ADAPTIVE MULTI-PATH CHANNEL ESTIMATION IN CDMA BASED ON PREFILTERING AND COMBINATION WITH A LINEAR EQUALIZER

E.Simon, L.Ros

LIS laboratory, Grenoble, BP 46 38402 St. Martin d'Hères cedex, France

ABSTRACT

This paper considers a downlink time-division CDMA multi-user system in the context of the vehicular-type multi-path channel. In this context, channel varies rapidly in time, and we should track directly the channel parameters in order to preserve the receiver performances. Since time-delays do not vary during a burst, only phases and magnitudes of the path coefficients have to be tracked. For this purpose, we propose to use an improved phase tracking algorithm based on prefiltering, introduced by the authors in [1], and to complete this algorithm by a magnitude tracking part. Data detection is then performed by means of a linear multi-user detector. Our analyses and simulation results show improved detection performance of the proposed adaptive receiver, as compared to the conventional adaptive LMMSE fractionally spaced equalizer.

1. INTRODUCTION

The channel estimation is a key element in many communication systems since it permits to compensate the distortions imposed by the channel on the transmitted information bearing signals. This paper adresses combined channel estimation and equalization in a multi-user linear receiver operating in the downlink of a time-division CDMA system. The receiver parameters are assumed to be well initialized (using a training sequence) so we will focus on the tracking.

When the channel varies within a burst, the parameters of the receiver must be corrected. In this case, the conventional adaptive LMMSE (Linear Minimum Mean Square Error) receiver [2][3], also called chip fractional equalizer, may be used under the standard decision-aided LMS algorithm. The standard adaptive receiver compensates channel fluctuations, and gives satisfactory performance when the rate of fading is low enough with respect to the symbol data rate. But in a rapidly time-varying multi-path channel, such an approach may not be well suited because of the great number of filter coefficients that have to be adapted, which sometimes leads to convergence problems.

In this case of rapidly changing environment, it is quite suitable to track directly the channel parameters, $i.e.$ path complex amplitudes (path time delays can in fact be assumed invariant within a burst). This tracking permits to make delay and phase compensation by channel matched filtering before performing equalization. The idea of jointly "adaptive equalization" combined with "phase compensation" was first introduced by Falconer in [4], and next used by Stojanovic [5] in underwater communications, where it was shown that it leads to good results, reducing the problem of "equalizer tap rotation".

In this paper we consider a mobile radio channel where phase tracking is assessed by means of a phase tracking loop, improved by the prefiltering technique proposed in [1]. For each path, a prefilter is designed to cancel the estimation bias due to other paths. Contrary to bias subtraction based method ([6] in section III.B.), our bias cancellation method is independant of the phase and magnitude variations, and so do not need to be updated. We propose here to use again the output of the prefilter to complete the previous algorithm in order to treat the magnitude recovery task as well. Then the proposed channel estimation algorithm is used, by way of illustration, with a linear equalizer working at a symbol rate after the matched filter.

The paper is organized as follows. The system model is given in Section 2. We introduce in the Section 3 the conventional adaptive LMMSE equalizer. The channel estimation algorithm is then presented in Section 4 and an application with an equalizer is presented in Section 5. Numerical results are given in Section 6 followed by a conclusion.

2. SYSTEM MODEL

The continuous-time baseband representation (complex envelope) of the transmitted signal is modeled as:

$$x(t) = T_s \sum_{k=1}^{K} \sum_{m} a_{k[m]} s_k(t - mT_s) \tag{1}$$

where $a_{k[m]}$ are the iid QPSK symbols with power A^2, transmitted by the k^{th} source at time mTs. K is the number of users. $s_k(t)$ is the signature of the k^{th} user, resulting from the convolution between the k^{th} spreading code $\{c_k\}$ and the half-Nyquist filter h_e (square root raised cosine filter):

$$s_k(\tau) = \sum_{q=0}^{Q-1} c_{k[q]} h_e(\tau - qT_c) \qquad (2)$$

where $T_c = T_s/Q$ is the chip duration, Q the spreading factor, and $c_{k[q]}, q = 0 \dots Q-1$ the chips. Let us assume that all K active codes are known. We suppose by convention one desired code only, the code number one. We note $\Gamma_{kj}(\tau)$ the cross-correlation function between user k and j: $\Gamma_{kj}(\tau) = \left(s_k * s_j^H\right)(\tau)$ [1].

