Reduced Complexity Iterative Multiuser Detector for beyond 3G CDMA Systems

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***Abstract*— A reduced complexity multiuser detector for DS-CDMA is proposed for beyond 3G (B3G) systems with large bandwidth. We first argue that the resulting increase in inter-symbol interference and data rate in B3G systems makes the computational load of multiuser receivers very high. Two solutions are proposed to reduce complexity. First, a frequency domain implementation of the RAKE receiver is described. Then, iterative methods for solving linear systems are used to reduce the complexity of the MMSE multiuser detector. Simulation results show that the proposed schemes achieve similar performance than a receiver with exact MMSE matrix inversion.**

I. INTRODUCTION

Emerging wireless services will demand higher data rates at lower cost, which in technological terms translates into more bandwidth, higher spectral efficiency, and reduced complexity terminals. This presents a major challenge to system designers, since each technology (e.g. TDMA, CDMA, or OFDM) has different tradeoffs in these terms. This paper addresses the design of an uplink multiple access scheme based on direct sequence code division multiple access (DS-CDMA) [1]. CDMA has been shown to have the potential to achieve the capacity of multiple access channels, albeit at the cost of higher complexity than TDMA and FDMA. The main challenge for DS-CDMA is dealing with a channel response spanning many symbols, resulting from the combination of large transmission bandwidth and channel time dispersion. The high bandwidth and corresponding high resolution of the receiver envisioned for beyond 3G (B3G) systems raise questions about the feasibility of the Rake receiver, since with bandwidths of 20-100 MHz [2], the signal energy is likely to be dispersed among many resolvable paths, and many Rake fingers may be needed. An alternative solution being proposed for this problem is a frequency domain (FD) Rake [3] which performs maximal ratio combining (MRC) in the frequency domain.

Besides the matched filter stage, implemented with a time or frequency domain Rake, multiuser detection will be required in order to achieve high spectral efficiency. The fundamental challenge of multiuser detection for B3G systems is that delay spread creates interference among several symbol intervals, resulting in very large correlation matrices. In order to make the MMSE receiver feasible, iterative methods for the solution of linear systems are proposed [4]. The implementation of successive over-relaxation (SOR) and the Chebishev method show that, in both cases, results close to exact matrix inversion are achieved in a few iterations.

The outline of this paper is as follows. First in Section II we describe continuous and block transmission schemes suitable for time domain (TD) and FD implementations of the Rake receiver, respectively. The reduced complexity receiver is described in Section III. The performance of the proposed reduced complexity receivers is evaluated in Section IV, and, finally, Section V contains the conclusions.

II. SYSTEM MODEL

Two transmission schemes are considered: continuous transmission (CT) and block transmission (BT). These two schemes are introduced in order to consider processing in time domain (TD) and frequency domain (FD), respectively.

For continuous transmission,

$$x_k\left(t\right)=\sum_{i=-\infty}^{\infty}\sum_{n=0}^{N-1}A_k b_k\left(i\right)c_k\left(n\right)p\left(t-nT_c-iT\right) \qquad (1)$$

where N is the processing gain, A_k is the signal amplitude, $b_k(i)$ denotes user k information symbols, c_k denotes its spreading sequence, $p(t)$ is the pulse waveform, T_c is the chip period and T is the symbol period.

For BT, symbols are arranged in blocks separated by a guard band or cyclic prefix. The transmitted DS-CDMA signal for block transmission is defined as

$$\begin{aligned}x_k\left(t\right)=\sum_{i=-\infty}^{\infty}\sum_{j=-G}^{M-1}\sum_{n=0}^{N-1}A_k b_k\left(i,j\right)c_k\left(n\right)p\left(t-nT_c\right.\\ \left.-jT-i\left(M+G-1\right)T\right)\end{aligned} \qquad (2)$$

where M is the block size, G is the prefix/guard interval length, $b_k\left(i,j\right),\ i=1,2,\cdots,M-1$ denotes user k data symbols in block j, and $b_k\left(i,j\right)=b_k\left(i,j+M\right),j<0$ denotes the block cyclic prefix,

