KALMAN FILTERING APPLIED TO CARRIER RECOVERY OF HIGH-ORDER QAM SIGNALS

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ABSTRACT

A carrier synchronization algorithm for high-order QAM modulated signals that uses Kalman filtering techniques shows fast acquisition times and good frequency tracking performances. The algorithm relies on a decision directed extended Kalman filter combined with a lock detector which determines the status of the synchronization process, modifying consequently the filter parameters. Simulations performed both on 64-QAM and 256-QAM show similar or better performances than usual PLL synchronization loops.

Keywords: *Quadrature Amplitude Modulation, Decision-Directed Carrier Synchronization, Extended Kalman Filter Algorithm.*

1. INTRODUCTION

High-Order Quadrature Amplitude Modulation (QAM) is a high-bandwidth efficient modulation scheme employed especially in the field of cable communications. For this type of modulation, one of the most important problems is carrier recovery to allow a coherent demodulation. To extract correctly the transmitted symbols as the carrier frequency offset between the transmitter and receiver is inevitable, the receiver must provide blind fast carrier frequency synchronization.

In order to track the frequency of the received signal, the receiver uses commonly a phase-locked-loop (PLL) technique [1]. The use of a PLL carrier recovery loop to synchronize a QAM receiver requires fast convergence rates and small steady-state phase tracking errors, two opposing requirements. Simple decision-directed phase error estimators used in this PLL loops provide limited performances for frequency acquisition, since the phase error range is very low for high-power symbols, e.g. ±3.7° for corner symbols in 256 QAM.

An alternative to PLL carrier recovery is the use of Kalman filter algorithms [2, 3]. The Kalman filter is known as the optimally recursive linear filter in the minimum mean squared sense. Whenever the system environment varies, the Kalman filter can adaptively cope with the system environment changes better than a traditional PLL can do. Linear Kalman filters or extended Kalman filters (EKF) based on a carrier phase dynamic model [3, 4], were successfully used in carrier synchronization of QAM signals. In all these applications, the system model incorporates as variables, the phase and frequency of the incoming signal. Since the relationship between these variables and the observations is

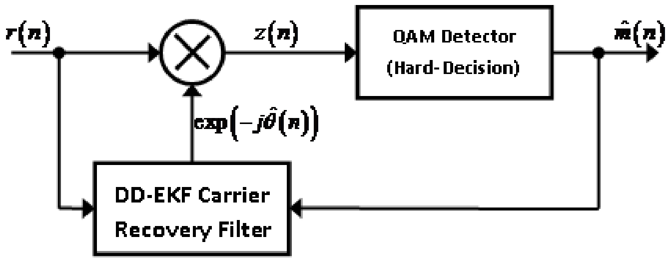

Figure 1 *Basic decision-directed EKF filter carrier synchronization system.*

nonlinear, EKF filters are used to solve the problems.

The paper develops a basic decision-directed EKF carrier synchronization algorithm (DD-EKF) for QAM modulation based on a Gaussian *random walk* model of the phase offset between the received signal and the reconstructed carrier [2]. Then, in order to improve the performances of the basic EKF algorithm, we adopt the architecture used in PLL synchronization loops [1] which rely on a coarse frequency acquisition mode and a fine phase tracking mode, selected by a lock detection module. Filter parameters are modified when system status changes from acquisition to tracking mode, thereby achieving a faster acquisition and a lower phase noise in tracking mode. To minimize the phase noise and to speed up acquisition, the adaptive carrier synchronization EKF algorithm uses different values of parameters in the acquisition mode as compared to tracking mode. Also, in order to avoid relatively large frequency offset and false synchronization, the algorithm uses a limitation mechanism to keep the estimated frequency in the region of interest.

Simulations were carried both on 64-QAM and 256-QAM. They show that the proposed adaptive decision-directed EKF algorithm has good acquisition and frequency tracking performances.