The multi-user signal from the transmitter travels through a multipath propagation channel with L independant paths and disturbed by an additive white complex gaussian noise $w(t)$, with a two-sided power spectral density $2N_0$. The received signal is modeled as:

$$r(t) = \sum_{l=1}^{L} \alpha_l\,(t)\, x\,(t-\tau_l) \quad + \quad w(t)$$

with $\alpha_l(t) = \rho_l(t)e^{j\theta_l(t)}$ being the l^{th} complex path tap and τ_l the respective path delay.

3. CONVENTIONAL ADAPTIVE LMMSE FRACTIONALLY SPACED EQUALIZER

The structure of this fractionaly chip-time-spaced receiver [2][3](called T_c-structure in [7]) noted b_1, makes directly the linear combination of the received signal samples using a discrete multi-rate filter, with the input sampled at T_c/S and the output sampled at T_s. The integer S is the number of samples per chip, fixed to $S = 2$ in this paper. The decision variable for code #1 is thus obtained by an inner product (N_{b_1} multiplications):

$$d_{1[m]} = \underline{b_1}^T_{[m]} \cdot \underline{r}_{[m]}, \text{ where}$$

$$\underline{b_1}_{[m]} = \left[b_{1[-N'_{b_1}]}, \dots, b_{1[N''_{b_1}]}\right]^T, \text{with } N_{b_1} = N'_{b_1} + 1 + N''_{b_1}$$

$$\underline{r}_{[m]} = \left[r\left(mT_s + N'_{b_1}\tfrac{T_c}{2}\right), \dots, r\left(mT_s - N''_{b_1}\tfrac{T_c}{2}\right)\right]^T$$

From the knowledge of the transmission initial parameters, the initial values for vector $\underline{b_1}$ with MMSE criterium are easily obtained, as given in [7]. Next, the iterative updating of the equalizer coefficients, at symbol time "$m.Ts$", is based on the instantaneous error $\epsilon_{1[m]}$ computed from the decisions $\hat{a}_{1[m]}$. Using the Least-Mean-Squares (LMS) algorithm, we form: $\underline{b_1}_{[m+1]} = \underline{b_1}_{[m]} - \frac{\mu}{2}.(\nabla_{\underline{b_1}}\{|\epsilon_{1[m]}|^2\})$ where μ is the positive real step-size parameter of the algorithm. The adaptation equations are:

$$\hat{a}_{1[m]} = \ hard\ decision\ from\ \{d_{1[m]}\}$$
$$\epsilon_{1[m]} = d_{1[m]} - \hat{a}_{1[m]}$$
$$\underline{b_1}_{[m+1]} = \underline{b_1}_{[m]} - \mu\,\epsilon_{1[m]}.\underline{r}^*_{[m]}$$

[1]The exponent $(.)^H$ represents hermitian transformation *i.e.* $f^H(t) = f^*(-t)$ for a given function f and hermitian transposition for vectors

4. ADAPTIVE CHANNEL ESTIMATION

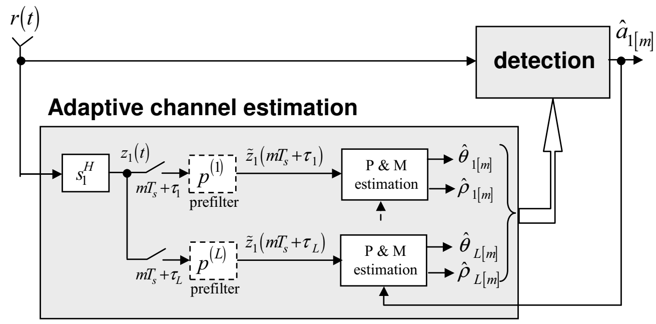

Fig. 1. Joint detection and channel estimation (conventional estimation without prefiltering, proposed estimation with prefiltering)

The topic of this section is the tracking of the channel phases and magnitudes. In [1], the authors proposed a new phase tracking algorithm, based on the concept of prefiltering, that can cope with irresolvable paths. We suggest expanding this algorithm to the magnitude tracking task, in taking use of the introduced prefiltering. The concept of prefiltering was studied before in [8] and by the authors in [9] for timing recovery in single path channel.

