A MIMO-HARQ Cross-Layer Throughput Metric applied to V-BLAST

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***Abstract*— This paper addresses the combination of Hybrid Automatic Retransmission Request (HARQ) processing with multiple input multiple output (MIMO) data transmission. The objective is twofold. First, we evaluate a reliable throughput performance metric that composes MIMO as well as HARQ protocol parameters in a single expression. Due to its generic property, this metric can be used by higher layer protocols for traffic engineering in all IP next generation networks. Hence, it facilitates cross-layer design. In particular, we perform an information theoretic consideration by employing the conditional cutoff rate for MIMO transmission while the HARQ processing is described by the renewal-theory. In a second step, we illustrate the generic property of this metric by its straight forward application to a V-BLAST scheme with MMSE detection. The V-BLAST scheme performs a HARQ process per transmit antenna (per data stream) where each data stream is transmitted via a SIMO (single input multiple output) channel. As a result, this paper also indicate that the proposed metric gives hints on designing MIMO-HARQ systems and it can also be employed for packet data MIMO modelling.**

I. Introduction

Next generation wireless systems will support high end-to-end service quality based on packet-data oriented data transmission. To guarantee high transmission quality over the highly error-prone mobile radio channel, advanced error handling and recovery are implemented in terms of retransmitting damaged data, often referred to as Hybrid Automatic Retransmission Request (HARQ). An analysis of HARQ protocols which is based on the renewal theory [1] has been done in [2]. In [3] Caire et. al. have evaluated a closed-form expression of the throughput of HARQ protocols for the Gaussian collision channel. It is also shown therein that the codeword error probability of random codes combined with HARQ processing can be reduced to the information outage probability. In [4] similar results are given for the case that always the maximum number of transmissions per packet is assumed. By applying random coding with typical set decoding and Gaussian codebooks, Caire et. al. also proves in [3] the existence of codes resulting asymptotically (very large code word length) in zero error codeword probability for each set encoder and any channel fading state if the transmitted rate is below the conditional mutual information over the retransmissions (information outage).

Recently, multiple input multiple output (MIMO) technology has been shown to tremendously increase the spectral efficiency of wireless communication systems [5], [6]. Meanwhile a multiplicity of different MIMO techniques have been proposed [7] that seek either to increase the diversity order (space-time coding) or the information rate of the system (spatial multiplexing).

In combination with HARQ schemes, many approaches exist mapping the HARQ processing on MIMO channels. A simplistic way is to apply a HARQ process for each data stream in a spatial multiplexing MIMO systems such as BLAST (Bell Labs Layered Space-Time) [8]. Several specifically selected MIMO-HARQ combinations has been investigated in [9] for linear spatial receivers or in [10] for space time block codes such as Alamouti coding [11]. However, both papers lacks due to an insufficient codeword error analysis for forward error channel coding which extremely determines the MIMO-HARQ performance.

Inspired by the Caire's work in [3] this paper gives an information theoretic consideration of combining HARQ processing with MIMO data transmission. The first objective is to evaluate a reliable throughput performance metric that composes MIMO as well as HARQ protocol parameters. To do this, we extend the results given in [3] to MIMO transmission and also carry out the analysis for finite symbol constellations such as QAM. Rather than employing the conditional mutual information as in [3], we evaluate an upper bound on the throughput of MIMO-HARQ processing by utilizing the conditional cutoff rate of a MIMO transmission. The conditioning is done on a specific MIMO Rayleigh fading channel realization.

It should be stressed that the evaluated metric is very generic and can be simply extended towards specific MIMO transmission schemes such as the V-BLAST scheme. In addition, the metric facilitates the inclusion of multiple user access in terms of code division or space division multiple access for instance. In order to indicate the generic fashion of that metric, in this paper, we apply the metric in a straight forward way to a V-BLAST scheme with minimum mean square error (MMSE) detection. The V-BLAST scheme performs a HARQ process per transmit antenna (per data stream) where each data stream is transmitted via a SIMO (single input multiple output) channel. As a result, this paper also indicate that the proposed metric gives hints on designing MIMO-HARQ systems.

