mathbf{F}_{N_{\mathrm{DFT}}^{(u)}} \\ \mathbf{D}_{\mathrm{IFFT}}^{(u)} &= \mathbf{I}_{N_{\mathrm{T}}^{(u)}} \otimes \left(\mathbf{F}_{N_{\mathrm{FFT}}}\right)^{-1},\end{aligned}$$

and using the circular MIMO channel matrix of the u-th user

$$\mathbf{H}_{\mathrm{cir}}^{(u)} = \left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \mathbf{P}_{\mathrm{rem}}\right)\mathbf{H}^{(u)}\left(\mathbf{I}_{N_{\mathrm{T}}^{(u)}} \otimes \mathbf{P}_{\mathrm{add}}\right),$$

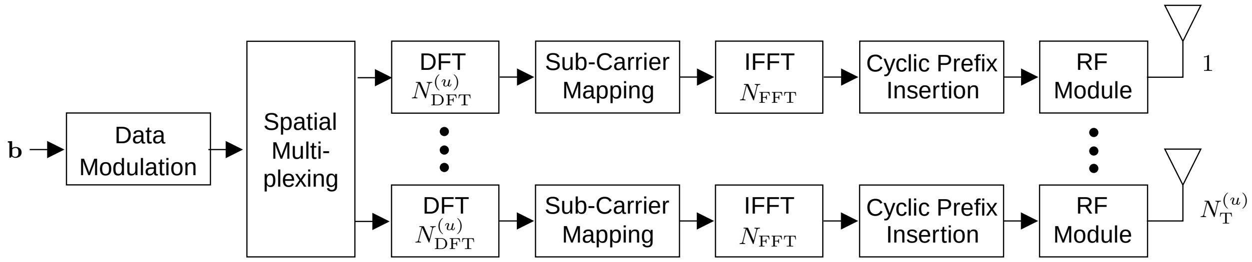

Fig. 2. MIMO SC-FDMA Transmitter for User u.

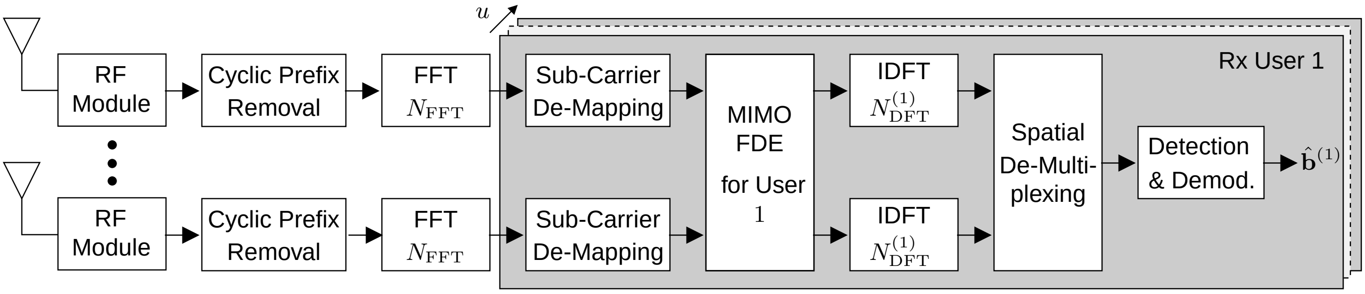

Fig. 3. MIMO SC-FDMA Receiver.

and $\breve{\boldsymbol{\nu}} = (\mathbf{I}_{N_\mathrm{R}} \otimes \mathbf{P}_{\mathrm{rem}})\,\boldsymbol{\nu}$, we obtain

$$\tilde{\mathbf{d}}^{(1)} = \left(\mathbf{D}_{\mathrm{DFT}}^{(1)}\right)^{-1} \mathbf{W}^{(1)} \mathbf{M}_{\mathrm{R}}^{(1)} \mathbf{D}_{\mathrm{FFT}} \left[\sum_{u=1}^{U} \mathbf{H}_{\mathrm{cir}}^{(u)} \mathbf{D}_{\mathrm{IFFT}}^{(u)} \mathbf{M}_{\mathrm{T}}^{(u)} \mathbf{D}_{\mathrm{DFT}}^{(u)} \mathbf{d}^{(u)} + \breve{\boldsymbol{\nu}}\right].$$