The conventional tracking approach uses the output of the matched filter $s_1^H(\tau)$ (matched to the desired code) as a first stage [10]. Let $z_1(t)$ be the matched filter output when $r(t)$ is applied, *i.e.* $z_1(t) = (r * s_1^H)(t)$. Then it consists of a bank of L fingers, one per path (see figure 1). On each finger the matched filter output is sampled in order to compensate the corresponding path delay (for example τ_l for the finger #l). If we assumed a flat fading channel ($L = 1$), the sampled output of the matched filter would be given by:

$$z_1(mT_s + \tau_1) = T_s\rho_1 e^{j\theta_1} a_{1[m]}\Gamma_{11}(0) + n(mT_s + \tau_1)$$

where $n(t)$ is the filtered version of $w(t)$. The system would be without interference, and both phase and magnitude estimations would be easily obtained. But in a multi-path context ($L > 1$), these estimations may be biased because $z_1(t)$ is strongly influenced by the additional paths [6]. In order to mitigate the multi-path degradation in such a way that will be explained further, we introduce a prefilter in each finger. The prefilter has a finite impulse response ($p_{[i]},\ i = -N, \dots, N$) and works at a rate of two samples per chip. The following notation will be used in the sequel: $\widetilde{f}(t) = \sum_{i=-N}^{N} p_{[i]} f(t - i\frac{T_c}{2})$, where f represents any desired function (*i.e.* $\widetilde{f}$ is the prefiltered version of f).

Let us consider the finger #l. If we assume a feedback estimation structure, data modulation is cancelled by multiplying with complex conjugate reconstructed symbols (noted $\hat{a}_{1[m]}$), and channel phase effect is corrected by multiplying with $e^{-j\hat{\theta}_{l[m]}}$, where $\hat{\theta}_{l[m]}$ is the estimate of θ_l at instant mT_s. We obtain the following sample, noted $\Omega^{(l)}_{[m]}$:

$$\Omega^{(l)}_{[m]} = e^{-j\hat{\theta}_{l[m]}}\hat{a}^*_{1[m]}\widetilde{z}_1\left(mT_s + \tau_l\right) \tag{3}$$

This sample, conditioned on the channel coefficients and the phase error $\epsilon_{\theta_{l[m]}} = \theta_l - \hat{\theta}_{l[m]}$, is on average given by:

$$\mathrm{E}\left\{\Omega^{(l)}_{[m]}\right\} = T_s A^2 \rho_l \widetilde{\Gamma}_{11}\left(0\right) e^{j\epsilon_{\theta_{l[m]}}} \tag{4}$$

$$+ \underbrace{T_s A^2 e^{-j\hat{\theta}_{l[m]}} \sum_{l_1 \neq l} \alpha_{l_1} \widetilde{\Gamma}_{11}(\tau_l - \tau_{l_1})}_{IPI} \tag{5}$$

The expectation not only consists of the desired term (4) as in the flat fading case, but it includes an additional term (5) called Inter-Path Interference (IPI). If the IPI term is non-zero, it can easily influence the signal at the finger output yielding a biased phase and magnitude estimation from $\Omega^{(l)}_{[m]}$. Notice that without a prefilter, equations (4) and (5) hold, by replacing the modified auto-correlation function $\widetilde{\Gamma}_{11}$ by the initial one Γ_{11}. The IPI term would be:

$$IPI_{nop} = T_s A^2 e^{-j\hat{\theta}_{l[m]}} \sum_{l_1 \neq l} \alpha_{l_1} \Gamma_{11}(\tau_l - \tau_{l_1}) \tag{6}$$