A look at equations (1) and (2) quickly reveals that the BT scheme has a penalty in spectral efficiency of $\rho=M/\left(M+G\right)$, assuming that the same pulse shape is used. At the receiver side, we have

$$y\left(t\right)=\sum_{k=0}^{K-1}\sum_{l=0}^{L_k-1}h_{lk}x_k\left(t-\tau_{lk}\right)+\eta\left(t\right) \qquad (3)$$

where K is the number of users, L_k is the number of paths for user k channel, with complex coefficients h_{lk} and relative delay τ_{lk}, and η denotes Gaussian noise.

III. TIME AND FREQUENCY DOMAIN STRATEGIES FOR THE RECEPTION OF SYNCHRONOUS DS-CDMA

A. TD-RAKE Receiver

The traditional approach for DS-CDMA reception is to combine the received signal energy by means of a time domain Rake receiver, consisting basically of a tapped delay line where the number of taps or Rake fingers must be designed to capture at least the most significant paths. Due to the continuous processing approach of the TD-Rake, CT is assumed. The computational complexity of the Rake receiver depends on the number of fingers, that is, on the dispersion of the symbol energy and the number of most significant paths. The latter is in turn determined by the channel time dispersion and by the receiver resolution or signal bandwidth.

The multiuser receiver front end consists of a matched filter bank with K Rake filters and de-spreading. The received signal, after de-spreading, is given by

$$\mathbf{z}\left(i\right)=\sum_{l=0}^{L}\mathbf{R}_l\mathbf{A}\mathbf{b}\left(i-l\right)+\xi\left(i\right) \tag{4}$$

where $\mathbf{A}$=diag(A$_1$,A$_2$,...,A$_K$) and $\mathbf{R}_l$ denotes the l-delay sequence cross-correlation matrix, given by

$$\left[R_l\right]_{i,j}=\rho_{i,j}\left(l\right)=\int\tilde{c}_i^*\left(t\right)\hat{c}_j\left(t-lT\right)dt \tag{5}$$

where

$$\tilde{c}_i\left(t\right)=\sum_{l=0}^{L_i-1}h_{li}c_i\left(t-\tau_{li}\right) \tag{6}$$

and

$$\hat{c}_j\left(t\right)=\sum_{l\in L_R}h_{lj}c_j\left(t-\tau_{lj}\right) \tag{7}$$

denotes the convolution of the j^{th} Rake filter (where only the subset L_R of paths are captured) and user j spreading sequence. For the ideal matched filter, with no restriction on the number of fingers, $\hat{c}_j\left(t\right)=\tilde{c}_j\left(t\right)$. In this model, L denotes the span (in symbol periods) of the channel delay spread, which is determined by the user with maximum delay spread.

The signal $\mathbf{z}$ is processed by a minimum mean square error (MMSE) linear multiuser receiver with S vector taps $\mathbf{w}_k(s)$, obtaining

$$\bar{z}_k\left(i\right)=\sum_{s=0}^{S-1}\mathbf{w}_k^H\left(s\right)\mathbf{z}\left(i-s\right) \tag{8}$$

where ^{H} denotes Hermitian transpose. The MMSE receiver is shown in Figure 1. Its taps can be obtained from the orthogonality principle as

$$\begin{array}{l} E\left[\bar{\mathbf{z}}\left(i\right)\mathbf{e}^*\left(i\right)\right]=\mathbf{0}, \\ \mathbf{e}\left(i\right)=\left(\bar{\mathbf{z}}\left(i\right)-\mathbf{b}\left(i\right)\right) \end{array} \tag{9}$$

