Time-Frequency Spacing Design for PACE in OFDM Systems

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Abstract— **This work explores the possibility to adapt the pilot location in an OFDM frame, by choosing the more appropriate pilot pattern, rectangular or hexagonal. Based on the prediction of some channel parameters and for a given pilot density, the transmitter seeks the most appropriate time-frequency pilot space that minimize the mean square error of the channel estimate obtained by the use of a general interpolator. A comparative analysis with conventional pilot patterns over different channel environments are developed and analyzed[1].**

Index Terms— OFDM, pilot pattern.

I. Introduction

In an orthogonal frequency division multiplexing (OFDM) system, channel estimation is usually performed by sending training pilot aided channel estimation (PACE) symbols on sub-carriers known at the receiver [1] [5]. The quality of the channel estimation depends on the pilot arrangement (i.e., on the density of pilots in an OFDM frame, or on their time-frequency collocation).

Several investigations have been undertaken with the objective to develop appropriate distributions of the pilots symbols in both time and frequency domain [1][2]. Results obtained by Tufvensson et. al. in [1] have demonstrated the direct influence of the pilot location within the OFDM on the system performance of channel estimation process.

The hexagonal pilot pattern has appeared as the most efficient strategy for pilot distribution in [4][2]. Since it requires lower samples, and lower pilot density than the rectangular pattern to represent the same signal. Evaluations upon different scenarios have demonstrated it good performance compared with other pilot patterns [3].

Usually, a simple interpolator such as linear, or Spline interpolation is often used in the practice. Note that when these interpolators are used, the performance of the channel estimation is affected by the pilot pattern. Almost of the previous researches on the design of pilot patterns for channel estimation in OFDM systems have been obtained based on the computer simulation results [1][2][3]. It is more than desirable to design the PACE pattern in an analytical form without a need of exhaustive computer simulation.

In this paper, we analytically derive the optimum time-frequency spacing by minimising the MSE of the estimated channel impulse response with the use of Wiener filter. Based on the two-dimensional sampling and the time-frequency pilot location in the OFDM frames, we have developed an algorithm which consist in optimising the time-frequency spaces of the pilots within the proper OFDM frames based on the maximum frequency Doppler and the delay profiles of the channel.

Following Introduction, the system description and the channel model are described in Section II. In Section III, the method for seeking the optimum time-frequency location is derived. The use of optimum pilot pattern and its performance is verified in Section IV. Finally, concluding remarks are summarized in Section V.

II. System Description

The system used in this proposal, is based on an OFDM scheme, where different data informations are transmitted over a set of N_c sub-carriers in the frequency domain, and modulated using the inverse fast Fourier transform (IFFT). A detailed description of the OFDM system can be found in [4]. We consider the transmission of the OFDM signal over a wireless channel with the impulse response

$$h(t,\tau)=\sum_{p=0}^{P} h_p(t)\delta(\tau-\tau_p) \qquad (1)$$

where P is total the number of multi-paths, $\delta(.)$ is the *Kronecker* delta function, τ_p and $h_p(t)$ denote the delay and the complex –valued of the channel impulse response. A cyclic prefix (*CP*) is inserted to prevent the orthogonality loss between the sub-carriers.

At the receiver side, the CP is removed before applying the FFT process. Assuming a perfect synchronization at the receiver, the *k-th* sub-carrier of the *l-th* symbol time is given by,

$$R_{k,l}=H_{k,l}\,S_{k,l}+N_{k,l} \qquad (2)$$

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where $H_{k,l}$ is the frequency response of the channel at the *k-th* sub-carrier and the *l-th* symbol time, $N_{k,l}$ is the noise (additive white Gaussian noise) component plus interference term with variance σ^2. The PACEs symbols are periodically introduced within the OFDM frame during each N_t time slots, and N_f sub-carriers in the frequency domain respectively (see Fig. 1 and Fig. 2).

