SVD ALGORITHMS AND QUANTIZATION APPLIED TO MIMO $\max - d_{\min}$ BASED PRECODER

Ancuta MOLDOVAN[1], Ghadir MADI[2], Baptiste VRIGNEAU[2], Tudor PALADE[1], Rodolphe VAUZELLE[2]

[1] Technical University of Cluj-Napoca, Communications Department, 26-28 G. Baritiu St., 400027 Cluj-Napoca
firstname.lastname@com.utcluj.ro
[2] University of Poitiers, XLIM-SIC UMR-CNRS 6172, Bvd Marie et Pierre Curie, 86962 Futuroscope Cedex
firstname.lastname@univ-poitiers.fr

ABSTRACT

In multiple-input multiple-output (MIMO) systems, precoders design pose significant implementation challenges due to the high computational complexity of the singular value decomposition (SVD) of the channel matrix. In this paper, we use the iterative COordinate Rotation DIgital Computer (CORDIC) algorithm to compute the SVD for different configurations of multi-antenna systems. The complexity/performance trade-offs involved in the design of the $\max - d_{\min}$ *precoder are also considered.*

Keywords: *MIMO, singular value decomposition, CORDIC, quantization.*

1. INTRODUCTION

MIMO techniques have the potential to provide significant performance enhancement, by efficiently taking advantage of random fading and multipath propagation environment. Spatial multiplexing allows a high spectral efficiency by transmitting independent data streams simultaneously through multiple transmit antennas [1]. Further, if channel condition is known at the transmitter, linear precoders that optimize pertinent criteria can be design, [2]- [5], in order to improve the transmission scheme.

One issue in the precoders design, relies on the singular value decomposition of the MIMO channel matrix, a numerically sensitive iterative process that involves high computational complexity. Moreover, the information that describes the channel has to be processed in real-time and sent from the receiver to the transmitter over a reliable feedback channel. Hardware-optimized SVD algorithms, based on CORDIC arithmetic [6], can be used to perform the diagonalization of the complex channel matrix and the precoder computation.

On the other hand, in many practical systems, the assumption of full channel state information (CSI) at the transmitter is not realistic, because of the limited capacity of the feedback channel. The amount of information sent from the receiver to the transmitter can be reduced by using partial or quantized CSI [7]. In this case, the receiver chooses the precoding matrix from a finite cardinality codebook, designed off-line and known at both sides of the communication link. The challenges associated with this system model are the design of the codebook and the criterion to choose the optimal precoding matrix from the codebook.

In this paper, we use solutions from literature [8], [9] to perform the SVD, based on CORDIC, for different configurations of MIMO systems. We focus on the design of the $\max - d_{\min}$ precoder [10] and adjust the arithmetic precision, as a trade-off between the complexity of the computation and the loss in the bit error rate (BER). This premise represents the first step in the design of a new quantization scheme for the $\max - d_{\min}$ precoder. The aim is to compute the SVD and to give the parameters for the selection criterion inside the codebook.

2. SYSTEM DESCRIPTION

A. Channel model

We consider a MIMO system with n_T transmit and n_R receive antennas, over which b independent data streams are transmitted. If channel state information is available at the transmitter, the channel can be virtually diagonalized, as in [5], and the received signal is given by:

$$\mathbf{y} = \mathbf{G}_D \mathbf{H}_v \mathbf{F}_D \mathbf{s} + \mathbf{G}_D \mathbf{n}_v \qquad (1)$$

where $\mathbf{s}[b \times 1]$ is the vector of transmitted symbols, $\mathbf{H}_v = \mathbf{G}_v \mathbf{H} \mathbf{F}_v = \mathrm{diag}(\sigma_1, ..., \sigma_b)$ is the $[b \times b]$ virtual channel matrix obtained through SVD, where σ_i represents each subchannel gain, in a decreasing order, $\mathbf{n}_v = \mathbf{G}_v \mathbf{n}$ is an additive white Gaussian noise (AWGN) vector of size $b \times 1$, $\mathbf{F}_D$ and $\mathbf{G}_D$ are the precoder and the decoder of size $b \times b$. A power constraint has to be satisfied, the average transmit power being limited to E_T:

$$\mathrm{trace}(\mathbf{F}_D \mathbf{F}_D^*) = E_T \qquad (2)$$

The solution of the $\max - d_{\min}$ precoder $\mathbf{F}_D$, is given in [5], for two independent data streams, $b = 2$ and 4-QAM with a spectral efficiency of $4bits/s/Hz$.