As used auto-correlation functions are not ideal ($i.e.$ such that $\Gamma_{11}(\tau) = \delta(\tau)$), the IPI term (6) is necessary non-zero. The purpose of the prefilter is so to cancel the term (5), and to maintain to zero this term even if the path complex amplitudes have strong variation. It is achieved by forcing to zero the correlation $\Gamma_{11}(\tau)$ at the locations of adjacent paths, $i.e.$

$$\widetilde{\Gamma}_{11}(\tau_l - \tau_{l_1}) = 0, l_1 = 1 \ldots L,\ l_1 \neq l \tag{7}$$

Note that $\widetilde{\Gamma}_{11}(0)$ must be real, leading to the following constraint: $\mathrm{Im}\left\{\widetilde{\Gamma}_{11}(0)\right\} = 0$. The prefilter coefficient expression is reported in the appendix. Then both phase and magnitude tracking tasks are based on the prefiltered sample $\Omega^{(l)}_{[m]}$ (3).

Concerning the phase, a conventional phase recovery loop, the "decision feedback loop", is inserted in each finger. The estimation of θ_l is thus updated at a symbol rate by a phase error signal $e^{(l)}_{[m]}$ filtered by the loop filter $g^{(l)}_{[m]}$:

$$\hat{\theta}_{l[m+1]} = \hat{\theta}_{l[m]} + (g^{(l)} * e^{(l)})_{[m]} \tag{8}$$

with $e^{(l)}_{[m]} = \mathrm{Im}\left\{\Omega^{(l)}_{[m]}\right\}$.

Concerning the magnitude, the method is the following: calculating $\mathrm{E}\left\{\mathrm{Re}\left\{\Omega^{(l)}_{[m]}\right\}\right\}$ yields the following useful term (by using (4)): $A^2 T_s \rho_l \widetilde{\Gamma}_{11}(0)\cos(\theta_l - \hat{\theta}_{l[m]})$. For small phase errors, $\cos(\theta_l - \hat{\theta}_{l[m]}) \approx 1$. Hence the magnitude estimation is given by:

$$\hat{\rho}_l = \frac{\mathrm{E}\left\{\mathrm{Re}\left\{\Omega^{(l)}_{[m]}\right\}\right\}}{A^2 T_s \widetilde{\Gamma}_{11}(0)} \tag{9}$$

The statistical expectation is practically computed by averaging $\xi_{l[m]} = \mathrm{Re}\left\{\Omega^{(l)}_{[m]}\right\}/A^2 T_s \widetilde{\Gamma}_{11}(0)$. The magnitude estimate at the instant mT_s is the output of a filter f when $\xi_{l[m]}$ is applied at the input:

$$\hat{\rho}_{l[m]} = (f * \xi_l)_{[m]} \tag{10}$$

We choose the following first order recursive filter: $f(z) = (1-\kappa)/(1-\kappa z^{-1})$, where κ is the filter parameter.

5. COMBINED USE WITH A LINEAR EQUALIZER

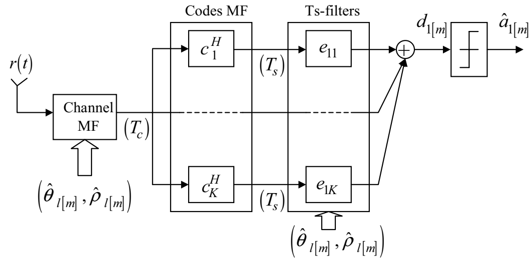

Fig. 2. Detection scheme with Ts-structure

In order to show the performance of the proposed channel estimation algorithm, we will use the estimated phases and magnitudes for the detection task, as seen in figure 1. Since the comparaisons for the simulations will be made with the conventional linear adaptive detector (LMMSE), we consider here also a linear detector. But we choose a structure of detector that makes use of the channel parameters directly in the front-end. This structure, called Ts-structure [7], is a finite length approximation of the well known infinite length theoretical optimum multi-user linear detector. It consists of a Bank of Matched Filter (MFB) applied to the global pulse shapes of all K active users, followed by a bank of synchronous discrete-time transverse filters $\{e_{1k[n]}, k = 1 \ldots K\}$, working at symbol-time, with P taps by branch (so a total of $K \times P$ coefficients) (see figure 2). The decision variable $d_{1[m]}$ is obtained by summing the K branch outputs. The channel estimation procedure is inserted in the channel matched filter. It is also used to update the filter coefficients $e_{1k[n]}$ (see [7] for the detailed computation). Those coefficients are updating every M symbols. The choice of M results from a trade-off between performance and cost.