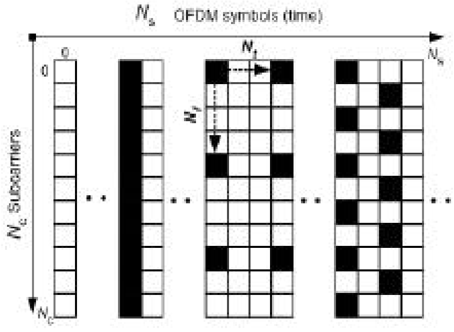

Fig. 1. Time-frequency pilot arrangement in an OFDM frame.

The structure of the pilot pattern is based on the theorem of the two-dimensional sampling [2], where any pilot pattern can be represented using two vectors, for instance **u** and **g** (*represented in the Cartesian coordinate*). These two vectors have each one two components that compose the pilot spacing in time dimension (*denoted with the sub-index* 1), and in frequency dimension (*denoted with the sub-index* 2). These two vectors have the following structure;

$$\mathbf{u} = [u_1, u_2]^T \qquad \mathbf{g} = [g_1, g_2]^T \tag{3}$$

the vectors **u** and **g** can be combined and represented by a matrix **V** such

$$\mathbf{V} = [\mathbf{u} : \mathbf{g}] \tag{4}$$

where **V** is a matrix of size (2×2) representing the two dimensional distribution of the pilots within an OFDM frame. The determinant of the matrix **V** will give us the pilot density *dp*,

$$dp = \left|\det(\mathrm{V})\right|^{-1} = \left|u_1 g_2 - u_2 g_1\right| \tag{5}$$

Although, u_1, u_2, g_1, and g_2 can be any value. For particular geometries of pilot patterns shown in Fig.1, the sampling matrix, **V,** can be written as,.

$$\mathrm{V} = \mathrm{V}_{\mathrm{fs}} = \begin{bmatrix} N_t & 0 \\ 0 & 1 \end{bmatrix}, \mathrm{V} = \mathrm{V}_{\mathrm{h}} = \begin{bmatrix} N_t & -N_t \\ N_{\mathrm{f}} & N_{\mathrm{f}} \end{bmatrix} \tag{6}$$

here the matrices V_{fs} and V_{h} denotes the block type and hexagonal pattern respectively. We assume that the value u_2 =0 without the loss of generality, since any parallelogram can be made to u_2 =0 by rotating the pattern, the pilot density can be rewritten as,

$$dp = \left|\det(\mathbf{V})\right|^{-1} = \left|u_1 g_2\right| \tag{7}$$

it can be emphasized from both equations (5) and (7), that the pilot density *dp* is inversely proportional to the pilot spacing.

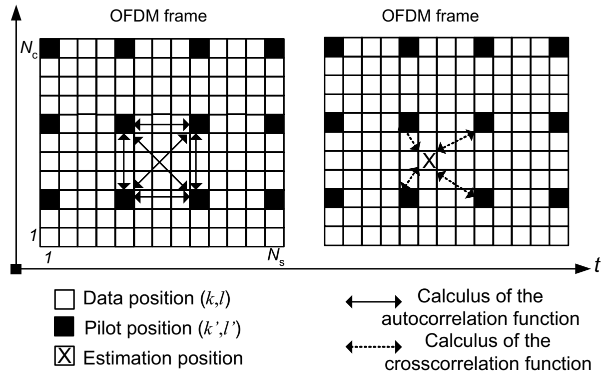

Fig. 2. A rectangular pilot spacing with N_c=11, N_s=13, N_f=5, N_t=5 and $_{tap}$=4.

The frequency response of the channel corresponding to the pilot inserted within the OFDM frame is first estimated as

$$\hat{H}_{k',l'} = \frac{R_{k',l'}}{S_{k',l'}} = H_{k',l'} + \frac{N_{k',l'}}{S_{k',l'}}, \forall\{k',l'\} \in \Omega \tag{8}$$

where *k'* and *l'* denote the sub-carrier and time symbol of the pilot symbol respectively. The final estimates of the complete channel transfer function belonging to the desired OFDM frame is obtained from the initial estimates $\hat{H}_{k',l'}$, and by a two-dimensional (2D) interpolation process. Using the 2D Wiener filtering the total frequency response of the channel is given by,

$$\hat{H}_{k,l} = \sum_{\{k',l'\}\in\Omega} w_{(k,l,k',l')}\, \hat{H}_{k',l'}, k = 1,\ldots,N_C, l = 1,\ldots,N_s \tag{9}$$

