Calibration—An open challenge in creating practical computational- and compressive-sensing systems

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***Abstract*—The goal of this manuscript (and associated talk) is not to present any recent experimental results from my laboratory. Rather, the purpose is to elucidate why I believe that *calibration* is one of the few remaining significant challenges in the struggle to create a wide range of practical computational sensing and compressive sensing (CS) systems. Toward this end, I briefly describe the fundamental and implementation difficulties associated with calibration as well as the existing calibration approaches and their associated limitations before sketching the theoretical question that must be addressed in order to solve the calibration challenge.**

I. Introduction

Computational sensing is the general term for a sensing approach in which estimation of the input signal $\mathbf{x}$ proceeds from a set of measurements $\mathbf{y}$ that result from the action of a linear measurement operator $\mathbf{H}$ (including the possibility of potential noise corruption). The specific form of the measurements depends on the physical nature of the system and the noise. For example $\mathbf{y} = \mathbf{H}\mathbf{x}+\mathbf{n}$ is the appropriate form for an optical system with post-measurement additive noise $\mathbf{n}$, while $\mathbf{y} = \mathbf{H}(\mathbf{x}+\mathbf{n})$ is the appropriate form for an RF system with pre-measurement additive noise (such as that which arises at the input to the first-stage amplifier).

Regardless of the specific form, for traditional isomorphic sensor systems operating with impulse-like sampling, the measurement operator (matrix) $\mathbf{H}$ is the identity matrix $\mathbf{I}$. Computational sensing then generalizes this to consider sensor systems that implement measurement matrices $\mathbf{H}$ that have *non-zero off-diagonal elements*. In this manner, the measurements $\mathbf{y}$ become *multiplexed* and estimation of $\mathbf{x}$ becomes a non-trivial inverse problem. In this picture, *compressive sensing* can then be described as a subset of computational sensing where the sensing matrix $\mathbf{H}$ not only has off-diagonal elements, but is also rectangular with fewer rows than columns. Thus, the number of acquired measurements in $\mathbf{y}$ is less than the number of native signal elements in $\mathbf{x}$.

A. The Importance of Calibration

Solution of the inverse problem—that is, estimation of the input signal $\mathbf{x}$ from the measurements $\mathbf{y}$ requires knowledge of the measurement matrix $\mathbf{H}$. While the measurement system will have been designed to implement a specific measurement matrix $\mathbf{H}_{\text{des}}$, experimental reality dictates that the implemented matrix $\mathbf{H}_{\text{imp}}$ will deviate from the design to some extent. An example of the possible deviation between $\mathbf{H}_{\text{des}}$ and $\mathbf{H}_{\text{imp}}$ for an experimental system is shown in Fig. 1. For reasons outlined below, high-quality recovery of $\mathbf{x}$ is frequently sensitive to these variations. Determining the actual form of $\mathbf{H}_{\text{imp}}$ is then the role of *calibration*.

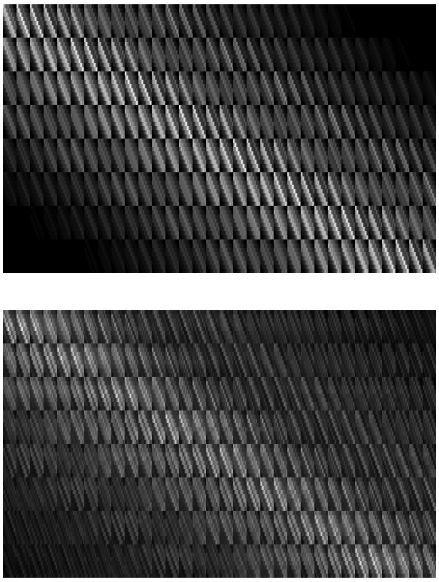

Fig. 1. The designed (top) and as-implemented (bottom) measurement matrices for an experimental compressive tracking system [1]. The implemented version of the matrix is an estimate created via a calibration process.

The sensitivity of system performance with respect to small deviations between $\mathbf{H}_{\text{des}}$ and $\mathbf{H}_{\text{imp}}$ can be understood by considering the multiplex nature of the measurement matrix. As mentioned above, the distinguishing feature of computational and compressive sensing approaches is that their measurement matrices contain non-zero off-diagonal elements. As a result, multiple signal elements are multiplexed together in each of the measurements. In cases where the input signal is dense in the native basis, this has the effect of encoding information about the input signal $\mathbf{x}$ into the variations of $\mathbf{y}$ about its mean.