2. THE CARRIER SYNCHRONIZATION SYSTEM

Assuming perfect timing synchronization and proper gain control, the receiver in Figure 1 [3] observes the discrete time down-converted signal

$$r(n) = m(n)e^{j(\Omega n+\varphi)} + v(n) = m(n)e^{j\theta(n)} + w(n) \qquad (1)$$

where $m(n) = a(n) + jb(n)$ is the n^{th} transmitted complex QAM symbol, φ and Ω are the carrier residual phase and frequency offset and $w(n)$ is a zero-mean complex Gaussian noise with distribution $N(0, \sigma_w^2)$.

The decision-directed EKF filter estimates the total phase of the incoming signal carrier $\hat{\theta}(n)=\hat{\Omega}(n)n+\hat{\varphi}(n)$ and brings to the input of hard-decision QAM detector the signal:

$$z(n)=m(n)e^{j(\theta(n)-\hat{\theta}(n))}+v_z(n)=m(n)e^{j\Delta\theta(n)}+w_z(n) \quad (2)$$

where $\Delta\theta(n)$ is a phase error and $w_z(n)$ is complex Gaussian noise with the same power as $w(n)$. The carrier synchronization system aim is to cancel $\Delta\theta(n)$. A hard decision is made on $z(n)$ to generate the detected complex symbol $\hat{m}(n)$, part of QAM constellation.

3. THE DECISION-DIRECTED EKF FILTER

3.1 The State-Space Model

The estimation of the phase and frequency of the carrier of a M-QAM modulated signal by Kalman filtering is based on a model of the signal in the state space. The parameters of the model are the phase $\theta(n)$ and frequency $\Omega(n)$, with

$$\Omega(n)=\theta(n)-\theta(n-1) \quad (3)$$

To synchronize the model with the received signal, the evolution of $\Omega(n)$ is driven by a random walk model:

$$\Omega(n)=\Omega(n-1)+v(n) \quad (4)$$

where $v(n)$ is a sequence of i.i.d. random scalars with distribution $N(0,\gamma^2)$. Thus the rate of evolution of the signal frequency estimation is controlled by the parameter γ.

By combining (3) and (4), the state evolution equation of Kalman filter is given by:

$$\begin{bmatrix}\theta(n)\\ \Omega(n)\end{bmatrix}=\mathbf{F}\begin{bmatrix}\theta(n-1)\\ \Omega(n-1)\end{bmatrix}+\mathbf{G}v(n) \text{ with } \mathbf{F}=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \text{ and } \mathbf{G}=\begin{bmatrix}0\\ 1\end{bmatrix} \quad (5)$$

The In-phase and Quadrature components of the received signal are the elements of the observation vector $\mathbf{r}(n)$:

$$\mathbf{r}[n]=\left[\operatorname{Re}(r(n)) \quad \operatorname{Im}(r(n))\right]^T \quad (6)$$

In this condition, eq. (1) expresses the observations vector as a nonlinear function of state vector $\mathbf{x}(n)=\left[\theta(n) \quad \Omega(n)\right]^T$:

$$\mathbf{r}(n)=\mathbf{h}(\mathbf{x}(n))+\mathbf{w}(n) \quad (7)$$

with

$$\mathbf{h}(\mathbf{x}(n))=\begin{bmatrix}a(n)\cos\theta(n)-b(n)\sin\theta(n)\\ a(n)\sin\theta(n)+b(n)\cos\theta(n)\end{bmatrix},\ \mathbf{w}(n)=\begin{bmatrix}\operatorname{Re}(w(n))\\ \operatorname{Im}(w(n))\end{bmatrix} \quad (8)$$

where the correlation matrix of the noise vector $\mathbf{w}(n)$ is

$$\mathbf{Q}=\frac{\sigma_w^2}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad (9)$$

