COMBINED PARTIAL TRANSMIT SEQUENCE AND COMPANDING FOR PAPR REDUCTION IN OFDM SYSTEMS

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ABSTRACT

High Peak-to-Average Power Ratio (PAPR) is one of the challenging issues for Orthogonal Frequency Division Multiplexing (OFDM). In this paper, we propose a PAPR reduction technique, which combines Partial Transmit Sequence (PTS) and companding techniques. Some simulation results are presented, commented, and compared with the results obtained applying PAPR reduction methods already proposed in literature.

Keywords: *OFDM, PAPR, PTS,* μ-law, A-law

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is an attractive technology for wireless communications. As one of multi-carrier modulation techniques, it offers considerable high spectral efficiency; multipath delay spread tolerance, immunity to the frequency selective fading channels and power efficiency [1], [2]. As a result, OFDM has been chosen for high data rate communications and has been widely deployed in many wireless communication standards such as Digital Video Broadcasting (DVB), mobile Worldwide interoperability for Microwave Access (mobile Wi-MAX), and 3GPP Long Term Evolution (LTE) [3].

Despite of its advantage, one of the major drawbacks of OFDM is the high PAPR value of the transmitted signals. This problem comes from the nature of the modulation itself, where multiple sub-carriers are added together to form the signal to be transmitted. Usually, the systems are constrained to a limited peak power due to the limitation of the dynamic range over which the transmitter amplifier operates linearly.

In OFDM systems, the PAPR is a random variable. Therefore it is important to search on the characteristics of the PAPR including its distribution and reduction in order to utilize the technical features of the OFDM.

One of the characteristics of the PAPR is its distribution. Often it can be expressed in terms of Complementary Cumulative Distribution Function (CCDF).

Several researchers have proposed schemes for reducing the peak amplitude of the transmitted signal, such as clipping [4], coding [5], Active Constellation Extension (ACE) [6], partial transmits sequence [7], Turbo Coded OFDM [8], and Companding technique [9].

The proposed scheme reduces the PAPR by combining the Partial Transmit Sequence (PTS) and Companding methods. This scheme will be compared with the constitutive schemes.

The paper is organized as follow: first we investigate the PAPR in OFDM systems in section 2 and then we describe the proposed technique in section 3. Simulation results will be presented in section 4 and finally we present some conclusions in section 5.

2. PAPR IN OFDM SYSTEMS

Starting from a group of N bits $X_k = (X_0, X_1, \ldots, X_{N-1})$ and a set of N subcarriers $\{f_k, k = 0,1, \ldots, N-1\}$, an OFDM symbol will be generated by the amplitude modulation of each subcarrier with a corresponding bit.

The subcarriers are chosen to be orthogonal, that is $f_k = k\Delta f$, where $\Delta f = 1/(NT)$ and T is the original bit period. Therefore, the complex envelope of the transmitted OFDM signals can be written as:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k\, e^{j2\pi f_k t}, \quad 0 \leq t \leq NT \qquad (1)$$

where $j = \sqrt{-1}$.

The PAPR of an OFDM signal is defined as the ratio between the maximum instantaneous power and its average value:

$$PAPR[x(t)] = \frac{\max\limits_{0 \leq t \leq NT}[|x(t)|^2]}{P_{av}} \qquad (2)$$

where P_{av} is the average power of $x(t)$ and it can be computed in the frequency domain because the Inverse Fast Fourier Transform (IFFT) is a (scaled) unitary transformation.

To better approximate the PAPR of the continuous-time OFDM signals the oversampling is recommended. It can be implemented by computing an LN-point IFFT of the data block with $(L-1)N$ zero-padding. Therefore, the oversampled IFFT output can be expressed as:

$$x[n] \triangleq \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k\, e^{\frac{j2\pi nk}{LN}}, \quad 0 \leq n \leq LN-1 \qquad (3)$$

The PAPR computed from the L-times oversampled OFDM signal can be defined as:

$$PAPR\{x[n]\} = \frac{\max\limits_{0 \leq n \leq LN-1}[|x[n]|^2]}{E[|x[n]|^2]} \qquad (4)$$

where $E\{.\}$ denotes the expectation operator.

When N is large, the distribution of the output time vector converges to Gaussian due to Central Limit Theorem.