The paper is organized as follows. Upon describing the

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HARQ processes, we also evaluate the cross-layer throughput metric based on the renewal-theory in section II. Section III extend this metric towards MIMO-HARQ transmission utilizing the conditional MIMO cutoff rate. To indicate the generic property of the metric section IV, we apply the metric in a straight forward manner to the V-BLAST system with MMSE detection. Results are shown in section V and section VI finally summarizes the paper.

II. HARQ-Throughput

In this section, we evaluate an expression of the throughput of HARQ processing by utilizing the renewal-theory [2]. The analysis is valid under the following assumptions. (1) An infinite buffer occupancy for each user. Thus, as soon as a packet is successfully delivered at the receiver the next packet is encoded and transmitted in the next available transmission interval. (2) We assume error- and delay-free feedback channels. (3) Each users accesses the channel with an average probability $\bar{p}$. That means, on each transmission interval[1] each user transmits a message with probability $\bar{p}$ and does not transmit with probability $1-\bar{p}$. This approach allows for simple modelling scheduling strategies [12]. (4) Only one single user transmits per transmission interval. Multiple user access can be included by extending the below described single user MIMO channel signal model to a multiple access model [7], [13]. Nevertheless, each user runs its own HARQ protocol independently of the other users likely to transmit.

Let us define the throughput per user to be

$$\eta = \lim_{l \to \infty} \frac{R(l)}{l} \quad \text{[b/s/Hz]} \tag{1}$$

with $R(l)$ the sum of number of bits per second per hertz (mutual information) successfully decoded up to slot l. Note that the stop of packet transmission is a recurrent random event associated with the random variable $\mathcal{R}$. By applying the renewal-reward theorem [1] we obtain for (1)

$$\eta = \frac{E\{\mathcal{R}\}}{E\{\mathcal{T}\}}, \tag{2}$$

where $\mathcal{R}$ is the random reward with $\mathcal{R} = R$ b/s/Hz if transmission stops due to successful decoding. If the transmission of the packet fails we have $\mathcal{R} = 0$ b/s/Hz. The random variable $\mathcal{T}$ describes the random time between two consecutive occurrences of the recurrent event and is called inter-renewal time [1]. Next we have to fix a stop of transmission. To do that, we assume a maximum number of transmissions per packet denoted by M, and in addition, a maximum acceptable number of slots N defining the delay constraint (note $M \leq N$). Transmission of a current packet stops if (1) the packet is successfully decoded at the m-th transmission in any slot n with $m \leq M$ and $n \leq N$, or (2) no successful decoding occurs up to transmission $m = M$ with $n \leq N$ or (3) no successful decoding occurs within the delay constraint $n = N$, and $m \leq M$.

[1]Herein, the expressions transmission interval and slot are used interchangeably.

Finally, let event $\mathcal{A}_m$ to be the event of successful decoding at the m-th transmission of the current packet. Due to the assumed HARQ protocol the events $\bar{\mathcal{A}}_m$ and $\mathcal{A}_m$ only occurs iff we have sequence $\{\bar{\mathcal{A}}_1, \ldots, \bar{\mathcal{A}}_{m-1}\}$. In other words, iff all preceding $m-1$ transmissions of the current packet failed. With these definitions the probability of unsuccessful decoding at the m-th transmission if all $m-1$ preceding decoding attempts failed can be expressed by

$$p(m) = P\{\bar{\mathcal{A}}_1, \ldots, \bar{\mathcal{A}}_m\}. \tag{3}$$

Respectively, the probability of having successful decoding at the m-th transmission is given by $q(m) = P\{\bar{\mathcal{A}}_1, \ldots, \bar{\mathcal{A}}_{m-1}, \mathcal{A}_m\} = p(m-1) - p(m)$.

Caire et. al. give in [3] a closed-form expression of (2) by computing $E\{\mathcal{R}\}$ and $E\{\mathcal{T}\}$. We leverage this result and assume hereafter for simplification that the transmission is restricted only by a maximum number of transmissions per packet. Thus, we have $N \to \infty$ but still $m \leq M$ which is named truncated HARQ [10]. Then, the per user throughput in (2) can be expressed by

$$\eta = \bar{p}\, \frac{R\,(1 - p(M))}{\sum_{m=0}^{M-1} p(m)}, \tag{4}$$

with $p(0) = 1$ per definition [3]. For the ALOHA protocol $p(m) = p(1)^m$ which leads to the expression $\eta = (1-p(1))\,R$ if the user always accesses the channel ($\bar{p} = 1$) [10]. Equation (4) clearly indicate to be a cross-layer throughput metric. On one hand the physical layer is captured by the probability of unsuccessful decoding $p(m)$. On the other hand parameter $\bar{p}$ represents the medium access control (MAC) layer since scheduling a number of users all likely to access the channel is applied in the MAC.

