Challenges in Optical Compressive Imaging and Some Solutions

Adrian Stern, [1]Yair Rivenson and Yitzhak August

Department of Electro-Optical Engineering
[1]Department of Electrical and Computer Engineering
Ben-Gurion University of the Negev, Israel
{stern, rivenson, augusty} @bgu.ac.il

***Abstract*—The theory of compressive sensing (CS) has opened up new opportunities in the field of optical imaging. However, its implementation in this field is often not straight-forward. We list the implementation challenges that might arise in compressive imaging and present some solutions to overcome them.**

I. Introduction

Compressive sensing (CS) theory introduced a new paradigm for sampling and, subsequently, stimulated interest in its application in various fields. Imaging is a natural field for the implementation of CS theory because typical images involve a large amount of data, which facilitates efficient compression. Compressive imaging (CI) techniques were developed for various purposes, such as reducing hardware [1, 2], shortening image scanning time [1, 3], increasing image resolution [4-6] [7] and improving other imaging performance parameters [8]. CI techniques have been developed for motion tracking [9], spectral imaging [10] and holography. A review of CI techniques may be found in [11].

Principles of CI system design differ drastically from the principles used for conventional imaging. Conventional imaging seeks to perform isomorphic mapping; that is, to create images that are exact replica of the object. Ideally, each object point is mapped to a single pixel sensor so that, besides simple geometrical transformation (e.g., inversion), the captured image is a sharp copy of the object. In contrast, CS acquisition guidelines prescribe some way of mixing the information so that multiple image points are projected onto a single pixel sensor. The preferred projection is a random one so that all object points are randomly spread on the image sensors.

When coming to apply the CS framework for optical imaging and sensing one needs to consider the special characteristics of the optical data collection systems. In Sec. II we discuss the special issues and implementation limitations arising in the application of CS for optical imaging and sensing. The implementation limitations can be significantly reduced by intelligently compromising the guidelines for optimal universal CS. For instance, instead of using random projections one may use some kind of structured pseudo random projection scheme. Random convolution [12] is such an example. In subsections III A, B we present another two examples. The CI implementation challenges may also be bypassed if a specific-task system is to be designed. For example, if the task is to track motion in the scene, a technique as described in Sec. IIIC can be efficiently applied. Fortunately, there are also cases in which the optical sensing mechanism fits the CS guidelines well. Such a case is demonstrated in Sec. IV.

II. Special aspects of application of CS for imaging

Let us consider a conventional CS measurement scheme:

$$\mathbf{g} = \mathbf{\Phi f} \tag{1}$$

where the signal $\mathbf{f}$ is assumed to be k-sparse (or at least compressible) in a domain defined by the sparsifying operation $\mathbf{a} = \mathbf{\Psi f}$. For universal imaging tasks, $\mathbf{\Psi}$ should perform some random projections. In incoherent imaging $\mathbf{f} \in \mathbb{R}^N, \mathbf{g} \in \mathbb{R}^M$ and $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ while in coherent imaging $\mathbf{f} \in \mathbb{C}^N, \mathbf{g} \in \mathbb{C}^M$ and $\mathbf{\Phi} \in \mathbb{C}^{M\times N}$. In the following, we shall consider the particular features of the components of (1) in the context of optical imaging and sensing.

A. The input signal

In optical sensing, the input signal $\mathbf{f}$ represents the features of the "object", such as the spatial, spatio-temporal, spectral or polarimetic distributions of the electromagnetic field or of the radiant power. We shall list the special features of $\mathbf{f}$ and their consequences.

1) Sparsity: In most imaging scenarios, the object is indeed highly compressiable, as required for CS. For instance, 2D images in the visible may be compressible by a factor of $10-50$. 3D images and hyprespectral images may be even more compressible.

2) Physical representation dimensions: The object is typically represented as a 2D or 3D distribution. Therefore, in order to adjust to the matrix-vector formalism of (1) the signal is converted into the form of a vector by lexicographic ordering. By this, analytic and computational tools developed for (1) can be directly applied; however, part of the structural information is lost. For efficient implementation of CS one should attempt to employ the structural information intelligently in the sparsifying operator $\mathbf{\Psi}$ and by introducing appropriate priors in the reconstruction process.