Hence, the probability that the PAPR is above a threshold λ can be written as [12]:

$$\Pr\{PAPR > \lambda\} = 1 - \left(1 - e^{-\lambda}\right)^{N}.$$

Most radio systems employ High Power Amplifiers (HPA) in the architecture of the transmitters to obtain sufficient transmission power. For the achievement of the maximum output power efficiency, the HPA is usually operated at or near the saturation region where its input-output characteristic is non linear. This non linearity makes the HPA very sensitive to the variations of the signal amplitude. It can be observed, analyzing Figure 1 that the tails of the spectrum of the signal obtained at the output of the HPA decrease slower than the tails of the spectrum of the signal at its input which proves that the non linearity of the HPA reduces the spectral efficiency of the OFDM based transmitter. Due to the variations of the instantaneous power of the OFDM signals their PAPR value is very high. Therefore, the HPA will introduce inter-modulation between the different subcarriers and as a result additional interference and the reduction of the spectral efficiency. This additional interference leads to an increase in Bit Error Rate (BER). In order to reduce the signal distortion and keep a low BER, the HPA must be exploited in its linear region where it has a large dynamic range. Unfortunately the linear HPAs have poor efficiency and are more expensive. Power efficiency is very necessary in wireless communication as it provides adequate area coverage, saves power consumption and allows small size terminals etc. It is therefore important to handle efficiently the non-linearity of the HPA and to try to provide possible solutions to the interference problem brought about. Hence, a better solution is to try to prevent the occurrence of such interference by reducing the PAPR of the transmitted signal with some manipulations of the OFDM signal itself. Large PAPR also demands the Digital to Analogue Converter (DAC) to have enough dynamic range to accommodate the large peaks of the OFDM signals.
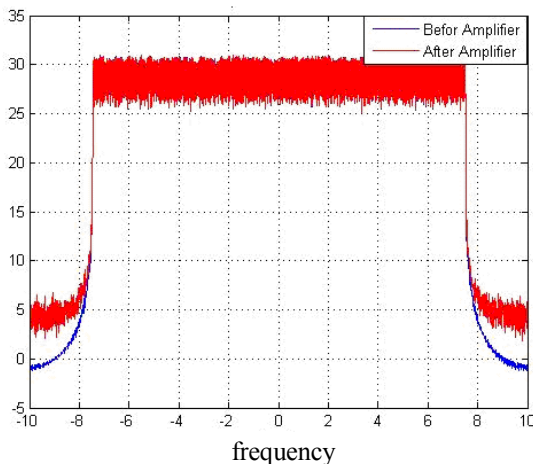

Figure 1: *Effects of the non linearity of the HPA on the spectrum of OFDM signals. The spectrum of the signal at the output of the HPA is normalized.*

Although, a high precision DAC supports high PAPR with a reasonable amount of quantization noise, but it might be very expensive for a given sampling rate of the system. Whereas, a low-precision DAC would be cheaper, its quantization noise will be significant, and as a result the signal Signal-to-Noise Ratio (SNR) will be reduced when the dynamic range of the DAC will be increased to support a high PAPR value. Furthermore, the OFDM signals show Gaussian distribution for a large number of subcarriers, which means that the peaks of the signals appear quite rarely, and that the uniform quantization is not desirable. If the OFDM signal is clipped, then band distortion and out-of-band radiation (adjacent channel interference) will be introduced into the communication system.

Therefore, the best solution is to reduce the PAPR before the OFDM signals are introduced into nonlinear HPA and DAC.

When a HPA has a high dynamic range, it exhibits poor power efficiency. It has been shown that PAPR reduction can significantly save the power, the net power saving being directly proportional to the desired average output power and highly dependent upon the clipping probability level.

3. PROPOSED TECHNIQUE

The proposed technique is shown in Figure 2 and the principles of Companding and PTS will be studied in the following.

3.1 Companding technique

The compander consists of a compressor and an expander. The compressor is a simple logarithm computation block. The inverse system of a compressor is called expander. In this paper, the compression is applied at the transmitter end, after the IFFT process and the expansion is applied at the receiver end prior to the FFT process. There are two types of companders that are used here which are described in details in [9]. These two types are the μ-law and the A-law companders [10].

3.1.1 μ-law Compander

The μ-law algorithm was primarily used in the digital communication systems of North America and Japan. The μ-law compander employs the logarithmic function at the transmitting side. In general a μ-law compression characteristic is expressed as:

$$y = \frac{V \log\left(1 + \mu \frac{|x|}{V}\right)}{\log(1 + \mu)} sgn(x)$$

where μ is the μ-law parameter which controls the amount of compression, x is the input signal and V is the maximum value of x.