III. Cutoff Rate based MIMO-HARQ metric

Consider a flat Rayleigh fading MIMO channel with M_t transmit antennas and M_r receive antennas. The received signal vector is given by

$$\mathbf{y} = \sqrt{\frac{P}{M_t}}\mathbf{H}\mathbf{x} + \mathbf{n}, \tag{5}$$

where $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is composing the channel characteristics, P denotes the transmitted power, $\mathbf{x}$ the transmitted signal vector composed of statistically independent space-time encoded symbols drawn from a scalar constellation (such as QAM) with unit energy and alphabet size A. Finally, $\mathbf{n}$ represents zero mean circularly symmetric complex Gaussian noise with covariance $E\{\mathbf{n}\mathbf{n}^H\} = N_0\mathbf{I}_{M_r}$. Superscript H stands for conjugate transpose and $\mathbf{I}_{M_r}$ is the identity matrix of dimension M_r. Hereafter, we assume perfect channel knowledge at the receiver and an unknown channel at the transmitter.

In order to evaluate the proposed throughput metric, we apply an information-theoretic consideration to compute the codeword error probability $p(m)$ given in (3) for the HARQ-protocol assumed. In case of SISO channels, Ozarow et. al. always performed decoding after M transmissions in [4] and showed therein that the probability of decoding failure for large codeword lengths L is given by the information outage probability [6]. The coding assumed is such that it conveys the information message in M blocks. The communication system transmits the message in L super symbols where a super symbol is a M-tuple constructed by joining together M symbols, each sent in a different block. For a given realization of the fading channel the channel is a time-invariant memoryless channel for which the coding theorem assures that transmissions with rates close to the conditional mutual information with high reliability are possible for very large L. The conditional mutual information is over M blocks, where the conditioning is on a specific channel realization (block fading channel).

Caire et. al. extended these results in [3] and proved the existence of random codes with set decoding property resulting asymptotically (very large L) in zero error codeword probability for each set encoder $(m \leq M)$ and any channel fading state if and only if the transmitted rate is below the conditional mutual information over the $m \leq M$ blocks. To summarize, the results given in [3], [4] facilitates to perform the codeword error analysis by investigating the information outage probability.

If we assume statistically independent block fading the conditional mutual information over $m \leq M$ transmissions is given by

$$I^m_{|\{\mathbf{H}\}} = \sum_i^m I_{|\mathbf{H}_i} \qquad (6)$$

with $I_{|\mathbf{H}_i}$ as conditional mutual information for the i-th transmitted block being subject to the channel realization $\mathbf{H}_i$. For circularly symmetric complex Gaussian inputs we have

$$I_{|\mathbf{H}_i} = \log_2 \det(\mathbf{I} + \frac{P}{M_t N_0}\mathbf{H}_i\mathbf{H}_i^H)$$

[6]. We now extend the investigations done in [3], [4] to MIMO transmission in a straight forward way. Thus, we assume random space-time encoding where each space-time codeword composes M set-codewords each of length L and express the probability of unsuccessful decoding in the m-th transmission block by the outage probability

$$P_{out} = P\{I^m_{|\{\mathbf{H}\}} < R\}. \qquad (7)$$

Here, R has to be interpreted as the transmitted rate in b/s/Hz with $R \leq M_t \log_2(A)$ due to the finite constellation size.