3) Size: The signal **f** and measurements **g** are typically large. For example, in incoherent imaging in the visible, N can be easily of order of 10^7 and in multidimensional imaging (such as in 3D images and hyperspectral images) it can be much larger. Obviously, this leads to computational implications in terms of reconstruction speed.

4) Non-negativity: In incoherent imaging, the signal **f** is non-negative. For efficient CI, this fact should be considered in the reconstruction process by introducing appropriate constrains in the reconstruction problem or by working with centralized signals (with the average subtracted).

B. The System Matrix

1) Size of the matrix: The size of the system matrix is $M \times N$, where N and M may be of order of $10^5 - 10^7$. Therefore the size of the system matrix is huge, leading to the following significant challenges:
Computational - $\mathbf{\Phi}$ may require hundreds of Gigabytes of storage and the application of reconstruction algorithms with such large matrices is very difficult and time-consuming.
Optical realization - Realization of random $\mathbf{\Phi}$ requires building an imaging system with a space bandwidth product (SBP) larger than $M \times N$. In other words, the imaging system needs to have at least $M \times N$ almost independent modes, or degrees of freedom. It is not trivial to design a system with such a large SBP. For example, spatial light modulators that are commonly used in CI, have an SBP of $\mathcal{O}(N)$. Therefore, in order to realize $\times M$ times larger SBP, multiple measurements are required.
Optical Calibration - Sensing systems with a large SBP also require exhaustive and time-consuming calibration processes. In order to calibrate $\mathbf{\Phi}$, one needs to measure N point spread functions, each having M samples.

2) Non-negativity: In incoherent imaging, it is impossible to realize a system matrix $\mathbf{\Phi}$ with negative entries. This means that $\mathbf{\Phi}$ spans only the positive orthant. As a result, the mutual coherence of $\mathbf{\Phi}$ is lower, indicating lower compressibility. This problem may be addressed by applying preconditioning in the reconstruction process [13] or by doubling the number of measurements to generate measurements equivalent to that of a bipolar system matrix.

C. Measured signal

1) Size: Although the dimension of the measured image **g** is smaller than that of the signal **f** $(M < N)$, in typical CI systems it is still large. Therefore, similar computation issues as with **f** (see subsection II.A) are relevant for **g** too.

2) Realness and non-negativity: Optical sensors measure irradiance, which is real and non-negative. Negative and complex values can be measured indirectly, typically by acquiring multiple measurements. For example, in compressive holography [10] complex field amplitude is measured with temporal or spatial multiplexing.

3) Dynamic range: The dynamic range of optical sensors is typically limited. For example, conventional, uncooled optoelectronic sensors in the visible have a dynamic range of 8-12 bits. At longer wavelengths, the dynamic range may be even smaller. This may set significant limitations, particularly in incoherent imaging, where $\mathbf{\Phi}$ is no-negative.

III. Feasible sampling operators for optical CS

A. Separable Sensing Matrix

One way to alleviate the complexity associated with implementing CI systems with random projections is by designing sensing operators $\mathbf{\Phi}$ that are separable in the physical dimension of the optical signal [14, 15]. For instance, for capturing a typical 2D image, one may use a sampling operator that is separable in the x-y directions. Mathematically, such a sensing operator can be expressed by means of the Kronecker product of the sensing operators in each direction, $\mathbf{\Phi} = \mathbf{\Phi}_x \otimes \mathbf{\Phi}_x$. The sensing operators in each direction,$\mathbf{\Phi}_x, \mathbf{\Phi}_y$, can be designed to perform random projections.
The SBP of an $x - y$ separable $\mathbf{\Phi}$, is $\mathcal{O}(\sqrt{N \cdot M})$; thus the matrix storage requirements and the optical sensing complexity is reduced from $\mathcal{O}(N \cdot M)$ to $\mathcal{O}(\sqrt{N \cdot M})$. Employing separable $\mathbf{\Phi}$ can be useful also in the reconstruction step as it permits using block-iterative algorithms.
The price to be paid by using a separable sensing technique is in reducing the compressibility performance. For instance, a theoretical analysis in [14] showed that for 2D images, approximately $\sqrt{N}$ times more samples are needed to achieve similar performance as with a non-separable random system matrix. An empirical study in [16] showed more relaxed requirements, indicating that the minimum number of samples required for perfect recovery is $M \approx 1.25K \log(N/k + 1)$. Analysis of compressibility of signals separable in more than two dimensions may be found in [17].
Compressive imaging with a separable sensing operator has been demonstrated for 2D images [14, 16]. Recently, an optical scheme implementing *hyperspectral* imaging with a separable sensing operator was presented in [18].