6. NUMERICAL RESULTS

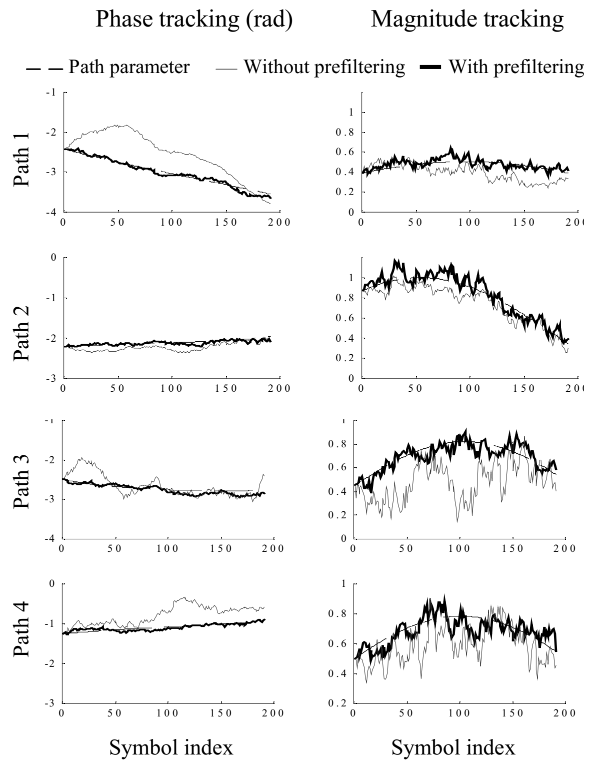

Fig. 3. Tracking, 300 Km/h, $K = 5$, $\frac{E_b}{N_0} = 20$ dB

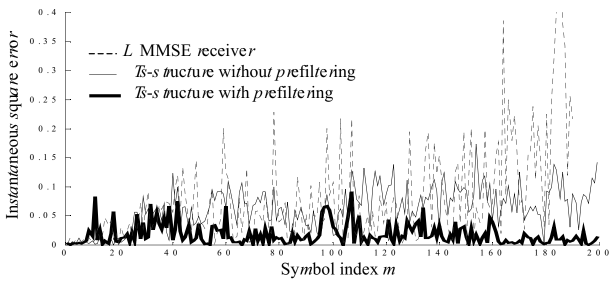

Fig. 4. Instantaneous square error, 300 Km/h, $K = 5$, $\frac{E_b}{N_0} = 20$ dB

In this section, we present a numerical analysis of the results obtained in the previous section. We use Hadamard codes of length $Q = 16$ with a scrambling sequence. The chip shaping filter is a square-root raised cosine with the roll-off factor of 0.22. For the channel model, a four tap Rayleigh fading model with the classical Jakes Doppler spectrum is used. The four channel paths are located at relative delays of $0, T_c, 2.5T_c$ and $6T_c$ with powers of $0dB$,

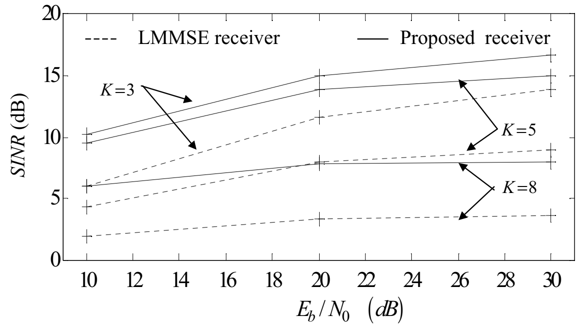

Fig. 5. Performance of both receivers, 300 Km/h

$-0.9dB$, $-4.9dB$, $-8dB$. The maximum Doppler spreading is set to $f_d = 555$ Hz, corresponding to 300 Km/h for a carrier frequency of 2 GHz. We choose a damping factor of 0.7 and a natural frequency of 800 Hz for each phase loop. Slot length is fixed to 200 symbols. The filter parameter κ is fixed to 0.8.

