LOCAL CRITERIA AND LOCAL ADAPTIVE FILTERING IN IMAGE PROCESSING: A RETROSPECTIVE VIEW

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ABSTRACT

Local adaptive linear and non-linear filters and local criteria for assessment of image processing quality that substantiate them date back to mid 1980-th. In this paper we review the basic ideas, support them with known facts on properties of human visual system and analyze contemporary tendencies

1. INTRODUCTION

Local adaptive linear and non-linear filters for image and signal processing and local criteria of image processing quality that substantiate their optimality and design date back to mid 1980-th ([1-7]) and were intensively investigated throughout the subsequent years ([8-21]). The filters scan signal/image with a sliding window and, at each window position associated with the running pixel, generate an estimate of the pixel on the base of measuring local statistics within window, such as local spectra in certain bases, local histograms and their moments and local order statistics. The local criteria assume assignment of an image quality measure to every pixel of the image. This is done on the base of evaluation, on average over "random" factors associated with image formation and intended utilization of the image, of quality losses over a subset of pixels in a certain neighborhood of the given pixel. In this paper we reiterate basic formulations of local criteria and illustrate the validity of basic assumptions using known facts on properties of human visual system.

2. LOCAL VS GLOBAL PROCESSING

Two approaches to the design and optimization of image processing algorithm are global and local ones. The global approach originates from the classical statistical communication theory and its concept of signal stationarity. In the global approach, processing algorithms are designed and optimized for image statistical ensembles and applied to images as wholes. The local approach treats images as spatially inhomogeneous and assumes local adaptivity of the processing algorithms. There is quite a number of arguments in favor of "local" approach versus "global" one:

• It is well known that, when viewing an image, human eye's optical axis permanently hops chaotically over the field of view ([22]) and that the human visual acuity is very non-uniform over the field of view. The field of view of a man is about 30°. Resolving power of man's vision is about 1′. However such a relatively high resolving power is concentrated only within a small fraction of the field of view that has size of about 2° (see, for instance, [23]). Therefore, the area of the acute vision is about 1/15-th of the field of view.



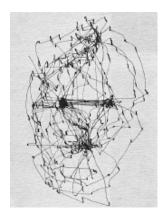


Figure 1. Test image (left) and results of recording eye fixation when observing this image (right) (adopted from [23])

Visual objects to be recognizable have to contain sufficiently large number of pixels. As an immediate illustration of this fact one can recall that, for the representing printed characters, one needs a matrix of at least 8x8 pixels. Even the smallest one pixel size object needs a neighborhood of 3x3 pixels to be detectable if not recognizable. The same and even to a greater degree holds for "texture" images. Texture identification is also possible only if texture area contains sufficiently large numbers of pixels. This means that image can be regarded as a composition of object domains

- with the linear size from several to several tens of resolution cells.
- Adaptive filter design assumes empirical evaluation of signal statistical parameters such as spectra (for local adaptive linear filters) or histograms (for rank filters). In global image statistics, parameter variations due to image non-homogeneity are hidden and are difficult if not impossible to detect. Therefore in global statistical analysis image local information will be neglected in favor of global one, which usually contradicts processing goals.

3. LOCAL CRITERIA

A mathematical framework for the optimal design, analysis and comparison of local adaptive filters is provided by local criteria of image processing quality ([14-18]).

Let $\{b_k\}$ be a set of N image samples (k=0,1,...,N-1) at the output of the imaging system and $\{a_k\}$ be a set of the system's input image samples that model a perfect, or "ideal" image. For the design of adaptive filters, the set $\{b_k\}$ is considered as a realization taken from a signal statistical ensemble generated by an ensemble Ω_N of random interferences caused by image acquisition devices. Let also $LOSS(a_m,b_m)$ be a measure of deviation of the observed m-th image sample from the ideal one. In these denotations, local criteria assign to every k-th image sample a quality loss measure:

$$AVLOSS(k) = AV_{\Omega_N} \left\{ \sum_{m} LOC[k;a(k)] LOSS(a(k-m),\hat{a}(k-m)) \right\} (1)$$

where LOC[k;a(k)] is a "locality" weight function that have positive non-zero weights for those samples that are to be involved in the spatial averaging of the loss function over the set of available samples and AV_{Ω_N} is an averaging operator over the ensemble Ω_N . Most frequently, the averaging operator assumes arithmetical averaging. However, in general, they might produce any global estimate of the set values such as median, alpha-