Since $\boldsymbol{\Lambda}^{(u)} = \mathbf{D}_{\mathrm{FFT}} \mathbf{H}_{\mathrm{cir}}^{(u)} \mathbf{D}_{\mathrm{IFFT}}^{(u)}$, we can write

$$\tilde{\mathbf{d}}^{(1)} = \left(\mathbf{D}_{\mathrm{DFT}}^{(1)}\right)^{-1} \mathbf{W}^{(1)} \mathbf{M}_{\mathrm{R}}^{(1)} \left[\sum_{u=1}^{U} \boldsymbol{\Lambda}^{(u)} \mathbf{M}_{\mathrm{T}}^{(u)} \mathbf{D}_{\mathrm{DFT}}^{(u)} \mathbf{d}^{(u)} + \breve{\boldsymbol{\nu}}\right]. \tag{2}$$

With $\mathbf{E}^{(1)} = \left(\mathbf{D}_{\mathrm{DFT}}^{(1)}\right)^{-1} \mathbf{W}^{(1)} \mathbf{M}_{\mathrm{R}}^{(1)}$ it follows

$$\tilde{\mathbf{d}}^{(1)} = \underbrace{\mathbf{E}^{(1)} \boldsymbol{\Lambda}^{(1)} \mathbf{M}_{\mathrm{T}}^{(1)} \mathbf{D}_{\mathrm{DFT}}^{(1)} \mathbf{d}^{(1)}}_{\text{equalized signal of user 1}} + \underbrace{\mathbf{E}^{(1)} \sum_{u=2}^{U} \boldsymbol{\Lambda}^{(u)} \mathbf{M}_{\mathrm{T}}^{(u)} \mathbf{D}_{\mathrm{DFT}}^{(u)} \mathbf{d}^{(u)}}_{\text{filtered interference caused by other users}} + \underbrace{\mathbf{E}^{(1)} \breve{\boldsymbol{\nu}}}_{\text{filtered noise}}. \tag{3}$$

The eq. (3) contains no restrictions concerning the sub-carrier mapping used on the transmit branches. One aim of MIMO systems is to employ the spatial domain to obtain higher data-rates without occupying additional bandwidth.

Hence, we have to restrict the sub-carrier mapping for the transmit branches to be equal. Therefore, we define $\mathbf{M}_{\mathrm{T}}^{(u)}$ to have the following structure:

$$\mathbf{M}_{\mathrm{T}}^{(u)} = \left(\mathbf{I}_{N_{\mathrm{T}}^{(u)}} \otimes \mathbf{M}_{0}^{(u)}\right),$$

where $\mathbf{M}_{0}^{(u)}$ describes the sub-carrier mapping for the user u. Corresponding to this, the de-mapping matrix must have the following structure

$$\mathbf{M}_{\mathrm{R}}^{(u)} = \left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \left(\mathbf{M}_{0}^{(u)}\right)^{\dagger}\right),$$

where $(.)^{\dagger}$ denotes the pseudo-inverse operation.

Ideally, all users use different sub-carriers, so that the users are orthogonal in the frequency domain. Under the assumption of perfect synchronization, perfect orthogonality of the users, and perfect inversion of the MIMO channel effects with help of $\mathbf{W}^{(1)}$, one can reduce (3) to

$$\tilde{\mathbf{d}}^{(1)} = \mathbf{d}^{(1)} + \mathbf{E}^{(1)} \breve{\boldsymbol{\nu}}, \tag{4}$$

which means that there will be no interference between the desired user and the other users.