B. Optical Radon Projections for Imaging

In [3], a CI technique is proposed that uses a cylindrical lens to perform a Radon projection of the object plane on a line array of sensors. The system performs a rotational scan to capture multiple Radon projections at various angles during the scanning process. By applying reconstruction algorithms based on ℓ_1 minimization, the image can be reconstructed from many fewer projections than are needed conventionally, e.g. with filtered back-projection algorithms.
The CI approach in [3] exhibits a very good trade-off between acquisition time and system complexity. Compared to the two other main CI approaches, it allows a much faster scan than with the "single pixel camera" [1], while, on the other hand, its implementation complexity is much lower than that of the "single shot compressive imaging camera" [4]. The imaging approach presented in [3] was further improved in [19], where it is shown that angular sampling with *golden angle* steps

allows progressive compressive image acquisition. Gradual improvement of the reconstructed image is obtained by adding new projections to the existing ones without re-sampling and recalculation. Each new measurement increases the quality of the previous reconstruction, as demonstrated in Fig. 1.

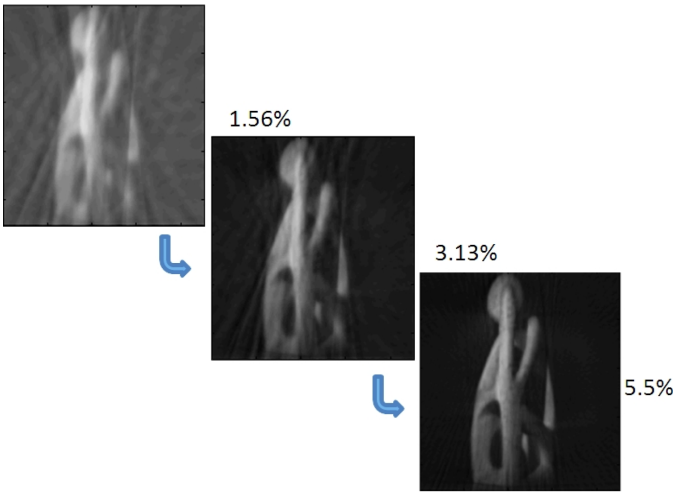

Fig. 1. Progressive compressive imaging with optical Radon projections, using obtained 1.56% (top), 3.13% (middle), 5.5% (bottom) of nominal samples (Nyquist). Image size 1280x1280 pixels.

The progressive compressive sensing approach is particularly useful when no prior knowledge about the required number of samples for good reconstruction is available. This means that the progressive Radon acquisition scheme is inherently adjustable to the type of the object imaged. The approach is also shown to be immune to sudden stopping of the scanning process, which otherwise would be intolerable with the uniform angular sampling scheme. An additional advantage of the approach is that it facilitates compressive imaging of large size images by employing ordered sets reconstruction algorithms on subsets of the data, thus remedying otherwise severe computation issues [19]. Note, for example, that the images in Fig. 1 are of megapixel size.

C. Optical Radon Projections for Motion Tracking

In the case that the task of the acquisition system is change detection or motion tracking, the signal is extremely sparse. Consider, for example, the task of tracking a point during 10 sec. with a temporal resolution of 20 milliseconds in a field of view of 1Megapixels. With conventional imagers, 500 Megapixels are acquired for this task, while here, the trajectory of the moving point can be described by only 500 pairs of Cartesian coordinates; thus $K/N = 0.5 \cdot 10^6$. Cartesian coordinates of moving objects can be obtained by measuring the temporal differences of two perpendicular Radon projections. As mentioned in Sec. IIIB, Radon projections can be obtained optically with anamorphic optical elements such as a cylindrical lens. Figure 2 depicts the concept behind change detection form two Radon projections. Consecutive temporal projections are subtracted from one another, indicating the projected location of the changes [Fig. 2 (c) and (d)]. Then the projections may be back projected to give the location of the changes on a Cartesian grid. Since the signal is extremely sparse, ℓ_1 minimization algorithms are particularly efficient.