The simplest examples of loss-functions are given in the Table 1. Obviously, these four examples do not exhaustively represent loss functions adequate to wide variety of types possible degradations of image quality. Images shown in Fig. 2 illustrate how different is visual evaluation of different types image degradation, that are all equivalent in terms of the quadratic loss-function *LOSS-2*.

trimmed mean, maximum and alike.

Some examples of locality functions that have found applications in local adaptive linear and rank filters are given in Table 2. Figs. 3 and 4 illustrate the validity and importance of the notion of neighborhood on the examples of "FLAT" and EV-neighborhoods.

Table 1. Examples of loss functions

LOSS-1	$ a(m)-\hat{a}(m) $
LOSS-2	$\left a(m)-\hat{a}(m)\right ^2$
LOSS-2p	$\left a(m)-\hat{a}(m)\right ^{2p}$
LOSS-Thr	$\begin{cases} 0, a(m) - \hat{a}(m) \le ThresholdValue \\ 1, a(m) - \hat{a}(m) > ThresholdValue \end{cases}$

Table 2. Examples of neighborhoods

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S-neighb	S-neighborhoods: pixel co-ordinates as attributes			
Spatial win-	$SwNBH(N_x, N_y)$: a rectangular win-			
dow	dow of (N_x, N_y) pixels			
Shape-	$SHwNBH(\{w_m\}, N_x, N_y)$: a spatial			
neighbor-	window of certain shape defined by weight			
hoods	coefficients $\{w_m\}$			
V-neig	hborhoods: pixel values as attributes			
"Epsilon-	$EVnbh(SwNBH; a_k; \varepsilon_v^+; \varepsilon_v^-)$: a subset			
V"-neighbor- hood	of pixel with values $\{a_n\}$ that satisfy ine-			
	quality: $a_k - \mathcal{E}_v^- \leq a_n \leq a_k + \mathcal{E}_v^+$.			
"K nearest	$KNVnbh(SwNBH; a_k, K)$: a subset of			
by value"-	K pixels with values $\{a_n\}$ closest to that of			
neighbor- hood of	element a_k .			
element	element u_k .			
Range-	$RNGnbh(SwNBH, V_{mn}, V_{mx})$: a subset of			
neighbor-	pixels with values $\{V_k\}$ within a specified			
hood:	range $\{V_{mn} < V_k < V_{mx}\}$			
	ghborhoods: pixel ranks as attributes			
"epsilon-R"- neighbor-	$ERnbh(SwNBH; a_k; \varepsilon_R^+; \varepsilon_R^-)$: a subset			
hood	of pixels with ranks $\{R_n\}$ that satisfy ine-			
	quality: $R_k - \varepsilon_R^- \le R_n \le R_k + \varepsilon_R^+$.			
"K-nearest by	$KNRnbh(SwNBH; a_k, K)$: a subset of K			
rank" neigh- borhood of	pixels with ranks closest to that of element			
element a_k	a_k over $Sw-nbh(N_x, N_y)$.			
Quantil-	$Qnbhig(SwNBH,R_{left},R_{right}ig)$ Elements			
neighbor- hood	(order statistics) whose ranks $\{R_r\}$ satisfy			
	inequality $R_{left} < R_r < R_{right}$			
	orhoods: pixel cardinalities as attributes			
"Cluster"	$CLnbh(SwNBH; a_k)$			
neighborhood	Neighborhood elements that belong to the			
of element	same cluster of the histogram over the			
a_k .	neighborhood as that of element a_k .			
	ghborhoods: Geometrical attributes			
"Flat"-	FLAT(SwNBH) - Neighborhood ele-			
neighborhood	ments with values of a certain measure of			
	local non-uniformity lower than a certain threshold			
	unesnoid			

