ASYMMETRIC TURBO CODES FOR LTE SYSTEMS WITH MEDIUM FRAME LENGTH

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ABSTRACT

The selection of the component codes of turbo codes has a major influence on the performance of these codes. In this paper we propose a new type of asymmetric turbo codes for medium length of interleaver, based on the selection algorithm for the generating polynomials matched to a largest spread QPP interleaver. From the performance results one can observe that the new asymmetric turbo codes perform better than the existing ones.

Keywords: *asymmetric turbo codes, QPP interleaver, LTE standard*

1. INTRODUCTION

Berrou, Glavieux and Thitimajshima provided in [1] that the use of iterative decoding and parallel concatenated codes or turbo codes can realize an error performance close to the Shannon limit. In [1], for a constraint length of 5, with an interleaver length of 65536 and 18 decoding iterations, a bit error rate (BER) of 10^{-5} is obtained.

Usually, the turbo codes described by Berrou consist of two or more recursive convolutional codes parallel concatenated separated by an interleaver. In order to obtain an improved error performance, the interleaver plays a major role leading to higher weight and lower multiplicities of codewords. The component codes of turbo codes are also important for the code's improvement.

In [2], it has been observed that for these component codes (different generating matrix or different memory order), the results obtained by simulations are improved, especially for high SNR values ("error floor" region), compared to the symmetric turbo codes. This is the case of asymmetric turbo codes.

The use of non-identical component codes for LTE (Long Term Evolution) [3] standard has not been studied yet in detail. In this paper, we investigate the performance of the proposed asymmetric turbo codes and then we compare the results with those used in LTE standard.

The LTE standard required a turbo code which uses a QPP (Quadratic Permutation Polynomial) interleaver. This is an algebraic type of interleaver and for a length L it is defined as [4]:

$$\pi(x) = (q_0 + q_1 x + q_2 x^2) \bmod L, \quad x = \overline{0, L-1} \qquad (1)$$

where q_1, q_2 are chosen so that the quadratic polynomial in (1) is a permutation polynomial and q_0 leads to a cyclic shift of the permutation elements.

The spread factor is defined as [4]:

$$D = \min_{\substack{i,j \\ i,j \in Z_N}} \{\delta_L(p_i, p_j)\} \qquad (2)$$

where $\delta_L(p_i, p_j)$ is the Lee metric [5] between the points $p_i = (i, \pi(i))$ and $p_j = (j, \pi(j))$:

$$\delta_L(p_i, p_j) = |i - j|_L + |\pi(i) - \pi(j)|_L \qquad (3)$$

where

$$|i - j|_L = \min\{(i - j)(\bmod L), (j - i)(\bmod L)\} \qquad (4)$$

The spread of an interleaver influences the minimum distance of the resulted turbo code.

The paper is organized as follows: Section 2 presents the proposed asymmetric turbo codes, the performance bounds and the corresponding simulations. Section 3 presents simulation results of both typical and proposed turbo code system in LTE standard. Section 4 concludes the paper.

2. THE PROPOSED ASYMMETRIC TURBO CODES

In order to obtain an improved error performance for the asymmetric turbo codes, we need to focus either on the design of the interleaver or the component codes. The component codes influence the determination of the codeword weight. In literature there are some algorithms to choose the generating polynomials of the component codes, especially for the symmetric case. In this paper, for the selection of those component codes, we use an extended version of the algorithm proposed in [6].

Following on the same steps, we extend the search for medium length of interleaver, i.e. 160, 256 and 320. The resulted generating polynomials are matched with the proper largest spread QPP interleaver. These types of interleavers

L	**π(x)**	**D**
160	$19x + 40x^2$	16
256	$15x + 32x^2$	16
320	$19x + 40x^2$	20

Table 1: *Largest Spread QPP interleaver*

are discovered by Takeshita in [2] and presented in Table 1 along with the spread of interleaver. For calculation of distance spectra, we have used the Garello's algorithm [7].

For each length of the interleaver and for some combinations of memory order , different generating polynomials matched to the largest spread QPP interleaver were obtained. Table 2 presents a selection of polynomials obtained by applying the algorithm, along with the parameters d_{min} and d_{free}. *m1* and *m2* represent the memory order of the first and the second encoder and CC1 and CC2 the two recursive systematic convolutional codes.