In order to use EKF, we apply the first order linearization procedure to $\mathbf{h}(\mathbf{x}(n))$ in (8) around the estimation of the state vector $\hat{\mathbf{x}}(n|n-1)$:

$$\mathbf{h}(\mathbf{x}(n))=\mathbf{h}(\hat{\mathbf{x}}(n|n-1))+\left.\frac{\delta\mathbf{h}}{\delta\mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(n|n-1)}(\mathbf{x}(n)-\hat{\mathbf{x}}(n|n-1)) \quad (10)$$

with

$$\mathbf{H}(n)=\left.\frac{\delta\mathbf{h}}{\delta\mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(n|n-1)} =\begin{bmatrix}-a(n)\sin\hat{\theta}(n|n-1)-b(n)\cos\hat{\theta}(n|n-1) & 0\\ -b(n)\sin\hat{\theta}(n|n-1)+a(n)\cos\hat{\theta}(n|n-1) & 0\end{bmatrix} \quad (11)$$

The calculation of $\mathbf{H}(n)$ requires the values of transmitted symbol $a(n)+jb(n)$ which is unknown at the receiver. If the receiver works with a sufficiently low error rate, then the output of the Hard-Decision device $\hat{m}(n)=\hat{a}(n)+j\hat{b}(n)$ can be used in place of transmitted symbol. Therefore, in the decision-directed EKF synchronization algorithm $\mathbf{H}(n)$ is replaced by:

$$\hat{\mathbf{H}}(n)=\begin{bmatrix}-\hat{a}(n)\sin\hat{\theta}(n|n-1)-\hat{b}(n)\cos\hat{\theta}(n|n-1) & 0\\ -\hat{b}(n)\sin\hat{\theta}(n|n-1)+\hat{a}(n)\cos\hat{\theta}(n|n-1) & 0\end{bmatrix} \quad (12)$$

Similarly, the nonlinear function $\hat{\mathbf{h}}(\hat{\mathbf{x}}(n|n-1))$ is calculated as:

$$\hat{\mathbf{h}}(\hat{\mathbf{x}}(n|n-1))=\begin{bmatrix}\hat{a}(n)\cos\hat{\theta}(n|n-1)-\hat{b}(n)\sin\hat{\theta}(n|n-1)\\ \hat{a}(n)\sin\hat{\theta}(n|n-1)+\hat{b}(n)\cos\hat{\theta}(n|n-1)\end{bmatrix} \quad (13)$$

3.2 The EKF algorithm

The procedure known as Extended Kalman Filter (EKF) algorithm [2] uses state-space equations (5) and (7) as well as the linearization of the observation function around the current vector estimate (12). Assume that the initial state $\mathbf{x}[1]$, the observation noise $\mathbf{w}[n]$ and the state noise $v[n]$ are jointly Gaussian and mutually independent. Let $\hat{\mathbf{x}}[n|n-1]$ and $\mathbf{R}[n|n-1]$ be the conditional mean and the conditional variance of $\hat{\mathbf{x}}[n]$ given the observations $\mathbf{r}[1],\ldots,\mathbf{r}[n-1]$ and let $\hat{\mathbf{x}}[n|n]$ and $\mathbf{R}[n|n]$ be the conditional mean and conditional variance of $\hat{\mathbf{x}}[n]$ given the observations $\mathbf{r}[1],\ldots,\mathbf{r}[n]$. Then, according [2] and using (12) and (13):

3.2.1. Measurement Update Equations

$$\mathbf{K}(n)=\mathbf{R}(n|n-1)\hat{\mathbf{H}}^T(n)\left(\hat{\mathbf{H}}(n)\mathbf{R}(n|n-1)\hat{\mathbf{H}}^T(n)+\mathbf{Q}(n)\right)^{-1} \quad (14)$$

$$\hat{\mathbf{x}}(n|n)=\hat{\mathbf{x}}(n|n-1)+\mathbf{K}(n)\left(\mathbf{r}(n)-\hat{\mathbf{h}}(\hat{\mathbf{x}}(n|n-1))\right) \quad (15)$$

$$\mathbf{R}(n|n)=\mathbf{R}(n|n-1)-\mathbf{K}(n)\mathbf{H}(n)\mathbf{R}(n|n-1) \quad (16)$$

3.2.2. Time Update Equations

$$\hat{\mathbf{x}}(n+1|n)=\mathbf{F}\hat{\mathbf{x}}(n|n) \quad (17)$$

$$\mathbf{R}(n+1|n)=\mathbf{F}\mathbf{R}(n|n)\mathbf{F}^T+\gamma^2\mathbf{G}\mathbf{G}^T \quad (18)$$

where $\mathbf{K}(n)$ is the Kalman gain matrix at moment n.