The term $\mathbf{M}_{\mathrm{R}}^{(1)} \boldsymbol{\Lambda}^{(1)} \mathbf{M}_{\mathrm{T}}^{(1)}$ in (2) can be interpreted as the effective MIMO channel matrix for user 1. In the ideal case

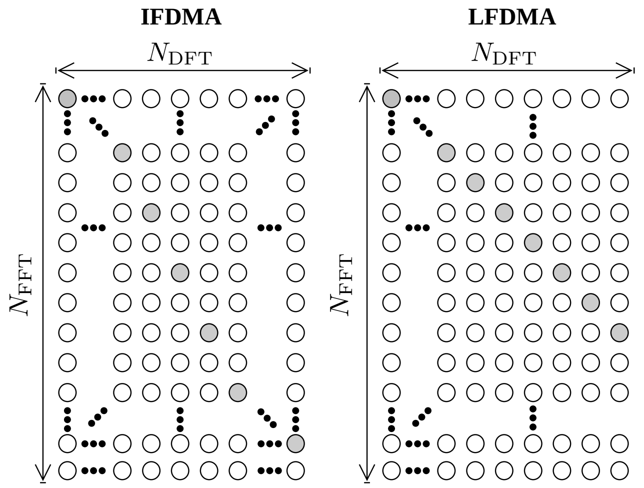

Fig. 4. Examples for Mapping Matrices in Case of LFDMA and IFDMA. The filled circles stand for '1', the unfilled circles for a '0'.

$\mathbf{M}_{\mathrm{R}}^{(1)}\boldsymbol{\Lambda}^{(2)}\mathbf{M}_{\mathrm{T}}^{(2)}$ results in a zero matrix. A MIMO equalization matrix for a Zero Forcing (ZF) equalizer can be given as $\mathbf{W}^{(1)} = \left(\mathbf{M}_{\mathrm{R}}^{(1)}\boldsymbol{\Lambda}^{(1)}\mathbf{M}_{\mathrm{T}}^{(1)}\right)^{\dagger}$.

Unfortunately, the orthogonality of the users can be destroyed by frequency offsets, which can occur for two reasons:

1. The different local oscillators in the UEs and the BS cause a user specific carrier frequency offset, which result in a continuous phase rotation of the received constellation and will introduce Inter-Carrier-Interference (ICI). ICI in SC-FDMA results in interference between the users.

2. The movement of the UEs relative to the fixed BS produces frequency shifts due to the so-called Doppler effect. Since the movements and speeds occur randomly, this will add time variations to the problem.

LFDMA and IFDMA have a different level of sensitivity concerning such frequency offsets, which we will investigate in the following section.

3. EFFECT OF USER-SPECIFIC CARRIER FREQUENCY OFFSETS

It is well-known that a Carrier Frequency Offset (CFO) $f_{\mathrm{CFO}}^{(u)}$ between UE u and BS results in a continuous phase rotation of the base-band received signal coming from user u.

In order to include the user-specific CFO effects a noise-free version of (1) can be modified to be

$$\mathbf{r} = \sum_{u=1}^{U} \left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \boldsymbol{\Delta}_{\mathrm{CFO}}^{(u)}\right) \mathbf{H}^{(u)}\mathbf{s}^{(u)}, \tag{5}$$

where $\boldsymbol{\Delta}_{\mathrm{CFO}}^{(u)}$ is a $L+N_{\mathrm{FFT}} \times L+N_{\mathrm{FFT}}$ diagonal matrix with the elements $[\boldsymbol{\Delta}_{\mathrm{CFO}}^{(u)}]_{n,n} = \mathrm{e}^{\mathrm{j}2\pi\Omega_{\mathrm{CFO}}^{(u)}n+\phi_0^{(u)}}$. Here, $\Omega_{\mathrm{CFO}}^{(u)} = f_{\mathrm{CFO}}^{(u)}/f_{\mathrm{s}}$ is the CFO normalized to the sampling frequency f_{s}. Note that the start phases $\phi_0^{(u)}$ can differ per user.

With $\check{\mathbf{r}} = (\mathbf{I}_{N_{\mathrm{R}}} \otimes \mathbf{P}_{\mathrm{rem}})\,\mathbf{r}$ we can rewrite

$$\begin{aligned} \check{\mathbf{r}} &= \sum_{u=1}^{U} \left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \mathbf{P}_{\mathrm{rem}}\boldsymbol{\Delta}_{\mathrm{CFO}}^{(u)}\right) \mathbf{H}^{(u)}\mathbf{s}^{(u)} \\ &= \sum_{u=1}^{U} \left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \check{\boldsymbol{\Delta}}_{\mathrm{CFO}}^{(u)}\mathbf{P}_{\mathrm{rem}}\right) \mathbf{H}^{(u)}\mathbf{s}^{(u)}. \end{aligned}$$