Note that we draw the code symbols from a scalar constellation in this paper. Rather than employing the corresponding mutual information of discrete modulations (6), we perform our throughput analysis on the conditional cutoff rate denoted by $R_{0|\mathbf{H}_i}$. The conditioning is on the fading channel and for equally likely transmit vectors $\mathbf{x}$ we have

$$2^{-R_{0|\mathbf{H}_i}} = \int_{\mathbf{y}} \left[\frac{1}{A^{M_t}} \sum_{\mathbf{x}} \sqrt{f_{\mathbf{y}}(\mathbf{y}|\mathbf{x}, \mathbf{H}_i)}\right]^2 d\mathbf{y}, \qquad (8)$$

where $f_{\mathbf{y}}(\mathbf{y}|\mathbf{x}, \mathbf{H}_i)$ denotes the channel transition pdf. For the assumed signal model (5) we can express the conditional cutoff rate by [14]

$$R_{0|\mathbf{H}_i} = \log_2 \left(\frac{1}{A^{M_t}} + \frac{\sum_{\mathbf{x}}\sum_{\mathbf{z}\neq\mathbf{x}} \beta(\mathbf{x}\to\mathbf{z}|\mathbf{H}_i)}{A^{2M_t}}\right)^{-1}. \qquad (9)$$

Here,

$$\beta(\mathbf{x}\to\mathbf{z}|\mathbf{H}_i) = \exp(-\frac{P}{4\,M_t\,N_0}||\mathbf{H}_i(\mathbf{x}-\mathbf{y})||_F^2) \qquad (10)$$

which is the Chernoff upper-bound on the probability that the receiver decodes transmitted codevector $\mathbf{x}$ as codevector $\mathbf{y}$ assuming ML decoding [7]. Analogous to (6) the conditional cutoff rate over $m \leq M$ transmissions is

$$R^m_{0|\{\mathbf{H}\}} = \sum_i^m R_{0|\mathbf{H}_i}. \qquad (11)$$

Since for any channel realization $I_{|\mathbf{H}_i} \geq R_{0|\mathbf{H}_i}$ we obtain with (6) and (11) an upper bound for the codeword error probability at the m-th transmission by

$$P\{I^m_{|\{\mathbf{H}\}} < R\} < P\{R^m_{0|\{\mathbf{H}\}} < R\}.$$

Using the outage probability expressed by the cutoff rate (uper bound) the event of an unsuccessful decoding in the m-th transmission block can be expressed by

$$\bar{\mathcal{A}}_m = \{R^m_{0|\{\mathbf{H}\}} < R\}. \qquad (12)$$

Up to now, we neglected the memory of the HARQ-protocol in our information-theoretic consideration. Remind that the probability of unsuccessful decoding at the m-th block for the HARQ protocol assumed is given by the probability of the sequence $\{\bar{\mathcal{A}}_1, \ldots, \bar{\mathcal{A}}_{m-1}, \bar{\mathcal{A}}_m\}$. For any specific $\mathbf{H}_i$ we have $R_{0|\mathbf{H}_i} \geq 0$. Thus, the random sequence $R^1_{0|\{\mathbf{H}\}}, \ldots, R^m_{0|\{\mathbf{H}\}}$ is nondecreasing with probability 1. Then, $\bar{\mathcal{A}}_m \subseteq \bar{\mathcal{A}}_l$ for all $m \geq l$ and we can write

$$p(m) = P\{\bar{\mathcal{A}}_1, \ldots, \bar{\mathcal{A}}_m\} = P\{\bar{\mathcal{A}}_m\}. \qquad (13)$$

Inserting (12) into (13) results in

$$p(m) = P\{R^m_{0|\{\mathbf{H}\}} < R\}, \qquad (14)$$

where $R \leq M_t \log_2(A)$. This result indicates that due to the monotone behavior the probability of unsuccessful decoding $p(m)$ for the HARQ process assumed equals the outage probability that occurs at the m-th transmission. Note again by (11) that $R^m_{0|\{\mathbf{H}\}}$ equals the sum of m conditional cutoff rates reflecting the memory of the HARQ process assumed. For the memoryless ALOHA protocol all events $\bar{\mathcal{A}}_i$, $i = 1, \ldots, m$ are i. i. d. leading to $p(m) = P\{R_{0|\{\mathbf{H}_i\}} < R\}^m$ for any $i \in \{1, \ldots, m\}$.