This mean value (or *baseline*) frequently utilizes a significant fraction of the available system dynamic range, limiting the dynamic range available for the variations—where the information about $\mathbf{x}$ is encoded. An example is shown schematically in Fig. 2. This issue is analogous to the *interfero-*

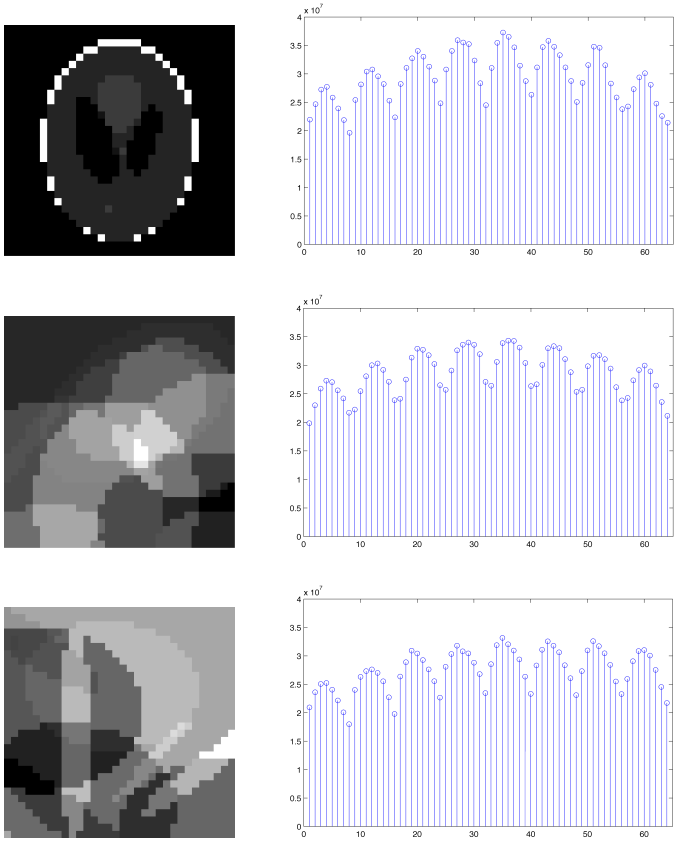

Fig. 2. An example of the multiplexing baseline. (Left) Three phantoms of size 32×32 composed of superposed partially-transparent ellipses. The mean value of the three phantoms is the same. (Right) Plots of the measurements produced by the phantoms when sampled with the measurement matrix in Fig. 1 (bottom). Information regarding the structure of the phantoms undergoes dynamic range compression and is encoded in the variations about the measurement mean.

metric baseline problem that arises in interferometric systems and is the primary manifestation of the so-called *multiplex disadvantage*.

It is true that the effect is mitigated somewhat in systems which are sparse in the native basis. However, practical situations that are natively sparse are rare (hence the need for the sparsifying transform in the majority of compressive sensing implementations).

The net effect of the dynamic range compression that results from the multiplex measurement that converts $\mathbf{x}$ into $\mathbf{y}$ is that accurate reconstruction becomes more sensitive to the specific form of $\mathbf{H}$ that encoded the measurements—thus driving the preference for $\mathbf{H}_{\text{imp}}$ (which is determined via calibration) over $\mathbf{H}_{\text{des}}$ (which is known from the intended system design).

B. Difficulties of Direct Calibration

The most direct calibration approach is what we might term *point-by-point*, and is effectively a Green's function (shift-variant impulse response) approach. The experimenter sequentially energizes each of the individual signal elements with unit amplitude. For each, the system response is recorded and then placed sequentially as the columns of a matrix. Once every signal element has been probed in this manner and the results integrated into the matrix, the result is an estimate of true measurement matrix $\mathbf{H}_{\text{imp}}$.

Although the point-by-point calibration approach is admirably direct, there are a number of potential difficulties that limit its practicality:

1) **Signal response too weak:** Energizing a single signal element at a time may produce a system response that is swamped by noise. Here we directly encounter the fact that *calibration is itself a measurement process*. Specifically, direct calibration estimates the measurement matrix $\mathbf{H}_{\text{imp}}$ via a traditional isomorphic approach where the estimate of each column of $\mathbf{H}_{\text{imp}}$ is taken as the measured system response for the corresponding single-element excitation. Any measurement noise is directly imposed on the estimate and may be non-negligible.
2) **Too many signal elements:** As computational and compressive sensing is applied to broader ranges of systems, the dimensionality (number of native signal elements) continues to grow. In the most advanced systems, the number of signal elements is of such a size that direct calibration is no longer practical—the time required is either beyond the patience of the experimentalist, or is on a timescale that is comparable with the timescales over which the measurement matrix $\mathbf{H}_{\text{imp}}$ varies (e.g. via thermal drift). For example, a compressive spectral imaging system under construction in my laboratory [2] has $\approx 8.4 \times 10^6$ native signal elements. Making the (optimistic) assumption that the apparatus will allow direct calibration at a rate of 10 Hz, we see that direct calibration would complete in just under 10 days of continual operation!
3) **Lack of desired control:** The direct approach requires the ability to isolate single signal elements and to control their amplitude. It is frequently the case that the experimentalist does not have a source that provides this level of control. For example, in a computational spectroscopic application, a tunable, narrowband spectral source may not be available. Instead, the experimenter may have access to a discrete set of spectral sources, each with a unique spectral profile that is a linear combination of the individual spectral channels.