$w_{k',l',k,l}$ denotes the coefficients of the Wiener filter for the desired estimate channel, and is closely linked with each pilot position $\{k',l'\}$, $\hat{H}_{n,i}$ is the estimated channel at any frequency and time position $\{k, l\}$ within the OFDM-frame. Note that, the set of the pilot positions in the OFDM frame is Ω. The discrete (*and continuous*) 2D Wiener filtering process is well developed in [2][3], and used in a Multi-carrier code division multiple access (CDMA) system in [5]. The two dimensional Wiener filter is an optimal linear estimator in the sense that it minimise the mean square error (MSE) The filter coefficients are obtained by applying the orthogonal principle in the linear mean square estimation. Note that it is interesting to estimate a wide-sense stationary uncorrelated scattering (WSSUS) 2D stochastic process $H_{k,l}$, and also assuming that $N_{k',l'}$ has zero mean which is statistically independent from the pilot symbol $S_{k',l'}$, the estimator is chosen to be linear. The two dimensional Wiener filter coefficients (*if the correlation matrix existent*) is given by,

$$\mathbf{w}_{k,l,k',l'} = \boldsymbol{\theta}^T{}_{k,l}\, \boldsymbol{\varphi}^{-1}_{k,l,k',l'} \tag{10}$$

where the $(N_{tap}\times N_{tap})$ matrix $\boldsymbol{\varphi}_{k,`,k',`l}$ represent the autocorrelation matrix function which depends only on the distance between the pilots positions, hence, it is independent of the actual channel estimation at the $\{k, l\}$ position. The vector $\theta^{T}_{k,l}$ of size ($1\times N_{tap}$) denote the cross-correlation vector function and depends only on the distance between the channel to estimate at the frequency time position $\{k, l\}$ position respectively. It is also worth noting that theoretically to fulfill the 2D sampling theorem, the pilot spacing for periodic rectangular sampling is given by [2][3][5],

$$N_f = \frac{1}{2\tau_{max} F_s}, N_t = \frac{1}{2 f_{D max} T_s} \quad (11)$$

where f_{Dmax} and τ_{max} denote the maximum frequency Doppler, and the maximum delay respectively. F_s are the subcarrier bandwidth, and T'_s the OFDM symbol duration including the *CP* interval.

III. Optimum Pilot Location

Assuming an ideal interpolator, equation (8) can be rewritten as,

$$H_{k,l} = \sum_{\{k',l'\}\in\Omega} w_{P(k',l',k,l)}\, \widetilde{H}_{k',l'}, k = 1,\ldots,N_C,\ l = 1,\ldots,N_s \quad (12)$$

where in this case $w_{P(k',l',k,l)}$ denote the two dimensional interpolator filter coefficient used when the channel is perfectly known (*i.e. when there is not errors in the channel estimation*). An ideal interpolator could be that used in [7],

$$w_{P(k',l',k,l)} = \frac{\sin\left(\frac{\pi k'}{u_1}\right)\sin\left(\frac{\pi l'}{g_2}\right)}{\left(\frac{\pi k'}{u_1}\right)\left(\frac{\pi l'}{g_2}\right)} \quad (13)$$

On the basis of (12), the error between the actual estimated channel and the ideal one can be calculated using the mean square error which can be represented as

$$\begin{aligned} J &= E\left\{\left|\hat{H}_{k,l} - H_{k,l}\right|^2\right\} \\ &= \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} DFT(\theta_{k,l})\left|DFT\left(w_{k,l} - w_{P(k',l',k,l)}\right)\right|^2 df_1\, df_2 \\ &+ \frac{\sigma_0^2 dp}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|DFT\left(w_{k,l}\right)\right|^2 df_1 df_2 \\ &= \boldsymbol{J}_1 + \boldsymbol{J}_2 \end{aligned} \quad (14)$$

where f_1 and f_2 are the new variables in the frequency domain such that $\pi/u_1 \geq |f_1|$ and $\pi/g_2 \geq |f_2|$ [3], the *DFT* means the discrete Fourier transform. The terms $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ represents the minimum mean square due to the self-distortion and interference. The first term $\boldsymbol{J}_1$ depends on the pilot pattern. On the other hand, the term $\boldsymbol{J}_2$ is due to the interference caused essentially by the own 2D interpolator. We assume that the second term $\boldsymbol{J}_2$ is an irreducible interference term. However, we focus our analysis only upon the $\boldsymbol{J}_1$term with the purpose to reduce it effect.