For normalized input signal: $|x| \leq 1$, the characteristic becomes:

$$y = \frac{\log(1 + \mu|x|)}{\log(1 + \mu)} sgn(x)$$

The μ-law expander is the inverse of the compressor and has an input-output relation of the form:

$$x = \frac{v}{\mu}\left(e^{\frac{|y|\log(1+\mu)}{v}} - 1\right) sgn(y).$$

3.1.2 A-law Compander

The characteristic of the A-law compander is given by:

$$y = \begin{cases} \dfrac{1 + \ln A \cdot \|x\|}{1 + \ln A} sgn(x), & \dfrac{1}{A} \le x \le 1 \\[2ex] \dfrac{A\|x\|}{1 + \ln A} sgn(x), & 0 \le \|x\| \le \dfrac{1}{A} \end{cases},$$

where A is the parameter which controls the amount of compression.

3.2 PTS

PTS is an important method for PAPR reduction; it is already known that the OFDMA signals generated by PTS are interdependent. PTS divides the frequency vector into some sub-blocks before applying a phase transformation. In a typical OFDM system with PTS, the input data block X is segmented into M disjoints sub-blocks, which are represented by the vectors $\{X^m, m = 0, 1, ..., M - 1\}$ [10]. Therefore we can get:

$$X = \sum_{m=0}^{M-1} X^{(m)}$$

where $X^{(m)} = [X_0^m \ X_1^m \ ... \ X_{N-1}^m]$ with $X_k^m = X_k$ or 0. The segmentation methods can be classified into three categories [10]: adjacent partition, interleaved partition and pseudorandom partition. Then, the sub-blocks $X^{(m)}$ are transformed into M time-domain PTSs:

$$x^{(m)} = [x_0^m \ x_1^m \ ... \ x_{LN-1}^m] = IFFT_{LN \times N}[X^{(m)}]$$

Then, these sequences are combined using complex phase weights: $b = [b_1 \ b_2 ... b_M]$ selected to minimize the PAPR. That is, the PAPR is reduced by the weighted combination of M sub-blocks. The resulting time domain signal is given by:

$$X'(b) = \sum_{m=0}^{M-1} b_m X^{(m)}.$$

Therefore, there are two important issues which should be solved in PTS: high computational complexity for searching the optimal phase weights and the overhead of the optimal phase factors as side information needed to be transmitted to receiver for the correct decoding of the transmitted bit sequence. The allowable phase weights are $b_m = e^{j\phi_m}$ where ϕ_m can be chosen freely within $[0, 2\pi]$, for convenience we can write:

$$X'(\phi) = \sum_{m=0}^{M-1} e^{j\phi_m} X^{(m)},$$

where $\phi = [\phi_0 \ \phi_1 ... \phi_M]$. Hence, the objective of the PTS scheme is to design an optimal phase weight for each sub-block that minimizes the PAPR. The problem can be formulated mathematically in the following form: minimize $max|X'(\phi)|$; subject to $0 \le \phi_m \le 2\pi$, $m=1, 2,..., M$.
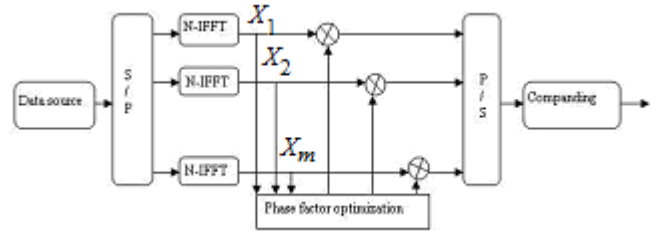

Figure 2: *Block diagram of the proposed PAPR reduction method based on combining PTS and Companding techniques.*
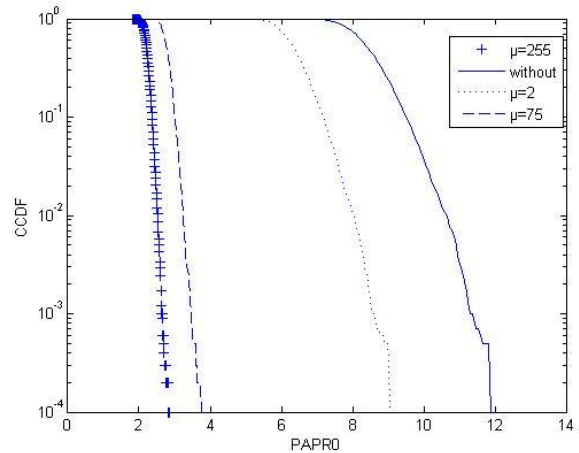

Figure 3: *CCDF performance of the proposed method for different values of μ.*

Some of the complexity of several full IFFT operations can be avoided in PTS [11]. Also, it is proved that the PAPR reduction based on PTS performs better than other similar technique such as Selected Mapping (SLM) scheme [12].