II. Existing Non-Direct Methods and Their Limitations

The realization (mentioned above) that calibration is itself a measurement process potentially provides the key to overcoming the pitfalls inherent in direct calibration. Direct calibration represents a traditional, isomorphic measurement approach to estimating the measurement matrix $\mathbf{H}_{\text{imp}}$. That is, if we imagine lexicographically reordering the elements of $\mathbf{H}_{\text{imp}}$ into a vector $\mathbf{H}_{\text{imp,vec}}$, the calibration process utilizes a measurement matrix $\mathbf{\Phi}_{\text{cal}}$ to capture the calibration measurements. For an optical system with post-measurement additive noise, this would have the form $\mathbf{y} = \mathbf{\Phi}_{\text{cal}}\mathbf{H}_{\text{imp,vec}} + \mathbf{n}$, with the

obvious extensions to other measurement models. For direct calibration, $\mathbf{\Phi}_{\text{cal}}$ is the identity matrix $\mathbf{I}$.

As with the original measurement problem, however, we can apply computational or compressive sensing ideas to the calibration process and consider more general forms of $\mathbf{\Phi}_{\text{cal}}$. There are a number of existing calibration approaches that make this generalizing step. The following subsections describe these approaches, their benefits with respect to direct calibration, and their limitations.

A. Multiplexed Calibration

Multiplexed calibration simultaneously energizes multiple signal elements for each measurement in the calibration process, resulting in a $\mathbf{\Phi}_{\text{cal}}$ that contains non-zero off-diagonal elements [3]–[5]. Estimation of $\mathbf{H}_{\text{imp,vec}}$ then proceeds through the solution of an inverse problem. If the column rank of the resulting $\mathbf{\Phi}_{\text{cal}}$ is equal to the number of native signal elements, then traditional algorithms can be brought to bear to yield the estimate. If the column rank is smaller than the number of native signal elements, compressive sensing techniques are more appropriate.

The plausibility of compressive methods for determining $\mathbf{H}_{\text{imp,vec}}$ can be understood by examining the structure of measurement matrices such as Fig. 1 (bottom) and noting the large degree of structure present. Obviously, this structure is a form of redundancy that indicates that $\mathbf{H}$ is fundamentally a lower-dimensional object than the native number of matrix elements would suggest. Note that this argument would not hold for *random* measurement matrices (although structure imposed as a result of implementation deviations would provide some reduction in the dimensionality)—a severe downside to random measurement in extremely high-dimensional systems.

1) Advantages:

- Multiplex calibration combines signal elements in every measurement. For systems dominated by *additive noise*, this reduces the impact of the noise, increasing the measurement SNR.
- *Compressive* multiplexed calibration—where the column rank of $\mathbf{\Phi}_{\text{cal}}$ is less than the number of native signal elements—reduces the number of measurement acquisitions and hence the required calibration time. This may prove helpful in situations where direct calibration is unfeasible as a result of the number of signal elements.

2) Drawbacks:

- Multiplex-based improvement in measurement SNR does not occur in systems that are *Poisson (shot) noise* limited. The mean SNR of such measurements remains constant upon multiplexing.
- Estimation via solution of the inverse problem results in *transform noise*—The total noise in the measurements is redistributed among the estimated signal elements in ways that can radically modify the noise statistics. For example in Poisson noise limited systems, noise is preferentially redistributed from strong to weak areas of the signal. This produces sub-Poisson noise statistics in the strong signal areas and super-Poisson statistics in the weak signal areas. Transform noise also frequently introduces correlations between the noise present at different signal elements, creating the appearance of structure when none is truly present. This redistribution results in errors in the estimated $\mathbf{H}_{\text{imp,vec}}$ that can potentially impact system performance.

B. Matrix Completion

A closely-related approach applies the techniques of *matrix completion* [6]–[8] to the problem. In this approach, assumptions regarding the low-rank nature of the measurement matrix $\mathbf{H}_{\text{imp}}$ allow its full structure to be estimated given knowledge of only a subset of its entries. In a recent paper, Vetterli et al. explore the use of matrix completion methods to the calibration problem in ultrasound imaging [9] and obtain promising results.

1) Advantages:

- The method is well-matched to the central task at hand—estimating a low-rank (structured) matrix from a set of possibly incomplete calibration measurements. In some cases this would allow the experimenter to achieve an accurate estimation of $\mathbf{H}_{\text{imp}}$ from a reduced number of calibration measurements and hence shorten the required calibration time.