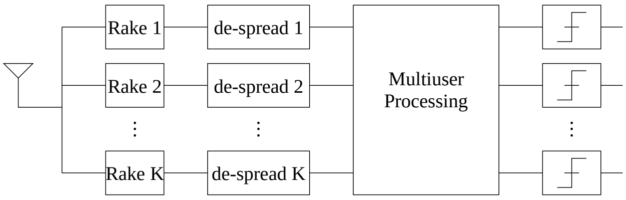

Fig. 1. Time domain processing receiver.

and, defining $\omega=\left[\mathbf{w}_1^T,\mathbf{w}_2^T,\cdots,\mathbf{w}_K^T\right]^T$ are given by

$$\omega_k=\mathbf{R}_{zz}^{-1}\mathbf{r}_{zb} \tag{10}$$

where

$$\mathbf{R}_{zz}=\left[\begin{array}{llll} \mathbf{r}\left(0\right) & \mathbf{r}\left(1\right) & \cdots & \mathbf{r}\left(S-1\right) \\ \mathbf{r}^H\left(1\right) & \mathbf{r}\left(0\right) & \ddots & \\ \vdots & \ddots & \ddots & \mathbf{r}\left(1\right) \\ \mathbf{r}^H\left(S-1\right) & & \mathbf{r}^H\left(1\right) & \mathbf{r}\left(0\right) \end{array}\right] \tag{11}$$

$$\begin{array}{l} \mathbf{r}\left(s\right)=E\left[\mathbf{z}\left(i\right)\mathbf{z}^H\left(i-s\right)\right] \\ \mathbf{r}_{zb}=\left[\mathbf{p}\left(0\right)\ \mathbf{p}\left(-1\right)\ \cdots\ \mathbf{p}\left(1-S\right)\right]^T \\ \mathbf{p}\left(s\right)=E\left[\mathbf{z}\left(i-s\right)b_k^*\left(i\right)\right] \end{array}$$

In order to calculate the MMSE filter taps matrix $\mathbf{R}_{zz}$, of size $KS\times KS$, must be inverted. The number of taps of the MMSE multiuser receiver is determined by the processing gain and the channel delay spread. Assuming that the number of filter taps is $S=2L$, Table I provides some examples for characteristic values of processing gain and delay spread for a system with 100 MHz bandwidth and a rolloff of 0.23.

TABLE I

NUMBER OF FILTER TAPS FOR MMSE MULTIUSER DETECTION

Delay spread	processing gain	S (100 MHz)	S (20 MHz)
25 ns	8	2	2
25 ns	32	2	2
10 us	8	204	22
10 us	32	52	12

Assuming a half-loaded system, one can see from the table that the dimension of $\mathbf{R}_{zz}$ may be considerable for channels with large delay spread.

B. FD-RAKE Receiver

The complexity of the Rake-based multiuser receiver is dominated by the calculation of the MMSE filter and by the filtering operation itself, as the number of coefficients can grow into the hundreds. An alternative approach follows the steps proposed for single-carrier, single-user systems of receiver-side frequency domain processing [5]. The scheme proposed is shown in Figure 2, and uses the BT scheme described in Section III, where a cyclic prefix (CP) is placed in the guard interval.

In the receiver, the CP is subtracted and data blocks are serial to parallel converted. Then the Fast Fourier Transform (FFT) converts it to the frequency domain, where a combiner

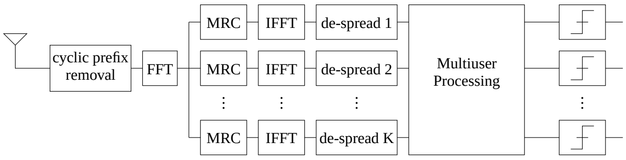

Fig. 2. Frequency domain processing receiver.

performs the function of the Rake receiver and implements the matched filter. Subsequent blocks perform an Inverse FFT and de-spreading for all of K users, while the last block performs multiuser processing.