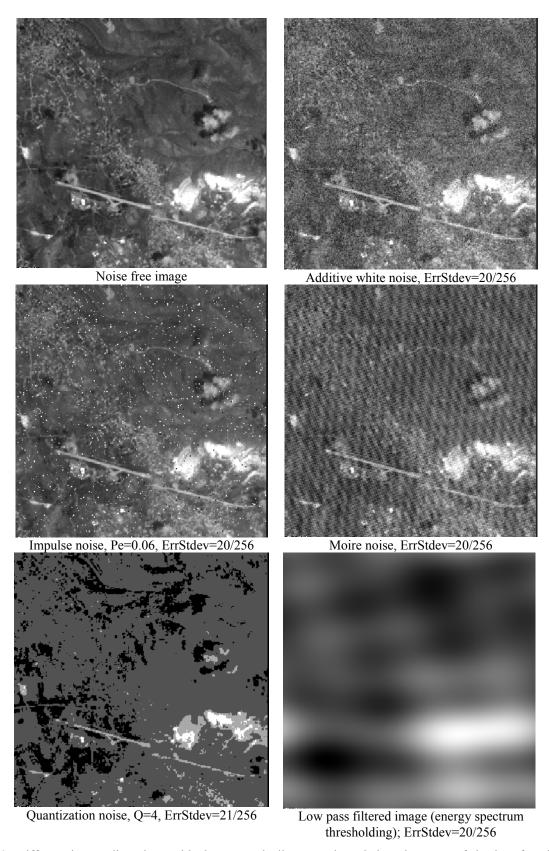


Figure 2. Different image distortions with the α numerically same degradations in terms of the loss-function α





Figure 3. Illustration of how different is visual perception of noise in images within "flat" and "non-flat" pixel neighborhoods. Images in the top row are a test image (left) and the result of the test image segmentation into "flat" (black) and "non-flat" (white) components according to whether values of local standard deviation in the spatial window 5x5 pixels is relatively low or high (standard deviation segmentation threshold 20). In images in the middle row additive white noise of the same standard deviation of 20 (in the image range 0-255) is added to "flat" (left) and to "non-flat" areas of the test image. One can see from these images that human vision is effectively not very sensitive to noise in "non-flat" image areas and is quite sensitive to noise in "flat" areas. Images in the bottom row shows noise present in images of the middle row.

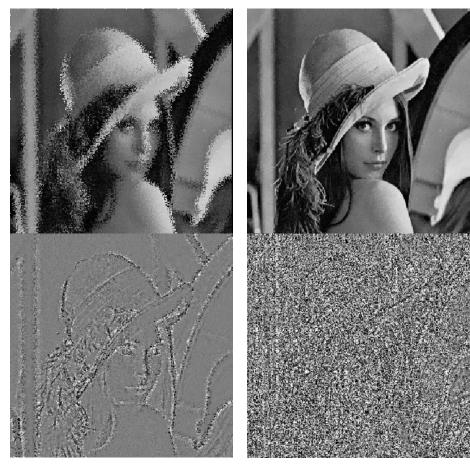


Figure 4. Illustration of the amount of information content of local histograms over spatial SwNBH - and EV-neighborhoods. Images in the top row are generated from pseudo-random numbers with the same distribution histogram as that of pixels of the initial test image shown in Fig. 3 in spatial window of 7x7 pixels (left) and of pixels in EV-neighborhood with EV-plus = EV-minus = 10 (right). Images in the bottom row show pixel-wise difference between the initial test image and corresponding images of the top row (the right image is, for display purposes, 12 times amplified with respect to the left image). One can see that while the left image preserves certain similarity with the original image thanks to common first order local statistics, the right image is practically indistinguishable from the original one, though pixel-wise difference between them is a random pattern with uniform distribution in the range ± 10 gray values (right bottom image).