B. Quantized CSI

The authors in [10] investigated the performances that are obtained if a limited-feedback unitary matrix is applied to the $\max - d_{\min}$ precoder. The quantization of the transmit beamforming vectors $\widetilde{\mathbf{F}}_v$ is based on [11], where the authors proposed a design method which maximizes the subspace distance between two codebook entries, a problem that relates directly to packing subspaces in the Grassmann manifold. The feedback information consists on the index of the selected unitary matrix from the codebook $\mathscr{F}_v$, and the channel angle, $\gamma = \tan^{-1}(\frac{\sigma_2}{\sigma_1})$, needed for the design of the $\max - d_{\min}$ precoder. It was shown that, a number of 8 quantization bits and a selection criterion based on a switch from a joint optimization of the max $-d_{\min}$ precoder and codebook to a unitary precoder and conversely, leads to performances very close to the theoretical ones.

Based on this approach, the first step in the design of a new quantization scheme for the max $-d_{\min}$ precoder, was to reduce the amount of information on the feedback channel. In the new codebook $\mathscr{F}$, each element is given by $\mathbf{F}_i = \mathbf{F}_{vi}\mathbf{F}_{di}$, where $\mathbf{F}_{vi}$ are the unitary matrices from $\mathscr{F}_v$ and $\mathbf{F}_{di}$ is the max $-d_{\min}$ precoder chosen in order to maximize the minimum Euclidean distance between the signal points at the receiver side.

The simulation shown that, with a 7 bits feedback information and the selection criterion based on maximizing the minimum distance, there is no performance loss compared to the results in [10]. The parameter that has to be sent to the transmitter is the index of the optimum selected pair inside the codebook. The major drawback of the quantization method is the demanding computational complexity of the selection criterion, influenced by the cardinality of the codebook and the computation of the minimum distance.

3. SVD ALGORITHMS

Since SVD algorithms are mainly based on Givens rotations [8], the use of CORDIC arithmetic allows easy computation of inverse tangents and vector rotations.

For the particular case of a 2×2 channel matrix with complex elements, a two steps diagonalization procedure, based on the Jacobi's method, was proposed in [9]. In the first step, a transformation performs the QR decomposition of the matrix, while the second one completes the diagonalization. In [12], the authors show that in MIMO-OFDM systems, the diagonalization of the arbitrary complex channel matrix can be efficiently implemented using multiple instantiations of low-area and hardware-efficient VLSI architecture. The singular value decomposition method is based on the Golub-Kahan algorithm (GK-SVD) [8] and it requires two phases in order to determine the singular values:

- Bidiagonalization

This phase consists in transforming the initial channel matrix $\mathbf{H}$ into a real-valued upper bidiagonal matrix $\mathbf{B}_0$, by alternately applying a sequence of Givens rotations. First, left-hand side (LHS) rotations are applied in order to eliminate the elements on the first column, followed by right-hand side (RHS) rotations to eliminate the first row, and so on, until the transformation is completed and $\mathbf{B}_0 = \widetilde{\mathbf{G}}_v\mathbf{H}\widetilde{\mathbf{F}}_v$. The resulting unitary matrices $\widetilde{\mathbf{G}}_v$ and $\widetilde{\mathbf{F}}_v$ are obtained by applying the same sequence of Givens rotations to the corresponding identity matrices.

- Diagonalization

To complete the decomposition, Givens rotations are applied from both sides to the bidiagonal matrix $\mathbf{B}_k$, such that all the off-diagonal entries t_i, with $i = 1,2,...,r-1$, converge to zero. This second phase consists of multiple diagonalization steps, repeated until the following constraint is satisfied:

$$|t_i| \leq 2^{-\varepsilon}(|d_i| + |d_{i+1}|) \tag{3}$$

where ε is the tolerance value and d_i are the diagonal elements that correspond to the unordered singular values.