After conversion back to the time domain through the FFT, and de-spreading, we obtain

$$\mathbf{z}\left(i\right)=\mathbf{R}_{zz}\mathbf{A}\mathbf{b}\left(i\right)+\xi\left(i\right) \tag{12}$$

where

$$\mathbf{R}_{zz}=\begin{bmatrix}\mathbf{R}_0 & \mathbf{R}_1 & \cdots & \mathbf{R}_{M-1}\\ \mathbf{R}_1^H & \mathbf{R}_0 & \ddots & \\ \vdots & \ddots & \ddots & \mathbf{R}_1\\ \mathbf{R}_{M-1}^H & & \mathbf{R}_1^H & \mathbf{R}_0\end{bmatrix} \tag{13}$$

and

$$\left[\mathbf{R}_m\right]_{i,j}=\rho_{i,j}=\sum_n \bar{c}_i\left(n\right)\bar{c}_j^*\left(n-mT\right) \tag{14}$$

where $\bar{c}_i(n)=c(n)\circledast h(n)$, and $\circledast$ denotes circular convolution. According to the design parameters, multi-access interference is limited to symbols within a block. The MMSE multiuser receiver is obtained by the transformation

$$\mathbf{F}=\left(\mathbf{R}_{zz}+\sigma^2\mathbf{A}^{-2}\right)^{-1} \tag{15}$$

which requires the inversion of the block correlation matrix $\mathbf{R}_{zz}$, of size $KM\times KM$. The block size cannot be reduced indefinitely, since the cyclic prefix is of fixed size and the spectral efficiency penalty increases, therefore the MMSE block of the FD receiver also represents a high computational burden.

C. Reduced Complexity Iterative MMSE Receiver

The complexity of the Rake-based multiuser receiver is dominated by the calculation of the MMSE filter, which requires a matrix inversion operation. Rather than direct inversion, we can use iterative methods for solution of the linear system, thereby reducing complexity and computational cost.

We concentrate on two iterative methods. First we consider the Successive Over-relaxation (SOR) method, associated with serial interference cancellation. Secondly, we consider the Chebyshev method, associated with parallel interference cancellation [4].

In general, the iterative solution for a linear system $\mathbf{M}\mathbf{x}=\mathbf{d}$ where $\mathbf{M}\in\mathbb{C}^{P\times P}$ is a known matrix, $\mathbf{d}$ is a known vector, and $\mathbf{x}$ is the unknown vector (for the MMSE filter under consideration $\mathbf{d}=\mathbf{z}$ and $\mathbf{M}=\mathbf{R}_{zz}+\sigma^2\mathbf{A}^{-2}$ for the FD-RAKE receiver and $\mathbf{d}=\mathbf{r}_{zb}$ and $\mathbf{M}=\mathbf{R}_{zz}$ for the TD-RAKE receiver) can be obtained by defining a matrix splitting $\mathbf{M}=\mathbf{S}-\mathbf{T}$ that leads to the following iteration

$$\mathbf{x}_{k+1}=\mathbf{S}^{-1}(\mathbf{T}\mathbf{x}_k+\mathbf{d}),\quad k=0,1,2,...$$

For convergence the spectral radius of the *iteration matrix* $\mathbf{B}=\mathbf{S}^{-1}\mathbf{T}$ is required to be less than one. For a fast iteration, one wishes to minimize this radius. Independent of the choice of $\mathbf{S}$ and $\mathbf{T}$, convergence is aided by selection of an initial guess $\mathbf{x}_0$ close to the solution.