Since the removal of the prefix, removes only rows from the received vector, the matrix $\check{\boldsymbol{\Delta}}_{\mathrm{CFO}}^{(u)}$ is a $N_{\mathrm{FFT}} \times N_{\mathrm{FFT}}$ diagonal matrix with the elements

$$[\check{\boldsymbol{\Delta}}_{\mathrm{CFO}}^{(u)}]_{n,n} = \mathrm{e}^{\mathrm{j}2\pi\Omega_{\mathrm{CFO}}n+\mathrm{j}2\pi\Omega_{\mathrm{CFO}}^{(u)}L+\phi_0^{(u)}}.$$

Since the start phases $\phi_0^{(u)}$ will be noticed by the channel estimation and compensated in the FDE, we can simplify our calculation by setting $\phi_0^{(u)} = -\mathrm{j}2\pi\Omega_{\mathrm{CFO}}^{(u)}L$.

Including $\boldsymbol{\Delta}_{\mathrm{CFOC}}^{(1)}$ for an CFO Correction (CFOC) for the user 1 and using $\mathbf{E}^{(1)}$ and $\mathbf{D}_{\mathrm{FFT}}$ in (2), we can write

$$\tilde{\mathbf{d}}^{(1)} = \mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \boldsymbol{\Delta}_{\mathrm{CFOC}}^{(1)}\right)\mathbf{r}.$$

Defining $\mathbf{s}^{(u)} = \left(\mathbf{I}_{N_{\mathrm{T}}^{(u)}} \otimes \mathbf{P}_{\mathrm{add}}\right)\check{\mathbf{s}}^{(u)}$ and inserting (5), we get

$$\begin{aligned} \tilde{\mathbf{d}}^{(1)} &= \mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \boldsymbol{\Delta}_{\mathrm{CFOC}}^{(1)}\right) \\ &\quad \sum_{u=1}^{U} \left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \check{\boldsymbol{\Delta}}_{\mathrm{CFO}}^{(u)}\right) \mathbf{H}_{\mathrm{c}}^{(u)}\check{\mathbf{s}}^{(u)}. \end{aligned}$$

Assuming that the matrix $\boldsymbol{\Delta}_{\mathrm{CFOC}}^{(1)}$ is perfectly compensating the CFO introduced by $\check{\boldsymbol{\Delta}}_{\mathrm{CFO}}^{(1)}$, one can conclude

$$\boldsymbol{\Delta}_{\mathrm{CFOC}}^{(1)}\check{\boldsymbol{\Delta}}_{\mathrm{CFO}}^{(1)} = \mathbf{I}_{N_{\mathrm{FFT}}} \Leftrightarrow \boldsymbol{\Delta}_{\mathrm{CFOC}}^{(1)} = \left(\check{\boldsymbol{\Delta}}_{\mathrm{CFO}}^{(1)}\right)^{\mathrm{H}}.$$

Including the noise, we can write similar to (3)

$$\begin{aligned} \tilde{\mathbf{d}}^{(1)} &= \underbrace{\mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\mathbf{H}_{\mathrm{c}}^{(1)}\check{\mathbf{s}}^{(1)}}_{\text{equalized signal of user 1}} \\ &\quad + \underbrace{\mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\mathbf{D}_{\mathrm{CFOC}}^{(1)}\sum_{u=2}^{U}\mathbf{D}_{\mathrm{CFO}}^{(u)}\mathbf{H}_{\mathrm{c}}^{(u)}\check{\mathbf{s}}^{(u)}}_{\text{filtered interference caused by other users}} \\ &\quad + \underbrace{\mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\mathbf{D}_{\mathrm{CFOC}}^{(1)}\boldsymbol{\nu}}_{\text{filtered noise}}, \end{aligned}$$

where we used the short-cuts

$$\begin{aligned}\mathbf{D}^{(1)}_{\mathrm{CFOC}} &= \mathbf{I}_{N_{\mathrm{R}}} \otimes \mathbf{\Delta}^{(1)}_{\mathrm{CFOC}}, \text{ and}\\ \mathbf{D}^{(u)}_{\mathrm{CFO}} &= \mathbf{I}_{N_{\mathrm{R}}} \otimes \breve{\mathbf{\Delta}}^{(u)}_{\mathrm{CFOC}}.\end{aligned}$$