Applying upon the interpolation error coefficient $\boldsymbol{J}_1$, the two-dimensional Fourier transform we obtain

$$\left|DFT\left(w_{(k,l,k',l')} - w_{P(k,l,k',l')}\right)\right|^2 = \left|DFT\left(w_{k,l,k',l'}\right) - u_1 g_2\right|^2 \quad (16)$$

In the case where the error made outside of the useful bandwidth is not considered, (16) can be approximated by the Taylor development to the following expression

$$\begin{aligned} Taylor\left[\left|DFT\left(w_{k,l} - w_{P(k',l',k,l)}\right)\right|^2\right] &= \frac{\left(u_1^2-1\right)^2}{144\,dp^2}\frac{1}{2\pi}\int_{-\pi}^{\pi} f_1^4 S_{H_k} df_1 \\ &+ \frac{\left(g_2^2-1\right)^2}{144\,dp^2}\int_{-\pi}^{\pi} f_2^4 S_{H_l} df_2 + \frac{\left(u_1^2-1\right)+\left(g_2^2-1\right)}{72\,dp^2} \\ &+ \frac{1}{2\pi}\int_{-\pi}^{\pi} f_1^2 S_{H_k} df_1 \frac{1}{2\pi}\int_{-\pi}^{\pi} f_2^2 S_{H_l} df_2 \end{aligned} \quad (17)$$

where S_{Hk} and S_{Hl} are the spectral density function of the channel in the frequency and time domain respectively. In fact the $\boldsymbol{J}_2$ term is independent of the pilot position. Therefore, the more appropriate pilot position must be chosen in such way that the value of $\boldsymbol{J}_1$ becomes the much reduced as possible.

Assuming that u_1 and g_2 are continues variables the optimum pilot spacing that minimise the term $\boldsymbol{J}_1$ is obtained by looking for the zero in it derives

$$\left.\frac{\partial J_1}{\partial u_1}\right|_{u_1=\tilde{u}_1} = 0, \quad \left.\frac{\partial J_1}{\partial g_2}\right|_{g_2=\tilde{g}_2} = 0 \quad (18)$$

the calculus of both functions in (18), arise the following equations

$$\begin{aligned} \tilde{u}_1 &= \frac{1}{\sqrt{dp}}\left(\frac{\frac{1}{2\pi}\int_{-\pi}^{\pi} f_2^4 S_{H_l}(f_2) df_2}{\frac{1}{2\pi}\int_{-\pi}^{\pi} f_1^4 S_{H_k}(f_1) df_1}\right)^{1/8} \\ \tilde{g}_2 &= \frac{1}{\sqrt{dp}}\left(\frac{\frac{1}{2\pi}\int_{-\pi}^{\pi} f_2^4 S_{H_l}(f_2) df_2}{\frac{1}{2\pi}\int_{-\pi}^{\pi} f_1^4 S_{H_k}(f_1) df_1}\right)^{-1/8} \end{aligned} \quad (19)$$

the values of the nominators in (19) are the 4th order moment of the Doppler spectrum and the power delay profiles respectively. It can be noted from (19) that the variables $\tilde{u}_1$ and $\tilde{g}_2$ are directly linked with the pilot density *dp*. Also it can be observed that both variables depend upon the channel parameters. Therefore they depend on the frequency Doppler and the power delay of the channel.

While the variable $\tilde{u}_1$ depends directly from the temporal axis, and indirectly from the frequency axis. However, the variable $\tilde{g}_2$ depends indirectly from the temporal axis and directly from the frequency axis. In the case where the mobility increase the value of $\tilde{u}_1$ raise, while $\tilde{g}_2$ diminish. Therefore, when $\tilde{u}_1$ goes to infinity, $\tilde{g}_2$ goes to zero, and the pilot location strategy resulting a block type of pilots as that usually used in WLAN systems or in indoor environment.