4. LOCAL ADAPTIVE FILTERS

Local criteria outlined in Sect. 3 serve for substantiation of local adaptive filtering for image perfection and enhancement. The filtering is performed within a filter window that scans input image pixel by pixel and, in each position k of the window, filters generate, from input signal samples $\{b_n^{(k)}\}$ within the window, an estimated output value \hat{a}_k for this position by means of a certain estimation operation **ESTM** applied to a certain subset $NBH\{b_n^{(k)}\}$ of window samples:

$$\{b_n^{(k)}\}\rightarrow \hat{a}_k: \hat{a}_k = \text{ESTM}(NBH\{b_n^{(k)}\}).$$
 (2)

Two families of local adaptive filters have been suggested: local adaptive linear filters and rank filters. A

comprehensive treatment of them can be found in Refs. [20, 21, 24].

4.1 Local adaptive linear filters

Local adaptive linear filters minimize variance of difference between filtered and "ideal" images over spatial window neighborhoods (LOSS-2 loss function)

$$AVLOSS(k) = AV_{\Omega_N} \left\{ \sum_{m \in SpW} |a(k-m) - \hat{a}(k-m)|^2 \right\}.$$
 (3)

They work in sliding window in a domain of a certain orthogonal transform such as DFT, DCT, Haar and the like, and have proved their high efficiency in image denoising. In case of DFT or DCT domain filtering, they are capable also of local adaptive image deblurring, including blind one, and resolution enhancement. They can also implemented as 3D filters for denoising and deblurring of multi-component images such color images or sequence of video frames, in which case proc-

essing is performed in the domain of the corresponding 3D transform ([12, 15, 21]). It was shown in Ref. 17 (see also [21]), that local adaptive transform domain filters can also be treated as implementations of empirical Wiener filtering of image sub-bands, which creates a base of their comparison with wavelet denoising methods (see, for instance, [25]).

4.2 Rank filters

Local rank filters implement methods of robust statistical estimations. They are optimized in terms of loss functions other than quadratic and over different types of neighborhoods exemplified in Table 2.

Typical estimation operations are listed in Table 3.

Table 3. Estimation operations

Denotation	Definition		
MEAN(NBH)	Arithmetic mean of samples of the neighborhood		
PROD(NBH)	Product of samples of the neighborhood		
K_ROS(NBH)	Order statistics: Value that occupies <i>K</i> -		
	th place (has $rank K$) in the variational		
	row over the neighborhood. Special cases:		
MIN(NBH)	Minimum over the neighborhood (the first term of the variational row)		
MEDNIAIDI	r		
MEDN(NBH)	Central element (median) of the variational row		
MAX(NBH)	Maximum over the neighborhood (the		
	last term of the variational row);		
MODE(NBH)	Histogram mode Value of the neighbor-		
	hood element with the highest cardinal-		
	ity:		
	MODE(NBH) = arg max(H(NBH))		
RAND(NBH): A	random (pseudo-random) number taken		
	with the same gray level distribution den-		
sity as that of elements of the neighborhood			
STDEV(NBH): Standard deviation over the neighborhood			
Interquantil distance $R_ROS(NBH) - L_ROS(NBH)$,			
where $1 \le L < R \le \text{SIZE}(NBH)$.			
Range: $MAX(NBH) - MIN(NBH)$			
SIZE(NBH): Num	nber of elements of the neighborhood		

Tables 4 to 6 represent examples of one-, two- and three- stage rank filters ([20]). Note that above mentioned transform domain local adaptive filters are also included in these table as special cases (RMSE optimal linear filters in Table 5 and transform domain filters in Table 6)