L	*m1*	*m2*	CC1		CC2		d_{min}	d_{free}
			n(D)	*d(D)*	*n(D)*	*d(D)*		
160	3	3	15	13	13	15	21	14
			17	11	11	15	20	12
	3	4	17	15	33	25	21	14
			17	13	23	25	22	14
	4	4	37	23	23	25	23	18
256	2	3	5	7	17	11	18	10
	3	3	13	15	15	13	18	14
	3	4	17	11	23	25	20	12
	4	4	37	23	23	25	22	18
320	2	3	5	7	17	11	21	10
	3	3	17	13	13	17	20	12
			15	13	13	15	20	14
	3	4	11	15	37	23	22	18
	4	4	37	23	23	25	24	18jih

Table 2: *The obtained generating polynomials*

When both component codes have the memory order 4, one can observe from Table 3 that identical generating polynomials are obtained, irrespective of the interleaver length. Also, when both component codes have a memory order 3, an identical generating matrix is obtained.

Based on these theoretical specifications and the applied algorithm, we search new generating polynomials and simulate the system. The simulations were performed for some asymmetric turbo codes with 1/3 coding rate, without puncturing, on an AWGN channel (Additive White Gaussian Noise), using a BPSK (Binary Phase Shift Keying) modulation and a post-interleaver trellis termination. The decoding algorithm is Log-MAP (Logarithmic Maximum Aposteriori), the iteration stopping criterion is LLR (Logarithm Likelihood Ratio) module, and maximum numbers of iteration was fixed at 16.

The simulations have been accomplished for the inteleavers lengths 160, 256 and 360, using different combinations of memory order or generating polynomials. In order to emphases the improvement of the proposed asymmetric turbo codes, we compare them to some turbo codes proposed in [8].

In Figure 1 the simulation results for the length of the interleaver L=160 are presented. The two convolutional component codes have memory order 4 and the generating matrix is G=[37/23, 23/25], according to Table 2. The memory of the two convolutional component codes is equal to 4 and the generator matrix is G = [37/23, 23/25], according to Table 2.

To see the improved performance, we compare the new codes with the asymmetric turbo code having the generator matrix G = [35/23, 27/31]. We note that for higher values of SNR, the coding gain at BER of 10^{-7} is 0.4dB in case of BER. The FER performances are similar.

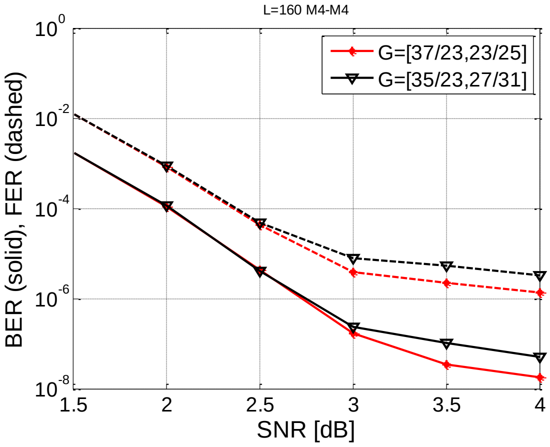

Figure 1: *BER and FER curves for the length of the interleaver L=160*

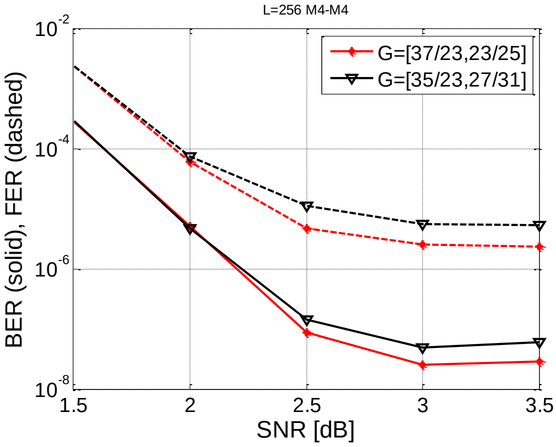

Figure 2: *BER and FER curves for the length of the interleaver L=256*

Figure 2 shows the comparison between the same two asymmetric turbo codes, with the difference that interleaver length is L = 256. Also in this case, the proposed turbo code results in better performance, the coding gain being 0.3 dB. For L = 320, the curves shown in Fig. 3 are obtained. The generator matrices are the same, but this time a higher improvement in BER curves is obtained.