2) Drawbacks:

- Matrix completion is generally posed in the context of *missing entries* that are distributed randomly throughout the matrix (see [9] for an example). This is suitable for systems where the output state of the system must be *sequentially acquired* in order to determine the full system response to a given calibration input. For systems where the output measurements are acquired in parallel, however, *skipping a calibration step* (to shorten the calibration time) would result in missing entries that are not arranged randomly throughout $\mathbf{H}_{\text{imp}}$, but rather are organized in *columns*, and existing algorithms perform poorly in this situation. Performing matrix completion after a basis change to redistribute the missing entries may possibly restore performance, but I am not aware of any work in that area.
- In their current form, matrix completion methods assume elements of the matrix are known in the native basis—implying single element excitation. This suffers from the same potential SNR issues as direct calibration. The previously mentioned idea of performing matrix completion after a basis change, should such an approach prove viable, would allow (require) multiplexed excitation. This would increase measurement SNR for cases which are limited by additive noise.

C. Parameterized Forward Model

The final (and most common) method of calibration is to create a *parameterized forward model*. In essence, it seeks to estimate $\mathbf{H}_{\text{imp}}$ through the creation of a more sophisticated $\mathbf{H}_{\text{des}}$. The system model is extended to include possible errors that could arise during implementation and the magnitude of

these errors are incorporated as *adjustable parameters*. Calibrating such a system is then a matter of determining certain experiments (excitation patterns) which reveal the appropriate magnitudes for these parameters.

1) Advantages:

- A well-developed parameterized forward model is extremely powerful. It incorporates a significant amount of prior knowledge regarding the intended structure of $\mathbf{H}_{\text{des}}$ as well as the physics of the likely effects that transform $\mathbf{H}_{\text{des}}$ into $\mathbf{H}_{\text{imp}}$. The resulting number of parameters captures the underlying dimensionality of $\mathbf{H}_{\text{imp}}$ with admirable efficiency.

2) Drawbacks:

- The parameterized forward model approach *trades* calibration acquisition time for model development time. The net benefit of this trade-off (if any) depends on the skill and insight of the person developing the model.
- Model mismatch is a serious concern; the model only incorporates terms that are explicitly included. Deviations between $\mathbf{H}_{\text{des}}$ and $\mathbf{H}_{\text{imp}}$ that arise from implementation errors that are not included in the model will not be captured during the calibration process.
- In advanced, high-dimensional systems, the number of necessary parameters can proliferate quickly (in correspondence with the increasing dimensionality of the underlying measurement structure of the instrument). The resulting models can become unwieldy and design of experiments to isolate the values of individual parameters may become difficult or impossible.

III. What is Needed

Although there are a number of non-direct calibration methods now in use, each has its own unique balance of advantages and drawbacks and none of the methods is ideal. In this section, I attempt to describe the properties that an ideal calibration approach would have and the theoretical questions that must be addressed in order to develop such an approach.

Over the past several years, there has been an evolution in the field of compressive sensing that emphasizes a move from random to designed sensing matrices. The various design strategies incorporate prior information regarding the statistical distributions of likely input signals, the nature of the sensing task, and the reconstruction/estimation algorithms that will be brought to bear. This design is then performed subject to the constraints of both physics and system architecture. The ideal calibration framework would provide a similar level of design by identifying the sequence of calibration measurements to be made subject to a variety of priors and constraints. Fundamental questions related to this goal include:

- What is the appropriate mathematical framework for the design of calibration sequences? Is there a mathematical reason to prefer the matrix, $\mathbf{H}_{\text{imp}}$ (matrix completion) or vector, $\mathbf{H}_{\text{imp,vec}}$ (computational/compressive sensing) form to the problem?
- Can constraints on the available input signals be incorporated in the design process? What if only a fixed set of inputs are possible?
- The experimenter will have general knowledge regarding the approximate form of $\mathbf{H}_{\text{imp}}$ (via knowledge of $\mathbf{H}_{\text{des}}$). How can this prior knowledge be incorporated into the design process?
- Clearly not all errors in estimating $\mathbf{H}_{\text{imp}}$ will be equivalent. The effect of specific errors is likely to depend on the ultimate *sensing task* of the system. How can prior information regarding this task be incorporated into the design of the calibration sequence? How can prior information regarding the likely input signals (in the course of the sensor task, not calibration) be incorporated into the design of the calibration sequence?
- Can the framework be made adaptive? Can the results of early stages in the calibration sequence influence the design of subsequent calibration measurements?
- Are there fundamental limits or guarantees that can be stated about designed calibration?

IV. Conclusion

Calibration is currently an open challenge with regards to developing advanced compressive and computational sensing systems. The fact that calibration is itself a measurement process provides a key opening through which to attack this problem. It is my hope that the rough ideas presented here can spark an engagement between the theoretical and experimental communities on this crucial issue.

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