The filtered interference to the desired user 1 caused by the other $U-1$ users is determined by

$$\mathbf{m}^{(1)} = \mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\mathbf{D}^{(1)}_{\mathrm{CFOC}}\sum_{u=2}^{U}\mathbf{D}^{(u)}_{\mathrm{CFO}}\mathbf{H}^{(u)}_{\mathrm{c}}\breve{\mathbf{s}}^{(u)}.$$

This can be formulated as

$$\mathbf{m}^{(1)} = \mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\sum_{u=2}^{U}\mathbf{D}^{(u,1)}_{\mathrm{rCFO}}\mathbf{H}^{(u)}_{\mathrm{c}}\breve{\mathbf{s}}^{(u)}, \qquad (6)$$

where $\mathbf{D}^{(u,1)}_{\mathrm{rCFO}} = \left(\mathbf{I}_{N_{\mathrm{R}}} \otimes \mathbf{\Delta}^{(u,1)}_{\mathrm{rCFO}}\right)$. The matrix $\mathbf{\Delta}^{(u,1)}_{\mathrm{rCFO}} = \breve{\mathbf{\Delta}}^{(1)}_{\mathrm{CFOC}}\breve{\mathbf{\Delta}}^{(u)}_{\mathrm{CFO}} = \left(\breve{\mathbf{\Delta}}^{(1)}_{\mathrm{CFO}}\right)^{\mathrm{H}}\breve{\mathbf{\Delta}}^{(u)}_{\mathrm{CFO}}$ describes the remaining CFO of user u, when the CFOC of user 1 is used. The elements of the matrix are given as

$$\left[\breve{\mathbf{\Delta}}^{(u)}_{\mathrm{rCFO}}\right]_{n,n} = \mathrm{e}^{\mathrm{j}2\pi(\Omega^{(u)}_{\mathrm{CFO}}-\Omega^{(1)}_{\mathrm{CFO}})n}.$$

Note, that the usage of the CFOC of user 1 can have positive and also negative effects to the remaining CFO of the other users. Since the CFOC of the desired user can cause additional interference to the other users due to the desired user, a CFOC should be maintained per user on the entire received signal.

We will now further analyze the interference to the desired user 1 caused by the $U-1$ users after usage of its CFOC.

The e-function can be developed in the series

$$\mathrm{e}^{x} = \sum_{k=0}^{\infty}\frac{x^k}{k!} \approx 1 + x,\ \forall x \ll 1.$$

Since $\Delta\Omega^{(1,u)}_{\mathrm{CFO}} = \Omega^{(u)}_{\mathrm{CFO}} - \Omega^{(1)}_{\mathrm{CFO}}$ is in practical systems in the order of 10^{-3}, we can write

$$\mathbf{\Delta}^{(1,u)}_{\mathrm{rCFO}} \approx \mathbf{I}_{N_{\mathrm{FFT}}} + \mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\mathbf{C},$$

where $\mathbf{C}$ is a diagonal matrix with 1 to N_{FFT} on its main diagonal.

Therefore, we get by rewriting (6)

$$\begin{aligned}\mathbf{m}^{(1)} \approx\ & \mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\\ & \cdot\sum_{u=2}^{U}\left[\mathbf{I}_{N_{\mathrm{R}}N_{\mathrm{FFT}}} + \mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\left(\mathbf{I}_{N_{\mathrm{R}}}\otimes C\right)\right]\\ & \cdot\mathbf{H}^{(u)}_{\mathrm{c}}\breve{\mathbf{s}}^{(u)}\\ =\ & \underbrace{\mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\sum_{u=2}^{U}\mathbf{H}^{(u)}_{\mathrm{c}}\breve{\mathbf{s}}^{(u)}}_{=0,\text{ due orthogonality of users in FD}}\\ & +\mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\sum_{u=2}^{U}\mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\left(\mathbf{I}_{N_{\mathrm{R}}}\otimes\mathbf{C}\right)\\ & \cdot\mathbf{H}^{(u)}_{\mathrm{c}}\breve{\mathbf{s}}^{(u)}.\end{aligned}$$

The first term is equal to zero due to the orthogonality of the users in frequency domain.