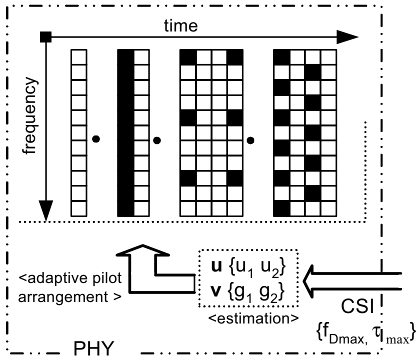

Fig.3. Description of different pilot system locations

This system is still not complete, because of both vectors **u** and **g** have each one two component (see (3)), and those used during the previous analysis have only three values $(\tilde{u}_1, \tilde{u}_2, \tilde{g}_2)$. The value of g_1 has not been determined. It can be checked in previous investigations [3][6] that, choosing g_1 equal to the half distance of $\tilde{u}_1$, it is possible to obtain an hexagonal pilot pattern, which is considered as the best distribution that minimizes the interpolation error. Because the pilots involved in the two-dimensional interpolation process have shorter time-frequency distances [3][5][6].

IV PERFORMANCE EVALUATION

In our simulations, we have considered an OFDM modulation with the system and channel parameters indicated in Tab.1. A range of the signal to noise ratio from 5 up to14 dB, and two $\tau_{\max}$ values equals to 50 and 100 ns have been use. Each figure legend depicts the used pilot pattern (*and whether the optimum spacing is used or no*t).

TABLE I
SIMULATION PROPERTIES

Parameters	Values
Total Bandwidth	20 MHz
Number of Carriers	N_c= 256
Frequency Carrier	5.8 GHz
Modulation	QPSK
Channel	Rayleigh
Number of Paths	8
Maximum channel delay $\tau_{\max}$	50 ns and 100 ns (*equi-spaced uniform*)
Velocity	60 Km/h and 100 Km/h
Cyclic prefix	2 µs
Filtering type	Two dimensional
Signal to Noise Ratio range [dB]	from 5 up to 14.
Equalization type	MMSE

It can be shown in Fig. (4) the bit error rate (BER) performances when different geometries of pilot patterns are used. During the simulation process it has been maintained the same density of pilot *dp*. Either, the rectangular and the hexagonal pilot patterns perform similar BERs when the optimum time-frequency spacing algorithm (*that means the use of (19)*) is used. However, the conventional hexagonal pattern without use of the optimum insertion provides worst performance as the SNR increase up to 14 dB.

The same behaviour can be observed in the figures (5) (6) and (7), even when different values of *v*, $\tau_{\max}$ and density of pilots are used. However, in Fig. (7), similar performances of BER are experienced with the three schemes for the SNR values proximally lower than 8 dB. Above 8 dB, the optimum-hexagonal scheme outperform the rest of the schemes, but with a small difference, except at SNR= 14 dB.
Note that in almost the figures, the schemes (*optimum rectangular and hexagonal*) that use the optimum time-frequency spacing based on (19) outperform the simply hexagonal spacing, except in Fig. (7).
This could be explained by the fact that, if we increase the pilot density, the optimum pilot time-frequency spacing based on (19) loses it interest. Since one main objective behind using different pilot pattern geometries is to be able to use the lowest possible density of pilot, and at the meantime locating the PACES using the most appropriate time frequency spacing, in order to achieve the best CSI acquisition at the receiver.

A general observation can be made, whish is; it doesn't matter the geometry of the pilot patterns whether the algorithm provided by the (19) is used. In the case where the pilot spacing is deflected from the optimum time-frequency location, a degradation of the system performance can be experienced, even when an hexagonal pattern is employed.

V. CONCLUSIONS

An adaptive algorithm (*base on* (19)) for the pilot insertion within the OFDM frames has been developed in this paper. The proper algorithm adapt the time-frequency pilot spaces according to the changes that experience the channel parameters $\tau_{\max}$ and $f_{D\max}$. Whether the previous investigations [2][3][5] have demonstrate the superiority and viability of the hexagonal pilot pattern, compared with the rectangular or the diagonal scheme, this work point out that, the performances of the hexagonal pattern geometry can be improved when the optimum time-frequency spacing of the pilots is chosen.
Note that during the simulations any kind of coding has been considered, which in any case will improve the system performance.