The Successive Over-relaxation (SOR) method [4, p.230] is a first order stationary method which is always convergent for symmetric positive definite matrices and $w\in[0,2]$[1]. The method applies the following matrix splitting: $\mathbf{S}=\mathbf{D}+\omega\mathbf{L}$, where $\mathbf{D}$ and $\mathbf{L}$ are respectively the diagonal and strictly lower triangular parts of $\mathbf{M}$ and $\mathbf{T}$ is defined by $\mathbf{M}=\mathbf{S}-\mathbf{T}$. Since matrix $\mathbf{M}$ is Hermitian (for both TD and FD processing receiver), $\mathbf{T}=(w-1)\mathbf{L}-\mathbf{L}^*$ and the iteration is given by

$$\mathbf{x}_{k+1}=(\mathbf{D}+w\mathbf{L})^{-1}((w-1)\mathbf{L}-\mathbf{L}^*)\mathbf{x}_k+\mathbf{d}),\quad k=0,1,2,... \tag{16}$$

The second reduced complexity approach considered is the Chebyshev method, since this parallel structure is particularly attractive for hardware implementation. The Chebyshev iteration is an optimized weighted parallel interference cancellation method such that, for a given number of iterations k_{max}, the first order Chebyshev iteration [4, p.179] is given by

$$\mathbf{x}_{k+1}=\mathbf{x}_k-\tau_k(\mathbf{M}\mathbf{x}_k-\mathbf{y}),\quad k=0,1,\ldots,k_{max}-1 \tag{17}$$

for some initial vector $\mathbf{x}_0$ and a sequence of iteration parameters $\{\tau_k\}$. This is a non-stationary method since the iteration parameter varies at each iteration step. Given that the total number of iterations k_{max} is fixed, the parameters τ_k are chosen to minimize the error norm after k_{max} iterations. A detailed derivation of the optimal parameter sequence can be found in [4]. They are given by the inverses of the zeros of Chebyshev polynomials,

$$\frac{1}{\tau_k}=\frac{\lambda_{\max}-\lambda_{\min}}{2}\cos\left(\frac{k-\frac{1}{2}}{k_{max}+1}\pi\right)+\frac{\lambda_{\max}+\lambda_{\min}}{2}, \tag{18}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the largest and smallest eigenvalues of the matrix $\mathbf{M}$, respectively. Calculation of the optimum parameters requires knowledge of the extreme eigenvalues of the iteration matrix $\mathbf{M}$ [6], [4]. However, simple methods exist to estimate the extreme eigenvalues which only involve matrix row sum operations and are applicable for any matrix in general, allowing efficient implementation of the Chebyshev iterative detector [6], [7]. For example, the largest eigenvalue can be found applying the following bound [7] based on the

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[1] The iteration parameter ω is kept constant for all iteration steps and can be optimized to minimize the spectral radius, thus optimizing convergence. For simplicity, the numerical results consider $\omega=1$ which results in a particularization of the SOR method known as Gauss-Seidel method

Gersgorin theorem

$$\lambda_{\max} \leq \max_{i} \left\{ \sum_{j=1}^{P} |m_{ij}| \right\} \tag{19}$$

and the minimum eigenvalue can be approximated by zero for large matrices.

D. Asynchronous DS-CDMA

In asynchronous DS-CDMA, each user's signal is received with different delays. Denote the received signal by

$$y(t) = \sum_{k=0}^{K-1} \sum_{l=0}^{L_k-1} h_{lk} x_k (t - \tau_{lk} - \delta_k) + \eta(t) \tag{20}$$

where δ_k denotes user k delay. For the purpose of analyzing the receiver, we assume the delays are uniformly distributed in an interval with length equal to the channel delay spread for CT and to the block size for BT. For CT, the received signal after de-spreading is

$$\mathbf{z}(i) = \sum_{l=-L}^{L} \mathbf{R}_l \mathbf{A} \mathbf{b} (i-l) + \xi(i) \tag{21}$$

with $[\mathbf{R}_l]_{ij} = \rho_{ij}(l)$. The receiver structure is similar to the synchronous case, with the main difference being the size of $\mathbf{R}_{zz}$, since now more filter coefficients are required..