The filtered interference to user 1 caused by the other $U-1$ users can be calculated as

$$\begin{aligned}\mathbf{m}^{(1)} \approx\ & \mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\sum_{u=2}^{U}\mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\left(\mathbf{I}_{N_{\mathrm{R}}}\otimes\mathbf{C}\right)\\ & \cdot\mathbf{H}^{(u)}_{\mathrm{c}}\breve{\mathbf{s}}^{(u)}\\ =\ & \mathbf{E}^{(1)}\mathbf{D}_{\mathrm{FFT}}\sum_{u=2}^{U}\mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\left(\mathbf{I}_{N_{\mathrm{R}}}\otimes\mathbf{C}\right)\\ & \cdot\left(\mathbf{D}_{\mathrm{FFT}}\right)^{-1}\mathbf{D}_{\mathrm{FFT}}\mathbf{H}^{(u)}_{\mathrm{c}}\breve{\mathbf{s}}^{(u)}.\\ =\ & \mathbf{E}^{(1)}\sum_{u=2}^{U}\mathbf{D}_{\mathrm{FFT}}\mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\left(\mathbf{I}_{N_{\mathrm{R}}}\otimes\mathbf{C}\right)\\ & \cdot\left(\mathbf{D}_{\mathrm{FFT}}\right)^{-1}\mathbf{\Lambda}^{(u)}\mathbf{M}^{(u)}_{\mathrm{T}}\mathbf{D}^{(u)}_{\mathrm{DFT}}\mathbf{d}^{(u)}.\end{aligned}$$

Of special interest is now the term

$$\mathbf{D}_{\mathrm{FFT}}\mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\left(\mathbf{I}_{N_{\mathrm{R}}}\otimes\mathbf{C}\right)\left(\mathbf{D}_{\mathrm{FFT}}\right)^{-1},$$

which realizes a cyclic convolution in the frequency domain, on each of the receiver branches. Thereby, it is responsible for the interference between user 1 and the others.

The effect is equal for all receive branches, so we only have to consider the cyclic convolution for a single receive branch

$$\mathbf{B}^{(1,u)} = \mathbf{F}_{N_{\mathrm{FFT}}}\mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\mathbf{C}\left(\mathbf{F}_{N_{\mathrm{FFT}}}\right)^{-1}.$$

Taking a closer look to the elements of $\mathbf{B}^{(1,u)}$, we can write

$$\left[\mathbf{B}^{(1,u)}\right]_{i,j} = \mathrm{j}2\pi\Delta\Omega^{(1,u)}_{\mathrm{CFO}}\sum_{k=1}^{N_{\mathrm{FFT}}}\sum_{l=1}^{N_{\mathrm{FFT}}}[\mathbf{F}]_{i,l}\mathbf{C}_{l,k}[\mathbf{F}]^{-1}_{k,j}.$$

Since,

$$
\begin{aligned}
[\mathbf{F}]_{i,j} &= \frac{1}{\sqrt{N_{\mathrm{FFT}}}} \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N_{\mathrm{FFT}}}(i-1)(j-1)}, \\
\left[\mathbf{F}^{-1}\right]_{i,j} &= \frac{1}{\sqrt{N_{\mathrm{FFT}}}} \mathrm{e}^{\mathrm{j}\frac{2\pi}{N_{\mathrm{FFT}}}(i-1)(j-1)}, \text{ and} \\
[\mathbf{C}]_{i,i} &= i,
\end{aligned}
$$

we can get

$$
\left[\mathbf{B}^{(1,u)}\right]_{i,j} = \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \frac{1}{N_{\mathrm{FFT}}} \sum_{k=1}^{N_{\mathrm{FFT}}} k \mathrm{e}^{\mathrm{j}\frac{2\pi}{N_{\mathrm{FFT}}}(k-1)(j-i)}.
$$

In the case $j = i$, we obtain

$$
\begin{aligned}
\left[\mathbf{B}^{(1,u)}\right]_{i,i} &= \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \frac{1}{N_{\mathrm{FFT}}} \sum_{k=1}^{N_{\mathrm{FFT}}} k \\
&= \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \frac{(N_{\mathrm{FFT}}+1)}{2}.
\end{aligned}
$$

Here, we deployed $\sum_{k=1}^{N_{\mathrm{FFT}}} k = \frac{N_{\mathrm{FFT}}(N_{\mathrm{FFT}}+1)}{2}$.

