Frequency domain iterative methods for detection and estimation

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Abstract— **Single-carrier transmission is one of the most potential options for uplink transmission in wide-area deployments. It combines the benefits of computationally simple transmitter processing and highly efficient power amplification at the terminal. Frequency-domain methods for efficient equalization can be utilized at the access point either in linear MMSE equalization or in iterative turbo equalization. We describe the potential solutions in frequency-domain receiver processing for equalization and channel estimation, and evaluate their performance. We demonstrate the potential for turbo processing to provide significant performance gain even in simple single-input-single-output channels. We note that extensions to multiple-input-multiple-output cases are straightforward.**

I. INTRODUCTION

Single-carrier or serial transmission has many qualities which make it a good candidate for uplink (reverse) transmission in wide-area cellular deployments of future networks. The generation of the transmitted signal requires only modest complexity and the resulting signal has a very good peak-to-average power ratio (PAPR), making it possible to use relatively cheap and efficient power amplifiers at the terminal. The cost savings due to simplicity are emphasized in future systems, where multiple transmission chains required by sophisticated spatial transmission methods will most likely be a norm rather than an exception.

The complexity in a single-carrier system is concentrated on the receiver, which must perform equalization to enable successful detection. Frequency-domain equalizers offer low-complexity implementation and potential hardware re-use with multicarrier -based downlink (forward) processing. In this paper we make an overview of frequency-domain linear and turbo-equalization methods for such a system. We compare two different approaches for turbo equalization, and conclude their equivalence in performance terms. We explore the linear and turbo equalizers' performance with a set of coding and modulation modes to demonstrate the viability of linear equalization and the performance gain through turbo iterations. A method for iterative channel estimation and its performance with turbo equalization are presented.

TABLE I
TRANSMISSION PARAMETERS

Parameter	Value
Bandwidth	20MHz
Symbol rate	16.25Msps
Symbols per block	832
Prefix symbols	80
Pulse shaping	Root-raised Cosine
Roll-off	0.23
Pulse filter length	8 symbols
Receiver sampling	symbol rate

II. STUDIED SYSTEM

We assume a block-cyclic transmission where a prefix is added to the signal prior to transmission. The transmitted signal is bit-interleaved-coded modulation with BPSK, QPSK or 16-QAM modulation and convolutional $(133, 171)_8$ codes of rate $1/2$ as outer codes. Random interleaving is assumed. The parameters of the transmission are listed in Table I. The propagation channel for an urban macro environment with a maximum propagation delay of 4.625μs is considered. The average power delay profile of the channel is given in Table II.

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TABLE II

5GHz URBAN MACRO CHANNEL

Power	0,000	0,010	0,030	0,360	0,370	0,385	0,250	0,260	0,280	1,040	1,045	1,065	2,730	2,740	2,760	4,600	4,610	4,625
Delay [μs]	-3,0000	-5,2200	-6,9800	-5,2204	-7,4404	-9,2004	-4,7184	-6,9384	-8,6984	-8,1896	-10,4096	-12,1696	-12,0516	-14,2716	-16,0316	-15,5013	-17,7213	-19,4813

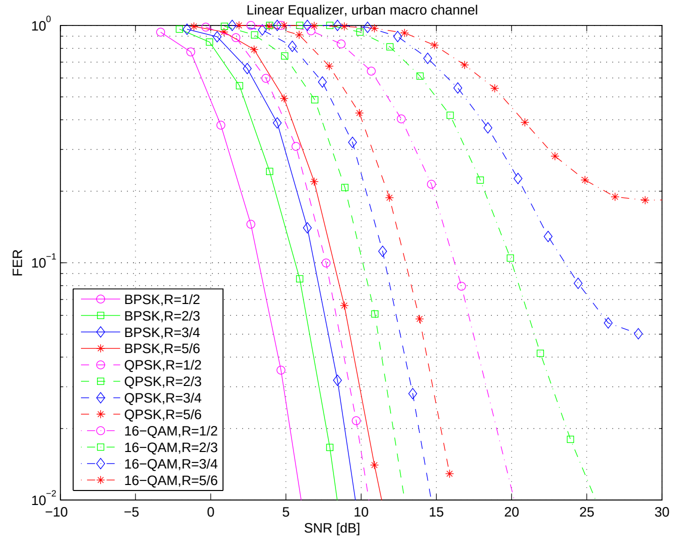

Fig. 1. Linear MMSE FDE.

III. FREQUENCY-DOMAIN EQUALIZATION

Frequency-domain equalization[1][2] enables the construction of high-performance equalizers at a reasonable complexity. In the case of a cyclic transmission (using a prefix) over a static channel, a linear frequency-domain equalizer (FDE) can be implemented with a single complex coefficient per frequency bin. Extensions to multiple-input-multiple-output channels are also known [3].