Table 4. One stage (Wnbh-based) filters

Moving average filter		
$\hat{a}_k = \text{MEAN}(SwNBH)$		
"Ranked order" ("percentile") filters		
$\hat{a}_k = K \text{ROS}(SwNBH)$		
Median filter	$\hat{a}_k = \mathbf{MEDN}(SwNBH)$	

MAX-filters	$\hat{a}_k = \mathbf{MAX}(SwNBH)$		
MIN- filters	$\hat{a}_k = MIN(SwNBH)$		
Adaptive Mode Quantization filter			
$\hat{a}_k = \text{MODE}(SwNBH)$			
Local histogram equalization $\hat{a}_k = \text{RANK}(SwNBH)$			
Quasi-range: $\hat{a}_k = \text{QSRNG}(SwNBH) = \text{R_ROS}(SwNBH) \cdot \text{L_ROS}(SwNBH)$			
$\hat{a}_k = \text{STDEV}(SwNBH)$			

Table 5. Two stage (NBH2-based) filters

Table 5. Two stage (NBH ² -based) filters					
General: $\hat{a}_k =$	General: $\hat{a}_k = MEAN(FUNC(NBH))$				
RMSE optimal	linear filters				
$\hat{a}_k = \text{MEAN}(\text{MULT}_{C}(SwNBH))$					
"L-filters", "Ra	nk Selection filt	ers"; "C-filters"			
$\hat{a}_k = MEAN$	MULT_R(Sw	(NBH)			
$\hat{a}_k = \text{MEAN}(\text{MULT_RC}(SwNBH))$					
F	REPL-A - neigh	borhood filters			
Weighted median filters	$\hat{a}_k = MED$	$\mathbf{DN}(\mathbf{REPL_C}(SwNBH));$			
Weighted K- ROS - filters	$\hat{a}_k = \mathbf{K}_{\mathbf{R}}$	OS(REPL_C(SwNBH))			
Morphologi- cal filters	Dilation filter	$\hat{a}_k = \text{MAX}(SHwNBH)$			
	Erosion fil- ter	$\hat{a}_k = \text{MIN}(SHwNBH)$			
	Soft Morph. filters	$\hat{a}_k = ROS(SHnbh)$			
	V-neighborh	nood filters			
KNN- filter	$\hat{a}_k = MEAN($	$(KNV(SwNBH;a_k;K))$			
	"Sigma				
$\hat{a}_k = \mathbf{M}$	EAN(<i>EVnbh</i> (S	$SwNBH; a_k; \boldsymbol{\varepsilon}_V^+; \boldsymbol{\varepsilon}_V^-)$			
	med Mean filter	'S			
$\hat{a}_k =$					
mean(ev(v	Vnbh; MEDN($SwNBH$ $(; \boldsymbol{\varepsilon}_{V}^{+}; \boldsymbol{\varepsilon}_{V}^{-}))$			
	R-neighb				
Alpha-trimmed mean, median					
$\hat{a}_k = \text{MEAN}(Qnbh(SwNBH, R_{left}, R_{right}));$					
$\hat{a}_k = \text{MEDN}(Qnbh(SwNBH, R_{left}, R_{right}))$					
Impulse noise filtering filters:					
$\hat{a}_k = \text{MEMB}(Qnbh(SwNBH, R_{left}, R_{right}), a_k) \cdot a_k +$					
$\left[1-\text{MEMB}(Qnbh(SwNBH, R_{left}, R_{right}), a_k)\right] \times$					
$SMTH(Qnbh(SwNBH, R_{left}, R_{right}))$					
where $MEMB(NBH) = 1$, if the pixel belongs					
to NE	to NBH ; otherwise $MEMB(NBH) = 0$				

Table 6. Tree stage (*NBH*³-based) filters

Transform domain filters

"Soft" thresholding
$$\hat{a}_k = \text{MEAN}(\mathbf{H} \cdot \mathbf{T}(SwNBH))$$
,