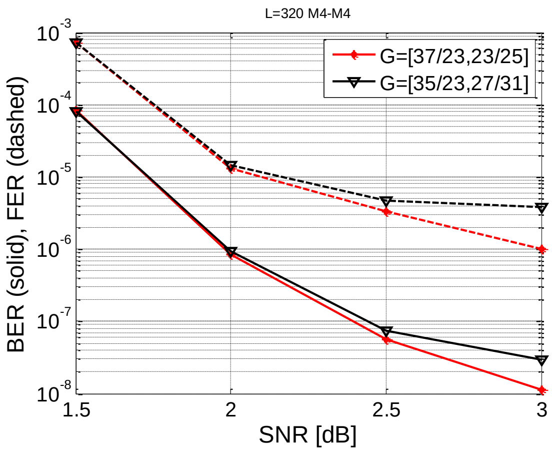

Figure 3: *BER and FER curves for the length of the interleaver L=320*

3. PERFORMANCE OF LTE VS. PROPOSED ASYMMETRIC TURBO CODES

LTE standard [3] is a project of the 3rd Generation Partnership Project (3GPP) and uses flexible bandwidths up to 20 MHz. LTE turbo encoder consists of a pair of rate 1/3 systematic convolutional encoders with constraint length K=4 separated by an interleaver. The generating matrix is G=[15/13,15/13] and the length of the interleaver is in the range of 40 to 6144 (involve 188 different lengths). It is important to know that the QPP interleaver was selected as interleavers for turbo codes.

From [3] we have selected the QPP interleavers corresponding to the interleaver's length 160, 256, 320; they are presented in Table 3.

L	**π(x)**
160	$21x + 120x^2$
256	$15x + 32x^2$
320	$21x + 120x^2$

Table 3: *Turbo code internal interleaver parameters for LTE standard*

Simulations were carried out for the interleaver length 256 and 320, coding rate 1/3 using Log MAP decoder over AWGN channel and the results are shown in Figure 4 and Figure 5.

Figure 4 compares the performance of a turbo code in LTE standard with the interleaver length equal to 256, according to Table 3, and that of the proposed asymmetric code, with the generator matrix G = [15/13, 13/15], according to Table 3. We note that the use of the proposed turbo code leads to an improvement of both BER and FER. , For instance, for SNR=4dB the ratio between the BER values is about 3.

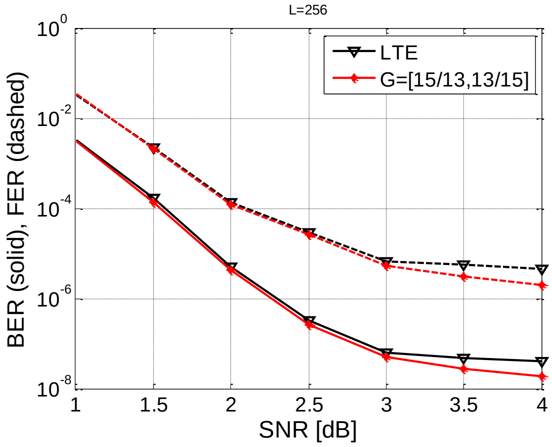

Figure 4*: Comparison of the BER and FER performance for typical and proposed asymmetric turbo code system in LTE standard with L=256*

Figure 5 illustrates the same type of comparison, but for the interleaver length of 320. Also in this case one can observe a better behavior of the proposed asymmetric code.

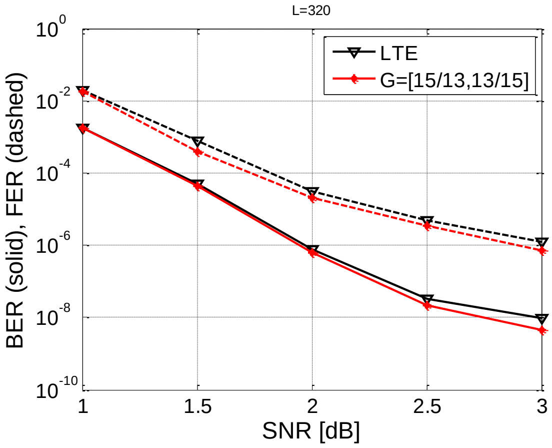

Figure 5*: Comparison of the BER and FER performance for typical and proposed asymmetric turbo code system in LTE standard with L=320*

4. CONCLUSIONS AND FUTURE WORK

In this paper we have proposed new asymmetric turbo encoders, which lead to better performance. Based on a previously proposed search algorithm, we expanded the search of generator polynomials for medium length of interleavers. The results we obtained were compared with those in LTE standard.

The simulation results illustrate that the performance of proposed asymmetric turbo codes is superior to the performance of typical one used in the LTE standard.

We intend to apply the same ideas of interleaver matched code for component codes of a MIMO turbo space time code.

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