In order to finally compute the MIMO-HARQ throughput we simply insert (14) into (4) and obtain

$$\eta = \bar{p}\,\frac{R\,(1 - P\{R^M_{0|\{\mathbf{H}\}} < R\})}{\sum_{m=0}^{M-1} P\{R^m_{0|\{\mathbf{H}\}} < R\}}. \qquad (15)$$

IV. V-BLAST-MMSE METRIC

In order to show the generic characteristic of the metric proposed, as an specific example, we will shortly explain its application to the V-BLAST scheme [15] with MMSE detection. The V-BLAST architecture incorporates multiple scalar encoders, one per transmit antenna. Thus, the data stream is de-multiplexed into multiple substreams being separately encoded and radiated from various transmit antennas. In this architecture we have per antenna HARQ processing. Within this paper, we assume V-BLAST detection at the receiver with linear minimum mean square error (MMSE) nulling algorithm to suppress inter-layer interference [7], [15]. The estimated signal vector on the transmitted layers, $\tilde{\mathbf{s}}$, is obtained by applying an equalizing matrix $\mathbf{W}$ to (5) as $\tilde{\mathbf{s}} = \mathbf{W}^H\mathbf{y}$. The MMSE matrix is given by $\mathbf{W} = \sqrt{\frac{M_t}{P}}\left[\mathbf{H}_i\mathbf{H}_i^H + (\frac{M_t}{P})N_0\mathbf{I}_{M_t}\right]^{-1}\mathbf{H}_i$ while the MMSE covariance matrix can be shown to be [7]

$$\mathbf{Q}_i = (\mathbf{I}_{M_t} + \frac{P}{M_t N_0}\mathbf{H}_i^H\mathbf{H}_i)^{-1}. \qquad (16)$$

Again, index i denotes the number of transmissions. Next, the post processing unbiased SINR for the l-th substream is given by

$$\gamma_{l,i} = ([\mathbf{Q}_i]_{l,l})^{-1} - 1. \qquad (17)$$

Here, $[\mathbf{A}]_{l,l}$ denotes the l-th diagonal element of a matrix $\mathbf{A}$.

After the MMSE detection each data stream reflecting one transmit antenna is fed into a scalar decoder. Thus we have M_t independent HARQ processes where each one can be described by a cutoff rate for single scalar transmission. Utilizing the SINR per data stream and per transmission (17) the cutoff rate for q-PSK modulation can be expressed by [16]

$$R_{0,l,i} = -\log_2\left(\frac{1}{q}\sum_{j=0}^{q-1}\exp(-\gamma_{l,i}\sin^2(\pi j/q)\right). \qquad (18)$$

With (18) we can compute the codeword-error probability per data stream

$$p_l(m) = P\{R_{0|\{\mathbf{H}\}}^{m,l} < \frac{R}{M_t}\} \qquad (19)$$

with $R_{0|\{\mathbf{H}\}}^{m,l} = \sum_{i=1}^{m} R_{0,l,i}$. Note that for simplification purposes we assume a uniform splitting of the data rate R onto the M_t data streams. So, we have R/M_t as per stream data rate. Nevertheless, the metric is also applicable to different per antenna data rates. By inserting (19) into (4) we obtain the per stream throughput η_l. Since we have M_t independent HARQ processes the total throughput is the sum of per stream throughputs, e.g. $\eta = \sum_{l=1}^{M_t}\eta_l$.

V. RESULTS

Fig. 1 shows the per user throughput η versus the transmitted data rate R for the MIMO-HARQ scheme. We applied BPSK and the signal-to-noise ratio (SNR) is set to be $P/N_0 = 0$ dB. The MIMO configuration is (2×2) while SISO (1×1) acts as reference. We choose $M = 1$ (only one transmission allowed) and for HARQ processing $M = 4$.

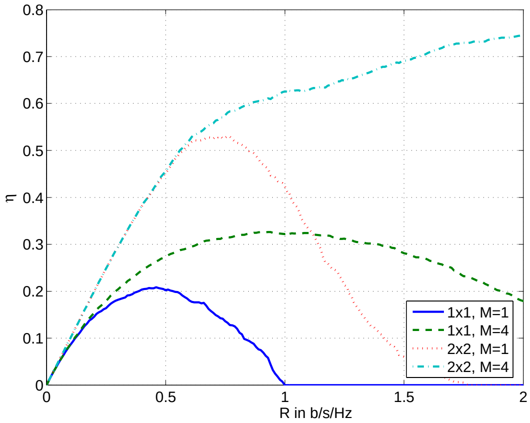

Fig. 1. Throughput vs. transmitted rate, antenna configurations (1×1) and (2×2), maximum # of transmissions $(M = 1, M = 4)$, BPSK, and $P/N_0 = 0$ dB