Simulations are carried out for both the conventional LMMSE equalizer and the proposed receiver. LMMSE equalizer is composed of $4Q$ coefficients which correspond to a length of $2T_s$. The step-size parameter μ is the optimal parameter for each slot. The equalizer bank used in our proposed receiver is composed of a total of $10 \times K$ coefficients, corresponding to a depth of $10T_s$. The filter coefficient update is such that the maximum phase change reaches 10^o. Figure 3 illustrates tracking performance over one slot for $K = 5$ users and $\frac{E_b}{N_0} = 20dB$. The tracked phases and the tracked magnitudes are shown for each path. We consider a 11 coefficient prefilter. We observe that a bias occurs without prefiltering, which is not the case with the prefilter. Figure 4 shows the resulting instantaneous square error ($i.e.$ $\left|d_{1[m]} - a_{1[m]}\right|^2$ at time mT_s, with $d_{1[m]}$ the decision variable) obtained over that slot. The Ts-structure with prefiltering outperforms the one without prefiltering. The instantaneous square error obtained with the conventional receiver is shown as well. We can see that after 150 symbols, the LMMSE receiver fails. Figure 5 shows the Signal to Interference and Noise Ratio (SINR) over different values of $\frac{E_b}{N_0}$ for $K = 3$, 5 and 8 users. The SINR values correspond to an average over 1000 slots. It is shown that the proposed receiver (the Ts-structure with prefiltering) outperforms the LMMSE one.

7. CONCLUSION

We proposed a new adaptive tracking algorithm for DS-CDMA systems operating with burst transmission in rapidly time-varying environment. We took advantage of the non-variable time delays within a burst and considered the estimation of the phase and the magnitude of propagation paths. We proposed to include a special prefiltering in the estimation of the parameters of each path, and demonstrated the satisfying obtained results. Without this prefiltering, we would have a bias on the estimates of the path parameters due to the adjacent propagation paths. We next used a simple linear detector together with the proposed estimator, and compared the receiver performance to that of the conventional linear adaptative detector (LMMSE). Simulation results confirmed the superior tracking capabilities of our receiver in fast fading conditions, over the conventional LMMSE receiver.

8. APPENDIX

The purpose of this appendix is to give an outline of the main steps leading to the prefilter coefficient expression. To this aim, let us first give the expression of the phase variance. Phase variance can be obtained from the error signal $e_{[m]}^{(l)}$: $\sigma^2 = \mathrm{E}\left\{e_{[m]}^{(l)} e_{[m]}^{(l)^*}\right\}$. σ^2 is a function of the prefilter coefficients. We note the vector $\underline{p} = \left[p_{[-N]} \dots p_{[N]}\right]^T$. σ^2 is composed of a Inter Symbol Interference (ISI) term, a Multiple Access Interference (MAI) term and a noise term:

$$\sigma^2(\underline{p}) = \sigma^2_{ISI}(\underline{p}) + \sigma^2_{MAI}(\underline{p}) + \sigma^2_{N}(\underline{p}) \tag{11}$$

where

$$\sigma^2_{ISI}(\underline{p}) = \frac{1}{2} T_s^2 A^4 \sum_{l_1=1}^{L} \rho^2_{l_1(t=0)} \sum_{n \neq 0} \left| \widetilde{\Gamma}_{11}(nT_s + \tau_l - \tau_{l_1}) \right|^2$$

$$\sigma^2_{MAI}(\underline{p}) = \frac{1}{2} T_s^2 A^4 \sum_{l_1=1}^{L} \rho^2_{l_1(t=0)} \sum_{k=2}^{K} \sum_{n} \left| \widetilde{\Gamma}_{k1}(nT_s + \tau_l - \tau_{l_1}) \right|^2$$

$$\sigma^2_{N}(\underline{p}) = A^2 N_0 \sum_{m=-N}^{N} \sum_{n=-N}^{N} \widetilde{\Gamma}_{11}\left((m-n)\frac{T_c}{2}\right) p_{[m]} p^*_{[n]}$$