We study the performance of a linear minimum mean-square-error (MMSE) equalizer with the modulations listed in Section II and the convolutional code punctured to rates $2/3$, $3/4$ and $5/6$. The corresponding results are listed in Fig. 1, where the SNR is defined as

$$\mathrm{SNR} = E_b/N_{0[\mathrm{dB}]} + 10\log_{10}(R\log_2 M)\,, \qquad (1)$$

where E_b/N_0 is the average received bit energy divided by the receiver noise energy, M is the cardinality of the modulation, and R is the code rate.

The linear FDE captures the channel diversity well, and provides a consistently good performance with different modes all the way up to $2/3$-rate 16-QAM. The two highest modes experience a saturation limiting the usefulness of these modes.

IV. FREQUENCY-DOMAIN TURBO EQUALIZATION

In this section, novel frequency domain turbo equalization techniques are proposed and investigated. These iterative techniques are chosen due to their common merit of being low-complexity and highly effective in minimizing the detection error to a very low level, an advantage cannot be easily matched by conventional non-iterative technique.We proceed by considering first the perfect CSI (channel state information) scenarios. Two iterative techniques, namely the frequency-domain equalization with time-domain decision feedback (FDE-TDDF) and the frequency-domain equalization with MMSE interference-cancellation (FDE-MMSE-IC), are presented. The situation when perfect CSI assumption is no longer valid will be considered in the next section.

A. Iterative Frequency Domain Equalization with Time Domain Decision Feedback (FDE-TDDF)

Fig. 2 depicts the general structure of the iterative FDE-TDDF equalizer with channel decoding. The iterative FDE-TDDF consists of a forward filter operating in the frequency domain and a backward filter processing the feedback signals in the time domain. The outputs generated by the FDE-TDDF are in the form of extrinsic information (log likelihood ratio, LLR), which will be fed to the binary MAP channel decoder as the intrinsic information (a priori LLR). The outputs from the binary MAP decoder will be in the form of extrinsic information (LLR) and be used to calculate the soft decisions as the feedback signals for the FDE-TDDF. After a certain number of iterations (assuming convergence is attained), the outputs from the MAP decoders, in the form of a-posterior probability ratio (APP), are utilized to generate hard decisions for the desired signals. Mathematically, the FDE-TDDF can be

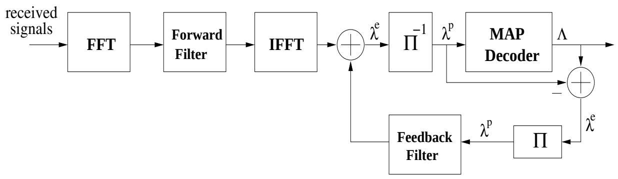

Fig. 2. Iterative FDE-TDDF Equalizer

described as follows. Denote $\mathbf{y} = [y(0), y(1), \cdots, y(N-1)]^T$ and $\mathbf{x} = [x(0), x(1), \cdots, x(N-1)]^T$ as the block of baseband received signals and input signals, respectively. Let $\mathbf{W}^{(i)} = diag\{w_0^i, w_1^i, \cdots, w_{N-1}^i\}$ and $\mathbf{G}^{(i)} = circ\{0, g_1^i, \cdots, g_{N-1}^i\}$ be the forward filter coefficients matrix and the feedback filter coefficients matrix, respectively, for the i^{th} iteration. Then, the outputs of the FDE-TDDF for the i^{th} iteration can be expressed as follows,

$$\tilde{\mathbf{x}}^{(i)} = \mathbf{F}^H \mathbf{W}^{(i)} \mathbf{F} \mathbf{y} + \mathbf{G}^{(i)} \hat{\mathbf{x}}^{(i-1)} \tag{2}$$

where $\hat{\mathbf{x}}^{(i-1)}$ is the vector containing the soft decisions provided by the MAP decoder during the $(i-1)^{th}$ iteration. The detailed derivation of the forward and backward filter coefficients can be found in [4]. The results are listed in below,

$$w_k^{(i)} = \frac{h_k(1 + \rho^{(i)} \sum_{l=1}^{N-1} (g_l^{(i)})^* \exp(-j2\pi lk/N))}{|h_k|^2 + \sigma^2} \tag{3}$$

$$\mathbf{g}^{(i)} = [g_1^{(i)}, g_2^{(i)}, \cdots, g_{N-1}^{(i)}]^T = -\frac{\mathbf{V}^{-1}\mathbf{v}}{\rho^{(i)}} \tag{4}$$

where $\rho^{(i)} = E\{x_j^* \hat{x}_j^{(i-1)}\}$ is defined as the reliability of the feedback decisions and,

$$\mathbf{V} = \begin{bmatrix} v_0 & v_{-1} & \cdots & v_{-(N-2)} \\ v_1 & 0 & v_{-1} & \ddots \\ \ddots & \ddots & \ddots & \ddots \\ v_{N-2} & \ddots & \ddots & v_0 \end{bmatrix} \tag{5}$$

and

$$v_k = \frac{1}{N} \sum_{l=0}^{N-1} \frac{|h_l|^2(1/\rho - 1) + \sigma^2/\rho}{|h_l|^2 + \sigma^2} \exp(-j2\pi \frac{lk}{N}) \tag{6}$$