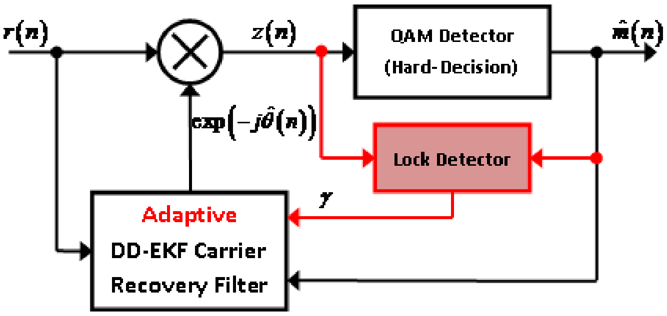

Figure 2 *Adaptive decision-directed EKF filter carrier synchronization system.*

3.2.3. Limiting Conditions

Viewing the four-quadrant symmetry of M-QAM signals, the action of the algorithm is confined to a carrier frequency offset between $-\Omega_S/8$ and $\Omega_S/8$, where Ω_S is the symbol rate. For an initial frequency offset near to $\Omega_S/8$, the algorithm can converge to $\pm\Omega_S/4+\Omega$ instead of Ω, the real value. This "false-lock" situation can be avoided by imposing to $\hat{\Omega}(n)$, the estimated frequency, the following limiting condition:

$$-\Omega_S/8 \le \hat{\Omega}(n) \le \Omega_S/8 \qquad (19)$$

4. ADAPTIVE SYNCHRONIZATION SYSTEM

Carrier synchronization process consists of two distinct stages: acquisition and tracking. Both stages involve different modes of action of the synchronization system in order to achieve both fast carrier synchronization in the acquisition stage and low phase error in the tracking stage. To meet these antagonistic requirements firstly, it is necessary to detect the synchronization moment and then, the EKF filter parameters should be dependent on the stage where the synchronization process is found. The model in Figure 2 fulfills these requirements, using a lock detector [1] to estimate the status of the synchronization process and an adaptive EKF filter.

The lock detector monitors the derotated signal $z(n)$ and the output of the Hard-Decision detector $\hat{m}(n)$ to determine whether or not the output constellation is phase-locked. The lock detector generates the binary signal

$$y(n)=\begin{cases}1, & \text{if } |z(n)-\hat{m}(n)|<\lambda \\ 0, & \text{otherwise}\end{cases} \qquad (20)$$

In presence of a frequency offset between Ω and $\hat{\Omega}(n)$, the received signal points in $\hat{m}(n)$ will not distribute closely around the expected points of constellation [1]. Hence, when the system is phase-locked, $y(n)$ will be equal to 1 in majority. Every N_1 symbols, the lock detector compares the average value of $y(n)$ taken on N_2 symbols with a threshold value β to determine if the system has acquired or not synchronization.

Lock detector output is used to control adaptively the performances of EKF filter. This control is exercised through the variance of Gaussian random walk model of estimated frequency, γ^2. The use of a large γ in tracking mode determines a rapid loss of synchronization, while with a low γ in the acquisition mode, the synchronization no longer occurs. Consequently, the use of a single value for the variance of estimated carrier frequency as in [3], cannot provide both a fast convergence rate in acquisition mode and a low frequency error in tracking mode.

In conclusion, the proposed synchronization system model uses the lock detector to toggle the estimated frequency variance γ^2 of EKF filter from a large value in acquisition mode to a very low value in tracking mode:

$$\gamma=\begin{cases}\gamma_1, & \text{in acquisition mode} \\ \gamma_2, & \text{in tracking mode}\end{cases}, \quad \gamma_1 \gg \gamma_2 \qquad (21)$$

5. SIMULATION RESULTS

The goal of performed simulations was to certify DD-EKF algorithms presented in the paper as viable alternatives to decision-aided PLL synchronizers. The basic DD-EKF algorithm is used in the structure of Figure 1 and the adaptive DD-EKF algorithm uses the system in Figure 2. Simulation trials consisting of 50,000 symbols were made with 64-QAM and 256-QAM signals having a DVB-C constellation and symbol rate Ω_S. The modulated signal propagates through a communication channel with additive white and Gaussian noise (AWGN) having zero-mean and variance σ_w^2. The received signal applied at the input of decision-directed synchronization systems has a carrier frequency offset Ω, one sample per symbol, perfect timing synchronization and proper gain control.