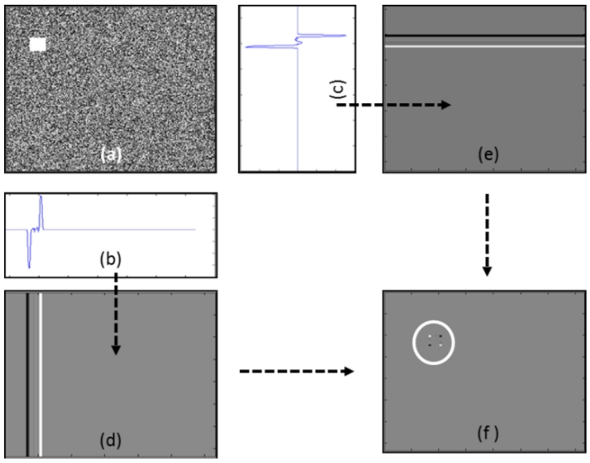

Fig. 2. Motion detection with two projections. (a) Original frame out of 2 consecutive frames; (b,c) difference between projection of two consecutive frames; (d,e) back projection of the frame difference; (f) intersection of the x,y back projections. The detected object is marked with white circle.

In practice, two projections are insufficient for detecting multiple moving objects in arbitrary directions. At least three projections are necessary to track objects moving in an arbitrary direction. In [9] we developed an optical system that essentialy perform, uses a superposition of four projections. Simulative experiments in [9] show that this system is able to track up to ten moving object points. Real experiments showed that objects can be tracked within a field of view of 500×500 pixels with approximately 250 times less samples than a conventional camera takes for the same task.

IV. Natural Optical Compressive Sensing Operators

There are cases in which the optical sensing operator fits the CS guidelines well. One such example is the free space propagation operator, described mathematically by the Fresnel transform. The Fresnel diffraction of the object field can be recorded by means of digital holography, which is found to be a physically realizable, quite simple and yet very efficient compressive sensing mechanism. Applying the CS paradigm for digital Fresnel holograms is attractive from the fact that the Fresnel and Fourier transforms are closely related. Therefore, Fourier subsampling schemes, studied extensively in CS literature, can be directly applied. In [20] it is shown that for a sufficiently large propagation distance the number of random samples in the hologram plane that is required for full reconstruction is $K \log N$, just like for the Fourier sensing case. Figure 3 shows an example of the dependence of the compressibility ratio M/N as a function of the imaging distance. From Fig. 3 it can be seen that the number of random Fresnel samples required to reconstruct the image exactly decreases with the imaging distance till it reaches an asymptote in the region where the Fresnel propagator behaves as a Fourier transform.

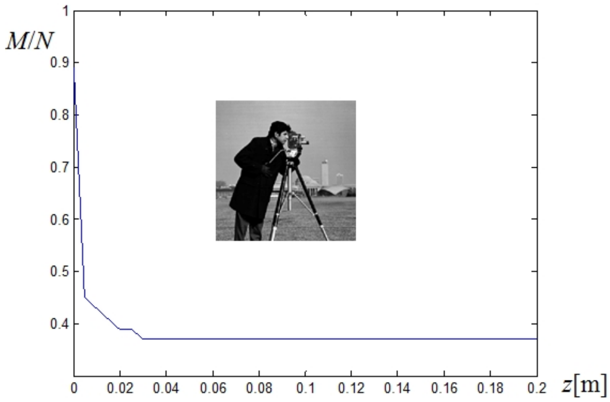

Fig. 3. Compressive sampling ratio required for full reconstruction of the Cameraman image (inset). M/N is the compressive sensing ratio and z is the recording distance.

For a recent review on compressive digital holography theory and applications the reader is referred to [10].

V. Conclusions

We have overviewed the characteristics of optical imaging that preclude straight-forward application of CS theory to imaging. In many cases, practical and physical limitations force the CI designer to deviate from basic CS guidelines. He has to compromise the randomness of the sensing operator required for universal CS by introducing some amount of structure. We presented two examples to demonstrate this.
The implementation limitations may be much less severe if a specific task is defined, as we have shown with our compressive motion detection and tracking system. In some particular cases, the particular optical sensing mechanism fits CS guidelines well. We have described compressive holography as an example of such a case.