The block transmission scheme together with frequency domain processing is better suited to synchronous or quasi-synchronous transmission. In a quasi-synchronous system, the sum of the largest time offset among users and the channel delay spread do not exceed the guard interval length, therefore the receiver design in Section III-B can still be used. On the other hand, for fully asynchronous systems, it is possible to use for BT the same receiver strategies as for asynchronous multi-carrier CDMA (see e.g. [8]).

IV. PERFORMANCE RESULTS

The described reduced complexity receivers have been evaluated through simulations of a B3G system. A single cell scenario was considered under the implicit assumption that inter-cell interference could be approximated as AWGN. Simulation parameters are summarized in Table II.

TABLE II

DS-CDMA SIMULATION PARAMETERS

Parameter	**Value**
Channel Model	SCME [9]
Delay Spread	up to 6 symbol periods
Power Control	Average (slow) power control
Modulation	BPSK
Block size (BT)	20 symbols
Spreading gain	15
Spreading sequences	Gold Codes
Number of users	8

Figures 3 and 4 show the performance of the time domain processing receiver in terms of raw (uncoded) bit error rate. The convergence of the iterative implementation of the MMSE time domain receiver in terms of BER versus number of iterations is shown in Figure 3 for SNR=8dB. Numerical results are based on the tapped-delay line channel model extension for B3G systems of 3GPP-SCM [9]. Convergence behaviour of the Chebyshev iterative method is shown for the iteration parameters obtained from their approximation based on the Gersgorin method. Numerical results show that after 10-15 iterations, SOR iterative implementation of the MMSE filter achieves similar performance to the exact MMSE matrix inversion receiver. Chebyshev implementation presents slower convergence for the time domain processing MMSE receiver, as seen in the Figure. In Figure 4, the performance is evaluated in terms of BER at different SNRs for different number of RAKE fingers. The number of iterations performed by the iterative implementation of the MMSE filter is 15 for the SOR method (also shown for comparison are results for 6 iterations) and 20 for the Chebyshev method. Shown for comparison is the BER of the matched filter (MF). Clearly the MF achieves very poor performance even at moderate

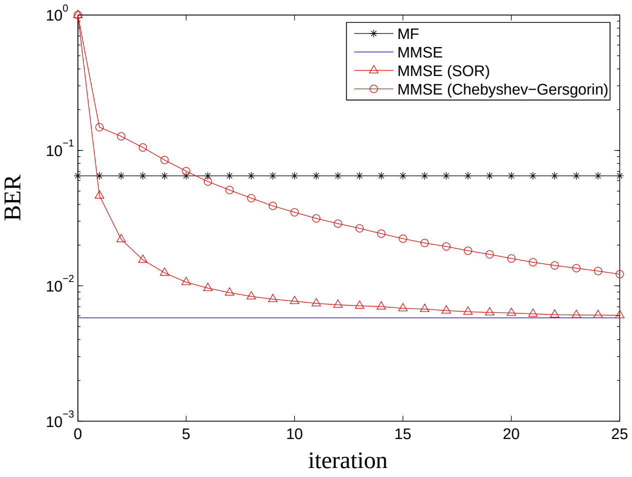

Fig. 3. Convergence of SOR and Chebyshev iterative implementations of the MMSE time domain receiver (SNR=8 dB).

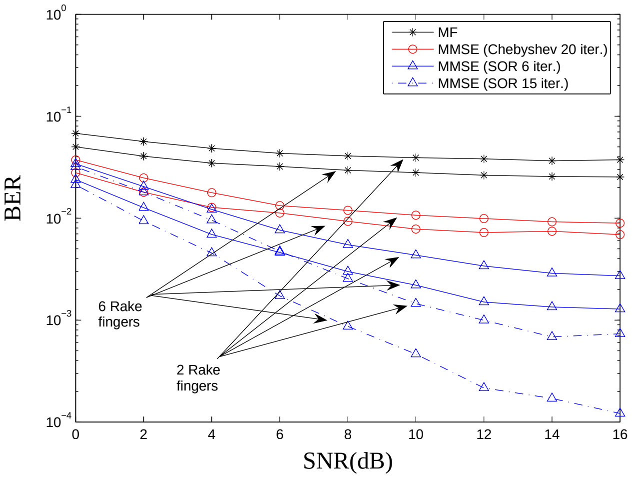

Fig. 4. Performance results of SOR and Chebyshev iterative implementations of the MMSE time domain receiver.