In the case where $j \neq i$, we get by setting $l = (j-i)$ and by substituting $x = \mathrm{e}^{\mathrm{j}\frac{2\pi}{N_{\mathrm{FFT}}}l}$

$$
\begin{aligned}
\left.\left[\mathbf{B}^{(1,u)}\right]_{i,j}\right|_{j\neq i} &= \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \frac{1}{N_{\mathrm{FFT}}} \sum_{k=1}^{N_{\mathrm{FFT}}} k x^{k-1} \\
&= \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \frac{1}{N_{\mathrm{FFT}}} \sum_{k=1}^{N_{\mathrm{FFT}}} \left(x^k\right)' \\
&= \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \frac{1}{N_{\mathrm{FFT}}} \left(\sum_{k=1}^{N_{\mathrm{FFT}}} \left(x^k\right)\right)' \\
&= \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \frac{1}{N_{\mathrm{FFT}}} \left(\frac{x - x^{N_{\mathrm{FFT}}+1}}{1-x}\right)' \\
&= \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \frac{1}{N_{\mathrm{FFT}}} \frac{1}{(1-x)^2} \\
&\quad \cdot \left[1 - (N_{\mathrm{FFT}}+1)x^{N_{\mathrm{FFT}}} \right. \\
&\quad \left. + N_{\mathrm{FFT}} x^{N_{\mathrm{FFT}}+1}\right].
\end{aligned}
$$

Here, we applied the first derivation of the geometric series.

By re-substituting $x = \mathrm{e}^{\mathrm{j}\frac{2\pi}{N_{\mathrm{FFT}}}l}$, and because of $\mathrm{e}^{\mathrm{j}2\pi l} = 1$, we can write

$$
\begin{aligned}
\left.\left[\mathbf{B}^{(1,u)}\right]_{i,j}\right|_{j\neq i} &= \frac{1}{N_{\mathrm{FFT}}} \sum_{k=1}^{N_{\mathrm{FFT}}} k \mathrm{e}^{\mathrm{j}\frac{2\pi}{N_{\mathrm{FFT}}}(k-1)(j-i)} \\
&= \frac{1}{\mathrm{e}^{\mathrm{j}\frac{2\pi}{N_{\mathrm{FFT}}}(j-i)} - 1}.
\end{aligned}
$$

At the end, we obtain

$$
\left[\mathbf{B}^{(1,u)}\right]_{i,j} = \mathrm{j}2\pi\Delta\Omega_{\mathrm{CFO}}^{(1,u)} \cdot \begin{cases} \frac{1}{\mathrm{e}^{\mathrm{j}\frac{2\pi}{N_{\mathrm{FFT}}}(j-i)} - 1}, & j \neq i \\ \frac{N_{\mathrm{FFT}}+1}{2}, & j = i \end{cases}.
$$

The elements of $\left[\mathbf{B}^{(1,u)}\right]_{i,j}$ can be seen as interference factors between sub-carrier i of user 1 and sub-carrier j of user u. Nevertheless, the sub-carriers must be occupied by the users to cause interference.

In order to include the occupation of the sub-carriers, we form the matrix

$$
\breve{\mathbf{B}}^{(1,u)} = \left(\mathbf{M}_0^{(1)}\right)^{\dagger} \mathbf{B} \mathbf{M}_0^{(u)},
$$

which includes the sub-carrier mapping of user u performed by the matrix $\mathbf{M}_0^{(u)}$ and the sub-carrier de-mapping for user 1 performed by $\left(\mathbf{M}_0^{(1)}\right)^{\dagger}$. This basically deletes all rows and columns of the matrix which are corresponding to unoccupied sub-carriers. Thereby, each element $\left[\breve{\mathbf{B}}^{(1,u)}\right]_{m,n}$ describes the interference factor of the n-th occupied sub-carrier of user u to the m-th occupied sub-carrier of user 1.