Acknowledgment

Adrian Stern wishes to thank the Israel Science Foundation (grant No.1039/09) and Israeli Ministry of Science for supporting this research.

Reference

[1] D. Takhar, J. Laska, M. B. Wakin, M. F. Duarte, D. Baron, S. Sarvotham, K. Kelly and R. G. Baraniuk. A new compressive imaging camera architecture using optical-domain compression. Presented at Proc. IST/SPIE Symposium on Electronic Imaging. 2006,.

[2] C. F. Cull, D. A. Wikner, J. N. Mait, M. Mattheiss and D. J. Brady. Millimeter-wave compressive holography. Appl. Opt. 49(19), pp. 67-82. 2010.

[3] A. Stern. Compressed imaging system with linear sensors. *Opt. Lett. 32(21)*, pp. 3077-3079. 2007.

[4] A. Stern and B. Javidi. Random projections imaging with extended space-bandwidth product. *Journal of Display Technology 3(3)*, pp. 315-320. 2007.

[5] S. Gazit, A. Szameit, Y. C. Eldar and M. Segev. Super-resolution and reconstruction of sparse sub-wavelength images: Erratum. *Optics Express 17(25)*, pp. 23920-23946. 2009.

[6] Y. Shechtman, S. Gazit, A. Szameit, Y. C. Eldar and M. Segev. Super-resolution and reconstruction of sparse images carried by incoherent light. *Opt. Lett. 35(8)*, pp. 1148-1150. 2010.

[7] Y. Rivenson, A. Stern and B. Javidi. Single exposure super-resolution compressive imaging by double phase encoding. *Optics Express 18(14)*, pp. 15094-15103. 2010.

[8] R. Horisaki, K. Choi, J. Hahn, J. Tanida and D. J. Brady. Generalized sampling using a compound-eye imaging system for multi-dimensional object acquisition. *Opt.Express 18(18)*, pp. 19367-19378. 2010.

[9] Y. Kashter, O. Levi and A. Stern. Optical compressive change and motion detection. Appl. Opt. 51(13), pp. 2491-2496. 2012.

[10] Y. Rivenson, A. Stern and B. Javidi, ”An overview of compressive sensing techniques applied in holography,” *Appl. Opt., vol. 52*, pp. A423-A432, 2013.

[11] R. M. Willett, R. F. Marcia and J. M. Nichols. Compressed sensing for practical optical imaging systems: A tutorial. *Optical Engineering 50pp*. 072601. 2011.

[12] J. Romberg. Compressive sensing by random convolution. *SIAM Journal on Imaging Sciences 2(4)*, pp. 1098-1128. 2009.

[13] A. M. Bruckstein, M. Elad and M. Zibulevsky. On the uniqueness of nonnegative sparse solutions to underdetermined systems of equations. *Information Theory, IEEE Transactions on 54(11)*, pp. 4813-4820. 2008.

[14] Y. Rivenson and A. Stern. Compressed imaging with a separable sensing operator. Signal Processing Letters, *IEEE 16(6)*, pp. 449-452. 2009.

[15] M. F. Duarte and R. G. Baraniuk. Kronecker compressive sensing. *Image Processing, IEEE Transactions on 21(2)*, pp. 494-504. 2012.

[16] Y. Rivenson and A. Stern. Practical compressive sensing of large images. Presented at Digital Signal Processing, 2009 16th International Conference on. 2009,.

[17] Y. Rivenson and A. Stern. An efficient method for multi-dimensional compressive imaging. Presented at Computational Optical Sensing and Imaging. 2009,.

[18] Y. August, C. Vachman, Y. Rivenson and A. Stern, ”Compressive hyperspectral imaging by random separable projections in both spatial and spectral domains,” Submitted for Publication,.

[19] S. Evladov, O. Levi and A. Stern. Progressive compressive imaging from radon projections. *Optics Express 20(4)*, pp. 4260-4271. 2012.

[20] Y. Rivenson and A. Stern. Conditions for practicing compressive fresnel holography. *Opt. Lett. 36(17)*, pp. 3365-3367. 2011.