SNRs. For instance, with a 6-finger RAKE at SNR=16 dB the SOR iterative implementation of the MMSE achieves a BER $O(10^{-4})$ with 15 iterations and $O(10^{-3})$ if 6 iterations are run instead. In both cases the performance achieved by the iterative implementation of the MMSE outperforms the $O(2*10^{-2})$ offered by the MF. Equivalent performance results for the frequency domain receiver are shown in Figures 5 and 6. In Figure 5 BER curves versus number of iterations are shown for the iterative implementation of the MMSE frequency domain receiver at SNR=4dB. In this case the rate of convergence of SOR and Chebyshev implementations are closer than for the time domain receiver.

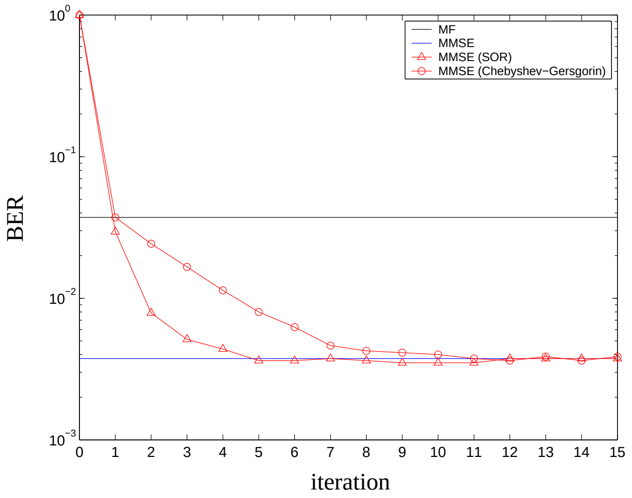

Fig. 5. Convergence of SOR and Chebyshev iterative implementations of the MMSE frequency domain receiver (SCME channel, SNR=4 dB).

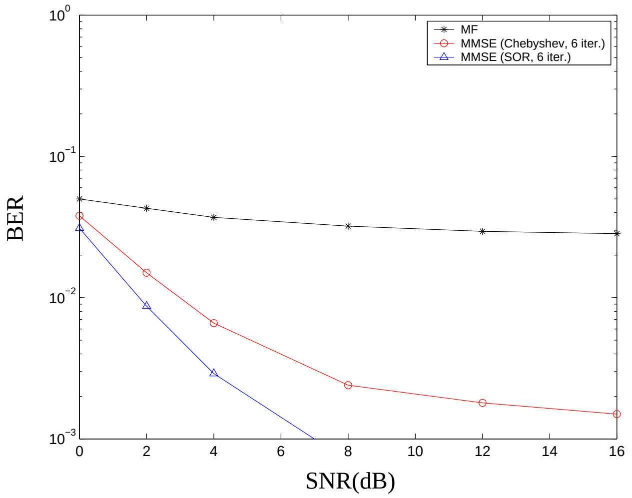

Fig. 6. Performance results of SOR and Chebyshev iterative implementations of the MMSE frequency domain receiver (6 iterations).

V. CONCLUSIONS

A reduced complexity DS-CDMA receiver for B3G systems was studied. Two possible transmission schemes were considered, suited for time and frequency domain matched filters, respectively. A reduced complexity MMSE multiuser detector based on iterative matrix inversion methods was used to remove multiuser interference. Results on a wideband channel show that the iterative MMSE receivers converge to the exact MMSE solution after a few iterations, achieveving good performance.

VI. ACKNOWLEDGEMENT

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