The adaptive DD-EKF algorithm uses a lock detector to determine when the synchronization system switches between the two working modes: acquisition and tracking. Based on results reported in [1] and our simulations in the case of 64-QAM signals the value of λ is set to 0.7 and β is 0.6 in the acquisition mode. For 256-QAM signals, the limit β increases at 0.7. The false lock-detection probability is easily found using binomial coefficients. For instance, using 64-QAM , with $\beta=0.6$ and $N_2=256$, the false lock-detection probability is less than 10^{-10}.

A PLL carrier recovery system uses two different time constants. In a similar way, the averaging parameters of lock detector, N_1 and N_2 change when the system switches between the two modes. They are lower in acquisition mode: $N_1=16$ and $N_2=64$ and higher in tracking mode: $N_1=64$ and $N_2=256$. The values of parameters $\gamma_1 \gg \gamma_2$ were determined experimentally.

A typical description of the way the carrier synchronization process works for a 256-QAM modulated signal is made by Figure 3. The received signal has the following parameters: $\text{SNR}=35$ dB and $\Omega=0.04\Omega_S$, where Ω is the carrier frequency offset. A first conclusion, taken from Fig. 3, concerns the utility of the lock detector to determine precisely the moment of transition from

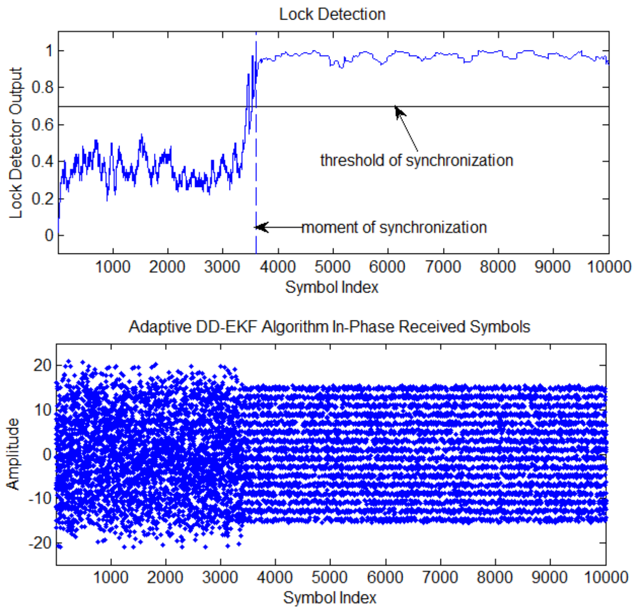

Figure 3 *Description of the synchronization process for a 256-QAM signal: (a) Output of Lock Detector, (b) In-Phase input signal to Hard-Decision detector.*

acquisition to tracking. The parameters of EKF filter are: $\gamma_1 = 1.25 \cdot 10^{-2} \Omega_S$ and $\gamma_2 = 3.125 \cdot 10^{-6} \Omega_S$.

The curves shown further are obtained by averaging the results of 100-run simulations. Figure 4 presents the acquisition time performances for various initial carrier frequency offsets of a 64-QAM signal with a signal-to-noise ratio $\text{SNR} = 30\ \text{dB}$. The parameters of EKF filter are: $\gamma_1 = 2 \cdot 10^{-2} \Omega_S$ and $\gamma_2 = 5 \cdot 10^{-5} \Omega_S$. The simulations result was compared with the results reported by Gagnon et al. [1] for a very fast PLL carrier synchronization loop in terms of acquisition speed. The adaptive DD-EKF algorithm has better performances than those reported by Gagnon et al.