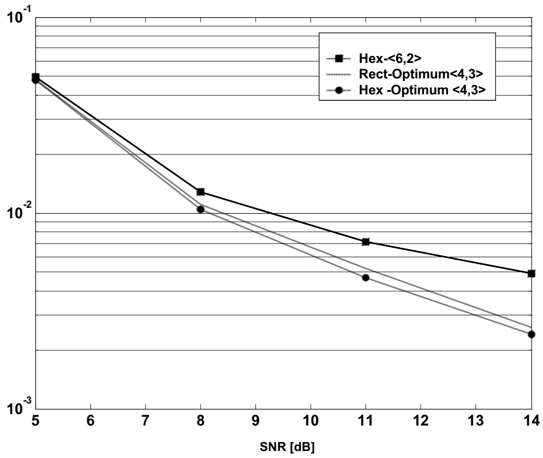

Fig. 4. OFDM system performance with velocity v=60 Km/h, τ_{max}=50ns, and dp=8.33%

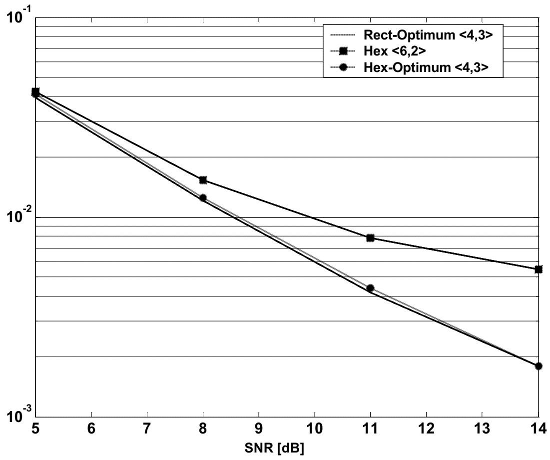

Fig. 5. OFDM system performance with velocity v=100 Km/h, τ_{max} = 50 ns, dp = 8.33%.

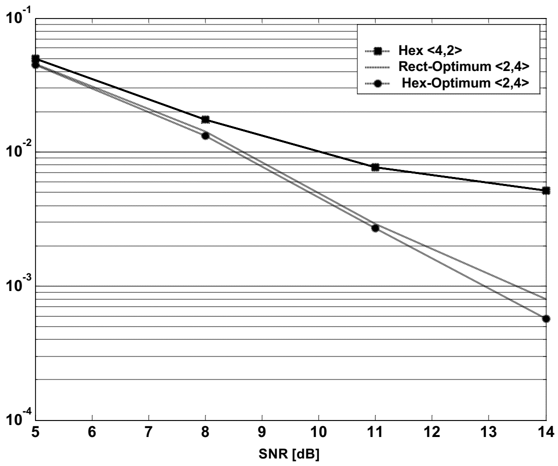

Fig. 6. OFDM system performance with velocity v=60 Km/h, τ_{max}=100ns, and pilot density dp=12.5%.

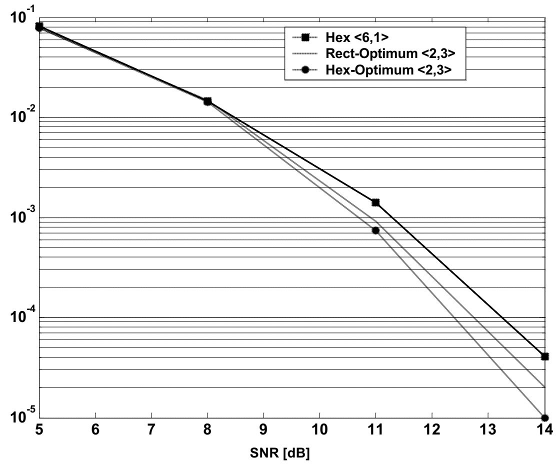

Fig. 7. OFDM system performance with velocity v=100 Km/h, τ_{max} =100ns, dp=16.6%

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