In order to ensure convergence of the diagonalization phase and to reduce the SVD computation time, the solution given in [8] was to perform the first Givens rotation, in each diagonalization step, with a modified input vector. To complete the diagonalization, the unitary matrices are determined by applying the same sequence of Givens rotations to the corresponding identity matrices.

4. IMPACT OF SVD IMPLEMENTATION ON BER

Arithmetic precision optimization is essential when converting the floating-point model to a fixed-point model. Since the implementation of the SVD methods bases on CORDIC arithmetic, the number of CORDIC rotations and the fixed-point precision, must be determined as a trade-off between the computational complexity and the loss in the error rate performance. For the GK-SVD method, the tolerance in (3) and the maximum number of diagonalization steps are adjusted in order to simplify the diagonalization phase.

For the adjustment of the parameters involved in the arithmetic precision optimization, BER simulations were performed for each method considered in the computation of the singular values and of the max $-d_{\min}$ precoder. A Rayleigh flat-fading channel model with AWGN is assumed and, for the design of the precoder, the entire channel matrix can be made available at the transmitter.

4.1 (2,2) MIMO systems

For the simulation results provided in Figure 1, we considered a number of 8 fractional bits and a varying number of CORDIC rotations. It can be depicted that, for a (2,2) system, a minimum number of 4 CORDIC rotations does not affect the link reliability.

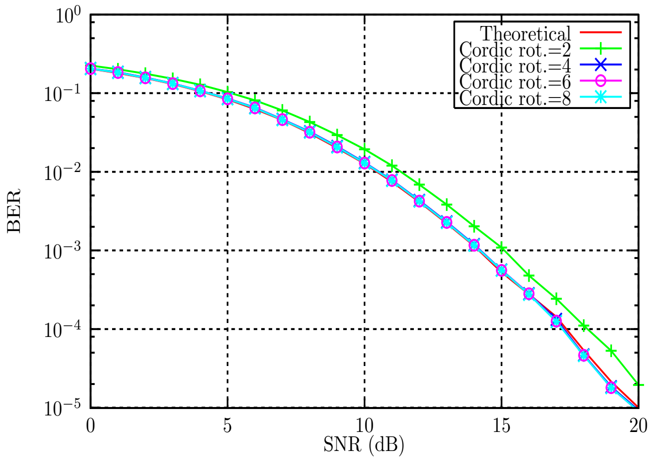

Figure 1*: Number of CORDIC rotations influence on BER in a (2,2) MIMO system.*

4.2 (2,4) MIMO systems

For this configuration, in order to reduce the complexity involved in the diagonalization of the channel matrix, we use a combination between the GK-SVD and the SVD for a 2×2 complex matrix. First, the initial channel matrix is converted to a bidiagonal matrix by applying Givens rotations, as in the first step of the GK-SVD procedure. To complete the diagonalization, the singular values are determined by performing the second phase from the 2×2 SVD method.

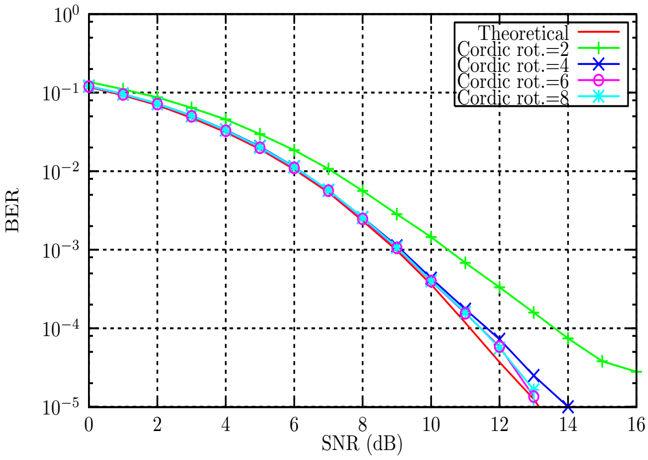

Figure 2*: Number of CORDIC rotations influence on BER in a (2,4) MIMO system.*

So, the arithmetic precision optimization also reduces at the adjustment of the number of CORDIC rotations and the number of fractional bits within the CORDIC. From the BER results provided in Figure 2, considering a number of 8 fractional bits, it can be concluded that a minimum number of 6 CORDIC rotations leads to no significant performance loss, compared to the optimal BER. Relatively to the (2,2) MIMO system, when we increase the number of received antennas from 2 to 4, for a BER=10^{-4}, the SNR gain is about 5dB, due to higher diversity order.