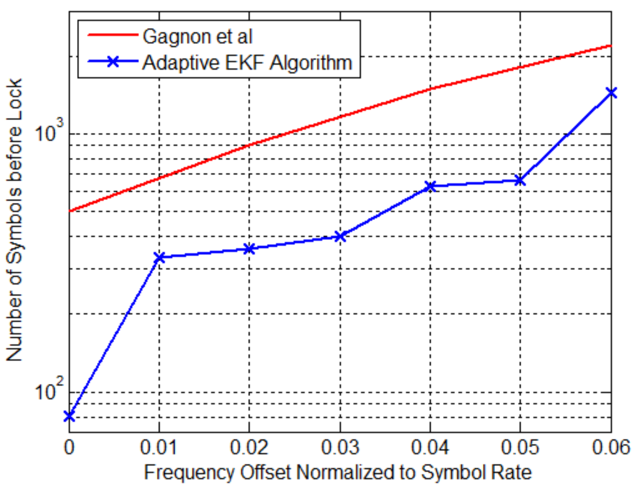

Figure 4 *Acquisition performances of the carrier synchronization systems for 64-QAM. For the PLL carrier synchronization loop, the results are taken directly from Gagnon et al.* [1].

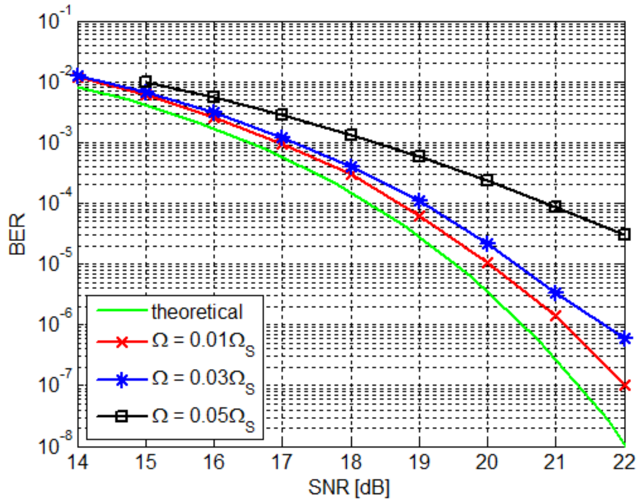

Figure 5 *The dependence of BER on SNR for 64-QAM at various initial carrier frequency offsets for the adaptive DD-EKF algorithm.*

In order to characterize the performance of the adaptive DD-EKF algorithm in tracking mode, the bit error rate (BER) of binary signal at the output of Hard-Decision device was compared with the theoretical curve for 64-QAM signals. Figure 5 reveals that for moderate carrier frequency offsets, the difference from the theoretical curve is lower than 1 dB.

6. CONCLUSIONS

This paper introduces a carrier synchronization system for M-QAM modulated signals, based on an adaptive DD-EKF algorithm. The algorithm uses a very efficient Lock Detector to assess whether or not the system attained synchronization. The Kalman filter modifies its parameters when system status changes from acquisition to tracking mode, thereby permitting a faster acquisition time and a lower phase noise in tracking mode.

REFERENCES

[1] G. Gagnon, F. Choquette, J. Belzile, and F. Gagnon, "A simple and fast carrier recovery algorithm for high-order QAM," *IEEE Trans. Commun. Lett.*, vol. 49, pp. 918-920, Oct. 2005.

[2] J. Gál, A. Câmpeanu, and I. Nafornită, "Kalman Noncoherent Detection of CPFSK Signal," in *The 8th International Conference on Communications "COMM 2010"*, Bucharest, June 10-12 2010, pp. 65-68.

[3] W.-T. Lin and D.-C. Chang, "Adaptive carrier synchronization using Decision-Aided Kalman filtering algorithms," *IEEE Trans.on Consumer Electronics,* vol. 53, pp. 1260-1267, Nov. 2007

[4] A. Patapoutian, "Application of Kalman filters with a loop delay in synchronization," *IEEE Trans. on Communications,* vol. Vol. 50, pp. 703-706, May 2002.