 $\mathbf{H} = diag \left| \max \left| \mathbf{T}(SwNBH) \right|^2 - \sigma^2 / \left| \mathbf{T}(SwNBH) \right|^2, 0 \right| \right]$

"Hard" thresholding

 $\hat{a}_k = \text{MEAN}(\text{STEP}\left\{ \mathbf{T}(SwNBH) \middle| -\sigma \right\} \cdot \mathbf{T}(SwNBH) \right),$

where σ is a filter parameter,

 $\text{STEP}(x) = \begin{cases} 0, x \le 0 \\ 1, x > 0 \end{cases}$

LLMMSE- filter

 $\hat{a}_k = \left(1 - \frac{\sigma^2}{(\text{STD}(SwNBH))^2}\right) a_k + \frac{\sigma^2}{(\text{STD}(SwNBH))^2} \text{MEAN}(SwNBH)$

where σ is a filter parameter

Stack filters

 $\hat{a}_k = \text{MAX} \begin{pmatrix} \text{MIN}(SubWnbh_1), \text{MIN}(SubWnbh_2), ... \\ ., \text{MIN}(SubWnbh_n) \end{pmatrix}$

5. CONTEMPORARY TRENDS

From the present author's perspective, at least three direction of modern developments of the local approach to synthesis and evaluation of image processing algorithms can be indicated:

- Adaptive selection of the filter window shape and size
- Extending notion of pixel neighborhoods to spatially disjoint neighborhoods and multiple window processing
- Introducing local visual image quality measures.

Appropriate selection of the shape and size of the primary spatial window of local adaptive filters is of crucial importance in their application. Obviously, image restoration capability of sliding window filtering will be higher, if window size is selected adaptively in each window position. To this goal, filtering can be carried out in parallel in windows of multiple sizes and shapes and, in each window position, the best filtering result should be taken as the signal estimate in this position. This can done using methods of statistical tests such as, for instance, intersection of confidence intervals method introduced by Katkovnik et al (see, for instance, [26]). A more general option is computing signal estimates by means of linear combination of filter outputs or using statistically robust data smoothing estimation operations such as those listed in Table 3.

Yet another possibility of improving image restoration capability of local adaptive filters, especially of their image denoising capability, is associated with admitting that pixel neighborhoods used for obtaining pixel gray value estimates must not necessarily be spatially connected with the pixel under estimation, which was an assumption implicitly or explicitly assumed in the described filters. In many practical applications, informa-

tion needed for estimation of every image pixel can be collected over all appropriate areas of the entire image frame and not necessarily only in a spatially connected neighborhood of the pixel. Fig. 5 illustrates this idea on an example of a spatially disjoint EV-neighborhood of the pixel marked by the cross.

This "non-local" approach was suggested and is being developed by J.M. Morel et all ([27 –29]). An extension of this approach by means of application of local adaptive transform domain multi-component image filtering to multiple windows "non-local" image denoising was recently presented in [30]. Note also that such a non-local image smoothing can be regarded as a variety of the method of correlational accumulation ([31]).

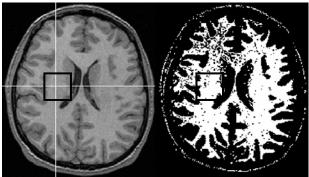


Figure 5. An illustration of "non-local" EV-neighborhood (right, white pattern) for the pixel on left image marked by the cross and $EV_{+} = EV_{-} = 10$. One can see from the figure that the EV-neighborhood of the marked pixel stretches over areas that are spatially very far from the pixel. Obviously, averaging by computing MEAN or MEDN over such an extended EV neighborhood will be much more efficient in terms of noise suppression than the averaging over any smaller spatial window such as that shown by the black box.

At last, an approach to evaluating image quality, which is essentially equivalent to the above-defined local criteria, is gaining now popularity for evaluation of image visual quality after recent publications by Zhou Wang and A. C. Bovik ([32]). It was successfully tested and confirmed in psycho-physical experiments intended for the evaluation and comparison of image compression methods.

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