Note that for SISO the rate is restricted to $R \leq 1$ b/s/Hz while for the MIMO configuration $R \leq 2$ b/s/Hz. The results clearly indicate the increase in throughput by applying MIMO transmission. Fig. 1 further shows the gain due to HARQ processing. Moreover, the results also give a first hint on designing a packet data oriented network. For larger maximum number of transmission M we observe a more flat maximum in throughput resulting in a more robust performance when computing the optimum rate (max. η) to be transmitted.

Assume in the following a scheduling strategy that allocates an optimum data rate R the user has to transmit to in order to get the maximum throughput for each signal-to-noise ratio P/N_0. Its worth mentioning that we have to deal with a constraint non-convex optimization problem with $0 \leq R \leq M_t\log_2(A)$ where the global maximum strongly depends on the signal-to-noise ratio. Fig.2 depicts the HARQ throughput

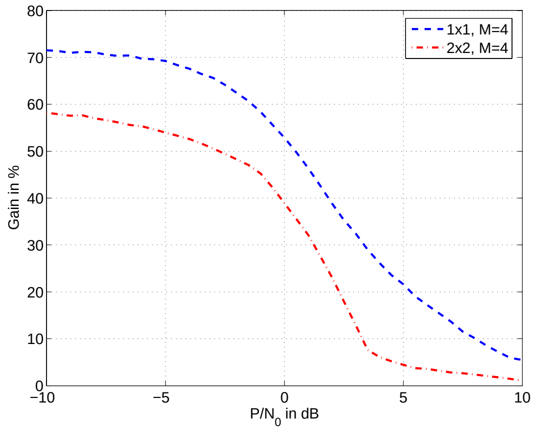

Fig. 2. Throughput gain of HARQ processing for different P/N_0 values: For each of the graphs the reference system is the one with $M = 1$.

gain for the MIMO as well as SISO system versus different P/N_0 values for such an optimum scheduling strategy. In case of the MIMO-HARQ scheme with $M = 4$ the reference system is the MIMO scheme with $M = 1$, and for the SISO-

HARQ scheme with $M = 4$ it is the SISO scheme with $M = 1$, respectively. Fig.2 shows that HARQ-transmission leads to a performance gain in the lower SNR region while less gain is observed for high SNR values. It is worth mentioning that for the SNR range analyzed we observe less throughput gain for HARQ processing when we apply MIMO compared with SISO. Especially high SNR ranges that favors MIMO application [7] indicate moderate HARQ-processing gains only (less than 10%).

Next, Fig. 3 compares the MIMO transmission scheme and maximum likelihood decoding, further denoted by MIMO, with the V-BLAST scheme and MMSE detection in terms of throughput vs. transmitted data rate R. Again, $P/N_0 = 0$ dB

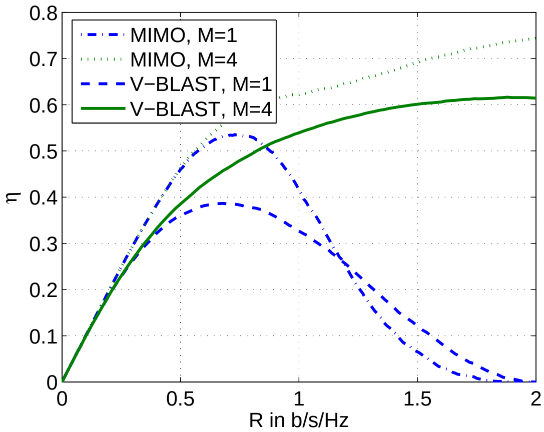

Fig. 3. Throughput vs. transmitted rate for MIMO and V-BLAST, (2×2) antenna configuration, maximum # of transmissions $(M = 1, M = 4)$, BPSK, and $P/N_0 = 0$ dB