Fig. 5 shows in an two user example, how the interference power caused by the sub-carriers of user u contribute to a single sub-carrier of user 1.

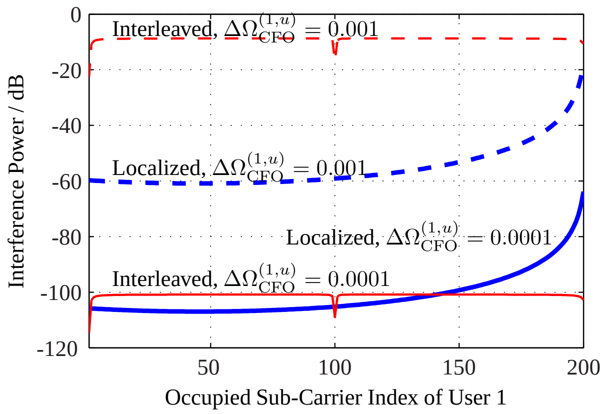

Fig. 5. Interference Power seen by the Sub-Carriers of the User 1. Parameters: $U = 2$, $N_{\mathrm{FFT}} = 512$, $N_{\mathrm{DFT}}^{(u)} = 200$, Zeroed DC Carrier.

In case of LFDMA user 1 sees significant interference power near to the border between the two user locations in the spectrum, whereas IFDMA result in a almost constant interference power for all sub-carriers of user 1. If the remaining CFO gets larger, LFDMA has clearly an advantage over IFDMA. Whereas for a lower remaining CFO IFDMA seems to be preferable, but the interference power is than already so low, that it can be neglected.

Note that, in case of SC-FDMA the power of each occupied sub-carrier is not necessarily equal. Instead it is dependent on the DFT result of the transmitted data block, which has of course a random content. Also, we have to point out, that due to PAPR reduction methods (e.g. pulse-shaping per

LFDMA band, optimized sub-carrier assignment methods) the robustness of LFDMA and IFDMA to CFO can be enhanced. This will be considered in further studies.

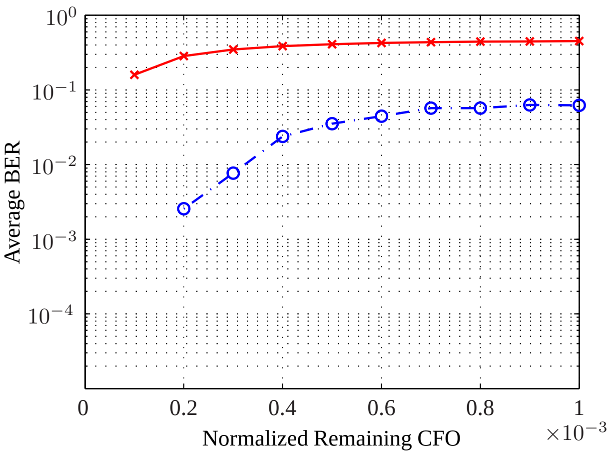

Fig. 6. Average BER of User 1 vs. Normalized Remaining CFO. Solid line: IFDMA, Dashed Line: LFDMA. Parameters: $U = 2$, Channel-Length: 120 taps, Channel Realizations: 100, $N_{\mathrm{FFT}} = 512$, $N_{\mathrm{DFT}}^{(u)} = 200$, Zeroed DC Carrier.

Fig. 6 shows the Average BER of the desired user over the normalized remaining CFO of user 2. The 2×2 MIMO SC-FDMA system employs 64-QAM modulation for each of the two users and a noise-free quasi-static Rayleigh fading channel. One can clearly see that LFDMA shows the better system performance.

4. CONCLUSION

Within this contribution we present a flexible mathematical framework in order to study MU MIMO SC-FDMA. We extended this framework towards multiple user-specific CFOs and studied the interference to an desired user for the two sub-carrier assignment strategies LFDMA and IFDMA, when a CFOC for the desired user is applied. The results show that LFDMA is more robust to carrier offset, since only the sub-carriers near to the border between local bands suffer significantly from interference. In case of IFDMA all sub-carriers of the desired user experience a significant interference, when the remaining CFO after the CFOC for the desired user is large enough.

5. REFERENCES

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