4. SIMULATION RESULTS

The main idea of the proposed scheme is to use a combination of two methods. First, the Partial Transmit sequences approach is used and second the signal with the lowest PAPR is submitted to the companding technique. The intention of this combination is to obtain a signal with lower PAPR than in the case of the PTS method. In this section we will present some simulation results obtained using different scenarios for PAPR reduction for OFDM signals. The results presented in the following were obtained by considering an Additive White Gaussian Noise (AWGN) channel, 512 subcarriers and a QPSK modulation. Figure 3 shows the CCDF performance of the proposed technique (PTS + companding) for different values of μ. The peak power at CCDF=10^{-4} is reduced by approximately 3dB, 8.5dB, 9.2 dB for μ= 2, μ=75, μ=255 respectively in comparison with the corresponding value obtained without any PAPR reduction technique. In Figure 4 are presented the CCDF performance of the proposed scheme compared with those of the system without any PAPR reduction, of the system equipped only with the PTS technique and of the system equipped only with the companding technique.
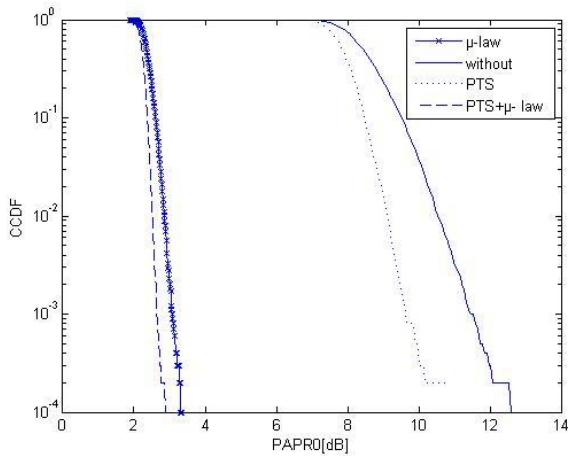
Figure 4: *CCDF performance for different PAPR reduction techniques. For the proposed method a value of* μ *=255 was used.*
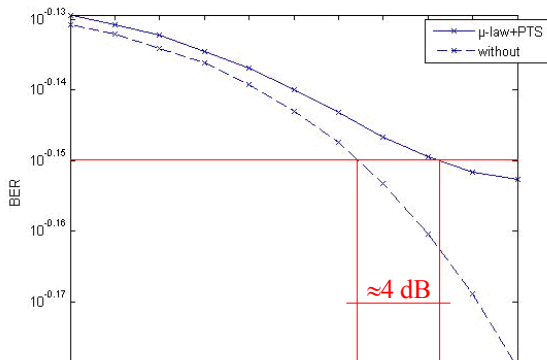

Figure 5: *BER* performance *comparison for OFDM system with and without PAPR reduction*

It can be observed that the PAPR value improves by 2dB for CCDF = 10^{-4} for PTS technique, by 8.5dB for the companding technique, and by 9.2dB for the proposed technique in comparison with the value obtained for the system without any reduction technique. In Figure 5 are presented the Bit Error rate (BER) performances of the proposed scheme compared with the original system over a flat fading Rayleigh channel without mobility and the same value of μ (μ=255). We can observe that BER increases for the system with PAPR reduction scheme because the signal is distorted. The mean BER degradation is of 4 dB. The BER results can be improved by channel coding, for example using turbo-codes.

5. CONCLUSIONS

In this paper, we introduced a new technique for PAPR reduction, which combines two classic PAPR reduction methods: the PTS and the Companding method. The Companding techniques use A-law or μ-law with suitable values of μ and A. The simulations results obtained prove the good performance of the approach proposed better than the performance which can be obtained using only one of the two composing methods applied separately.

We intend to develop a new technique to reduce the PAPR of OFDMA and Single Carrier OFDMA without BER performance degradation and additional Power needed.

6. ACKNOWLEDGEMENT

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