The prefilter coefficients minimize this variance under the constraints defined in (7). This problem of minimization under constraints may be solved with the Lagrange multipliers. To avoid the zero solution, we choose to normalize the S-curve slope at 0. We obtain the following solution (for notational simplicity, the desired path is the first path, $i.e.\ l = 1$):

$$\underline{p}_{opt} = \frac{1}{T_s A^2 \rho_{1(t=0)}} \underline{\underline{\Gamma}}^{-1}\, \underline{\underline{G}}\, \underline{\underline{M}}^{-1} \underline{1} \tag{12}$$

where

- the vector $\underline{1}$ is $(2L \times 1)$: $\underline{1}^T = [1, 0 \dots, 0]$

- the matrix $\underline{\underline{\Gamma}}$ is $(2N+1 \times 2N+1)$. $\underline{\underline{\Gamma}}$ is such that $\sigma^2 = \underline{p}^H\ \underline{\underline{\Gamma}}\ \underline{p}$. The matrix $\underline{\underline{\Gamma}} = \underline{\underline{\Gamma_{ISI}}} + \underline{\underline{\Gamma_{MAI}}} + \underline{\underline{\Gamma_{N}}}$ contains the three disturbance terms:

$$\begin{aligned} \underline{\underline{\Gamma_{ISI}}} &= \tfrac{1}{2} T_s^2 A^4 \textstyle\sum_{l_1=1}^{L} \rho^2_{l_1} \sum_{n \neq 0} \underline{u}^{1,l_1}_{1,n} \underline{u}^{1,l_1\,H}_{1,n} \\ \underline{\underline{\Gamma_{MAI}}} &= \tfrac{1}{2} T_s^2 A^4 \textstyle\sum_{l_1=1}^{L} \rho^2_{l_1} \sum_{k=2}^{K} \sum_{n} \underline{u}^{1,l_1}_{k,n} \underline{u}^{1,l_1\,H}_{k,n} \\ \underline{\underline{\Gamma_{N}}} &= A^2 N_0 \underline{\underline{B}} \end{aligned} \tag{13}$$

where $\underline{\underline{B}} = [b_{mq}]$ with $b_{mq} = \Gamma_{11}\left((m-q)\,\frac{T_c}{2}\right)$

- the matrix $\underline{\underline{M}}$ is $(2L \times 2L)$:

$$\underline{\underline{M}} = \begin{bmatrix} \mathrm{Re}\left\{\underline{u}^{1,1}_{1,0} \underline{\underline{\Gamma}}^{-1} \underline{\underline{G}}\right\} \\ \vdots \\ \mathrm{Re}\left\{\underline{u}^{1,L}_{1,0} \underline{\underline{\Gamma}}^{-1} \underline{\underline{G}}\right\} \\ \mathrm{Im}\left\{\underline{u}^{1,1}_{1,0} \underline{\underline{\Gamma}}^{-1} \underline{\underline{G}}\right\} \\ \vdots \\ \mathrm{Im}\left\{\underline{u}^{1,L}_{1,0} \underline{\underline{\Gamma}}^{-1} \underline{\underline{G}}\right\} \end{bmatrix} \tag{14}$$

- the vector $\underline{u}^{l,l_1}_{k,n}$ is $(2N+1 \times 1)$:

$$\underline{u}^{l,l_1}_{k,n} = \begin{bmatrix} \Gamma_{k1}(nT_s + N\frac{T_c}{2} + \tau_l - \tau_{l_1}) \\ \vdots \\ \Gamma_{k1}(nT_s - N\frac{T_c}{2} + \tau_l - \tau_{l_1}) \end{bmatrix} \tag{15}$$

- the matrix $\underline{\underline{G}}$ is $(2N+1 \times 2L)$:

$$\underline{\underline{G}} = \left[\underline{u}^{1,1^*}_{1,0} \dots \underline{u}^{1,L^*}_{1,0},\ j\underline{u}^{1,1^*}_{1,0} \dots j\underline{u}^{1,L^*}_{1,0}\right] \tag{16}$$

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