4.3 (4,4) MIMO systems

The singular value decomposition for a (4,4) MIMO system is performed based on the GK-SVD algorithm. The first step consists in the determination of the minimum tolerance value, as a trade-off between the computational complexity and the performance loss. The CORDIC floating-point model was considered and the simulation results have shown that a minimum tolerance value $\varepsilon = 3$ provides near optimal BER. For the fixed-point implementation, 16 fractional bits are required for an appropriate computation of the channel's singular values.

The next step in the arithmetic precision optimization is to determine the maximum number of diagonalization steps. The results depicted in Figure 3 show that a small number of steps $k = 3$, has the advantage of a reduced computational complexity, but leads to an error-rate degradation, without reaching the BER floor for the considered range, as it does when $k = 1$. The number of CORDIC rotations has a significant influence on the transmission performance, especially if low BER is required. From the simulation results in Figure 4 it can be concluded that, at least 10 CORDIC rotations must be performed, in order to avoid the BER floor.

It must be stated that, for a (4,4) max $-d_{\min}$ precoded MIMO system, the modifications applied to the GK-SVD algorithm, in order to reduce its complexity, lead to a degradation of BER. The SNR loss is about 1dB, compared to

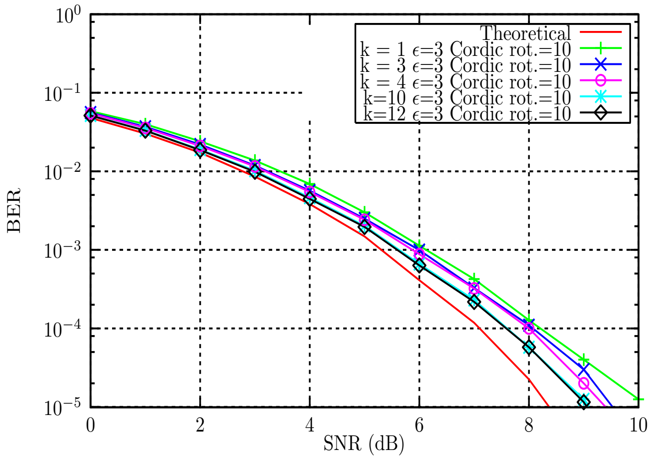

Figure 3*: Number of diagonalization steps influence on BER in a (4,4) MIMO system.*

the theoretical BER, for a BER=10^{-4}. For the same error performance, the SNR gain compared to the (2,4) and (2,2) systems, is near 4dB, respectiv 9dB.

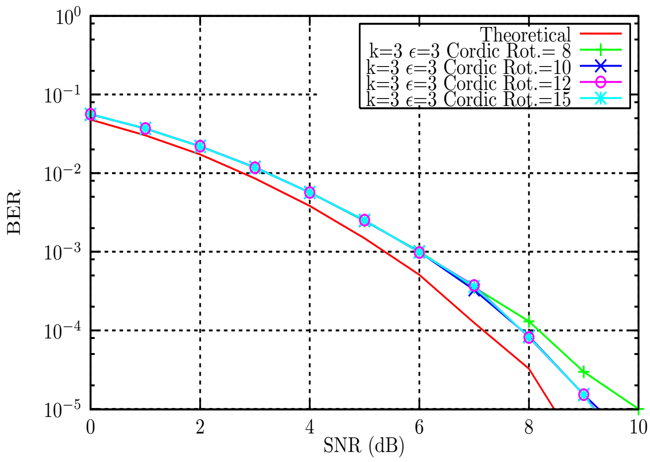

Figure 4*: Number of CORDIC rotations influence on BER in a (4,4) MIMO system.*

5. BER LOSS WITH LIMITED FEEDBACK

In this section, we will quantify the effect of quantization of the precoding matrix, with a finite number of bits, on the bit error rate, for different configurations of MIMO systems.