and $M = 1$ as well as $M = 4$. The results indicate a significant higher maximum throughput for the MIMO scheme. Especially for $M = 4$ the MIMO scheme outperforms the V-BLAST with MMSE versus the entire data rate range. Note that the MIMO scheme applies maximum likelihood decoding and one single HARQ process via the MIMO transmission while the V-BLAST scheme with MMSE performs multiple HARQ processes (one per data stream) where each data stream is transmitted via a SIMO transmission degraded by inter-layer interference. But for $M = 1$ (no HARQ), it is interestingly to observe a steeper descent for the MIMO scheme which can be explained by a steeper increase in codeword error probability. This requires a more sensitive system design in terms of defining the optimum data rate to be transmitted with. Also note that due to the steep descent, in a high R range, the V-BLAST scheme with MMSE outperforms the MIMO scheme.

VI. Conclusion

Utilizing the results in [3], [4] we evaluated a throughput metric for MIMO-HARQ processing by employing the conditional cutoff rate for MIMO transmission while the HARQ-processing is described by the renewal-theory. We illustrated in this paper that the proposed MIMO-HARQ metric is very generic and can be applied by simple modifications to specific MIMO-HARQ schemes. Herein, we chose the V-BLAST scheme with MMSE detection as an example. As results, the metric gives hints on designing MIMO-HARQ transmission schemes avoiding highly computational intensive simulations. Furthermore, the metric can be employed by higher layer protocols for a cross-layer design. Thus, traffic engineering algorithms can utilize this metric for efficient data transmission in all IP next generation networks.

References

[1] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, Singapore, 3-rd edition, 1991.
[2] M. Zorzi and R. R. Rao, "On the use of renewal theory in the analysis of ARQ protocols," *ITC*, vol. 44, pp. 1077–1081, Sept. 1996.
[3] G. Caire and D. Tuninetti, "The Throughput of Hybrid-ARQ Protocols for the Gaussian Collision Channel," *IEEE Trans. on Information Theory*, vol. 47, no. 5, pp. 1971–1988, July 2001.
[4] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic consideration for cellular mobile radio," *IEEE Trans. on Vehicular Technology*, vol. 43, no. 2, pp. 359–378, May 1994.
[5] G. J. Foschini, "Layered Space-Time Architecture for Wireless Communication in a Fading Environment When using Multi-Element Antennas," *Bell Labs Technical Journal*, vol. 1, no. 2, pp. 41–59, Autumn 1996.
[6] E. Telatar, "Capacity of multi-antenna gaussian channels," *European Trans. on Telecommunications*, vol. 10, no. 6, pp. 585–595, Nov.-Dec. 1999.
[7] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge, University Press, 2003.
[8] H. Zheng, A. Lozano, and M. Haleem, "Multiple ARQ Processes for MIMO Systems," in *Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC)*, 2002, vol. 3, pp. 1023–1026.
[9] E. N. Onggosanusi, A. G. Dabak, Y. Hui, and G. Jeong, "Hybrid ARQ Transmission and Combining for MIMO Systems," in *Proc. IEEE International Conference on Communications (ICC)*, 11-15 May 2003, vol. 5, pp. 3205–3209.
[10] A. Maaref and S. Aissa, "A Cross-Layer Design for MIMO Rayleigh Fading Channels," in *Electrical and Computer Engineering*, 2004, vol. 4, pp. 2247–2250.
[11] S. M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, October 1998.
[12] J. M. Holtzmann, "CDMA forward link waterfilling power control," in *VTCs*, 2000, pp. 1663–1667.
[13] H. Dai, A. F. Molisch, and H. V. Poor, "Downlink Capacity of Interference-Limited MIMO Systems With Joint Detection," *ITWC*, vol. 3, no. 2, pp. 442–453, March 2004.
[14] D. Rainish, "Diversity Transform for Fading Channels," *IEEE Trans. on Communications*, vol. 44, no. 12, pp. 1653–1661, December 1996.
[15] G.D. Golden, C.J. Foschini, R.A. Valenzuela, and P.W. Wolniansky, "Detection Algorithm and Initial Laboratory Results Using V-BLAST Space-time Communication Architecture," *Electronics Letters*, vol. 35, no. 1, January 1999.
[16] J.G. Proakis, *Digital Communications*, McGraw–Hill, 3-rd edition, 1995.