Figure 6 illustrates the BER simulations in a (2,2) MIMO system, with $b = 2$ and 4-QAM symbols. The theoretical assumption is that, the SVD is optimal, and that, for the design of max $-d_{\min}$ precoder, the entire channel matrix is available at the transmitter. As it can be depicted from the results, with the proposed quantization method and 7 bits on the feedback channel, there is a reduced BER loss. For a BER=10^{-4}, the SNR gain is near 0.5dB. If the SVD is performed with the CORDIC algorithm, with 6 rotations and 8 fractional bits for the fixed-point implementation, there is no added loss in the BER performance.

The same simulations were performed for a max $-d_{\min}$

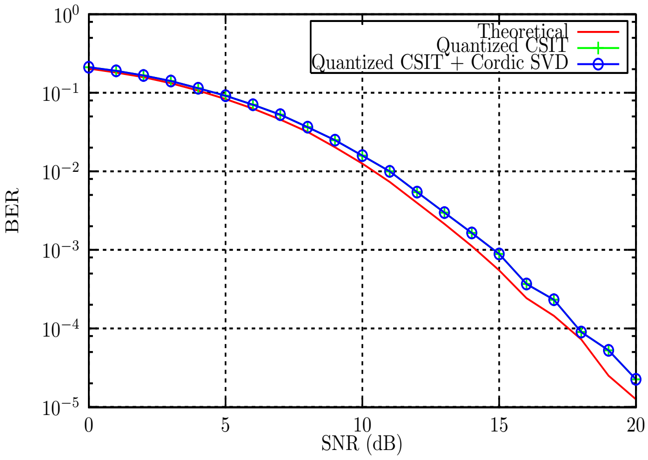

Figure 5: *Quantization and CORDIC loss in a (2,2) MIMO system.*

precoded (4,4) MIMO system, with $b=2$ and 4-QAM symbols. For the SVD implementation algorithm we consider: a tolerance value $\varepsilon=3$, $k=3$ diagonalization steps, 16 fractional bits and 10 CORDIC rotations. From the results in Figure 6, it seams that, in this case, the quantization leads to a SNR loss of almost 2dB, for a BER=10^{-4}. The degradation is even higher when the CORDIC algorithm in applied.

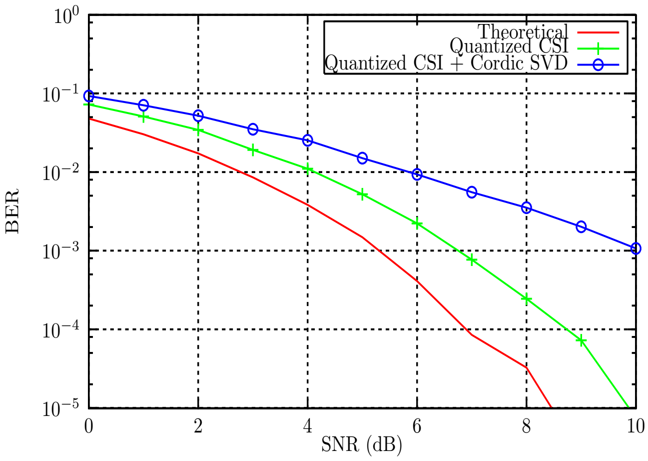

Figure 6: *Quantization and CORDIC loss in a (4,4) MIMO system.*

6. CONCLUSION

In this paper, we used the fixed-point model of the CORDIC algorithm to compute the SVD for different configurations of MIMO systems. The arithmetic precision optimization is a trade-off between the computational complexity and the loss in the BER, in a max $-d_{\min}$ based precoded system. We also made the first steps in the problem of quantization, associated with the max $-d_{\min}$ precoder, with finite-rate feedback. The maximization of the minimum Euclidean distance involves high computational complexity and the error performance are satisfactory only for MIMO systems with 2 transmit antennas. This requires consideration towards the development of a new quantization scheme, based on the parameters from the SVD computation.

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