In deriving the backward filter coefficients, there is a simply way to avoid calculating the inverse $\mathbf{V}^{-1}$, see [4]. Next, the outputs fed to the binary MAP decoder, in the form of LLR, can be expressed as follows,

$$\lambda_j^e = \frac{2\tilde{x}_j^{(i)} \gamma^{(i)}}{E(|\epsilon_j^{(i)}|)^2} \tag{7}$$

where $\gamma^{(i)}$ and $E|\epsilon_j^{(i)}|^2$ are the useful signal's amplitude and residual noise variance contained in $\tilde{x}_j^{(i)}$, respectively, see [4] for the computation of these two quantities. The MAP decoder generates the LLR (also denoted as $\lambda_j^e)$) as its outputs and they are converted to the soft decisions (assuming BPSK modulation) required by the FDE-TDDF,

$$\hat{x}_j^{(i)} = tanh(0.5\lambda_j^e) \tag{8}$$

B. Iterative Frequency Domain Equalization using MMSE-based Interference Cancellation Technique (FDE-MMSE-IC)

Frequency-domain MMSE turbo equalization has been proposed for single-input-single-output (SISO) and MIMO systems in [5][6], respectively. Fig. 3 depicts the structure of the FDE-MMSE-IC scheme with channel decoding. The main difference between the FDE-MMSE-IC and the FDE-TDDF described above is that the former contains only one forward filter while the backward filter is replaced with the channel coefficients modulator (which is used to re-create the ISI, using the symbol estimation from the decoder, so that they can be subtracted off from the received signals later). Unlike the FDE-TDDF, subtraction of ISI in FDE-MMSE-IC takes place prior to the forward filtering in the frequency domain. For lower complexity, all known signal components are cancelled from the received signal and the desired signal component, in the form of symbol estimates $\hat{b_k}$ from the previous iteration, are combined with the forward filter outputs with suitable weighting ($\bar{\gamma_k}$,d_k^{-1} in Fig. 3). The outputs (in the form of extrinsic information) from the FDE-MMSE-IC equalizer will be fed to the binary MAP decoder, which will in turn generate the soft decisions as the feedback signals for the FDE-MMSE-IC. For more details regarding the derivation of filter coefficients for the FDE-MMSE-IC, please see [6].

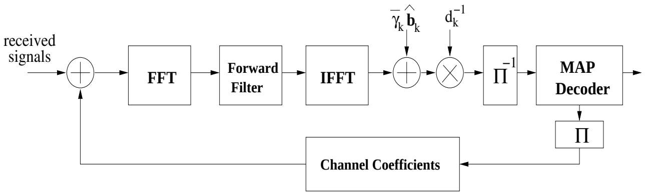

Fig. 3. Iterative FDE-MMSE-IC Equalizer

V. ITERATIVE CHANNEL ESTIMATION

In this section, perfect channel knowledge is not available and channel estimation is required. We consider a pilot-assisted channel estimation method in conjunction with the FDE-TDDF described in the previous section.

A. Iterative Frequency Domain Equalization with Time Domain Decision Feedback and Channel Estimation (FDE-TDDF-CE)

In Fig. 4, the proposed method, FDE-TDDF-CE, is shown. Basically, the FDE-TDDF-CE is identical to the original FDE-TDDF, only with the channel estimation as an additional process in each iteration. Prior to the first turbo iteration, the pilot signals are used to generate the estimated channel statistics (the mean and the variance of each random channel coefficient). The FDE-TDDF filters' coefficients are updated accordingly taking into consideration the estimated channel statistics. For the subsequent turbo iterations, the soft outputs from the decoders are used as the intermediate code symbols estimation, which are processed together with the pilot signals by the channel estimator to produce a new set of refined channel statistics estimation. The new channel information is then used by the FDE-TDDF in the next iteration and so on. The channel estimation is carried out in the time domain based on the least-square criterion. The maximum channel length is assumed to be known or else it is assumed to be equal to the length of the cyclic prefix. In this paper, the latter assumption can be used, i.e., the receiver considers the channel length to be equal to the cyclic prefix.

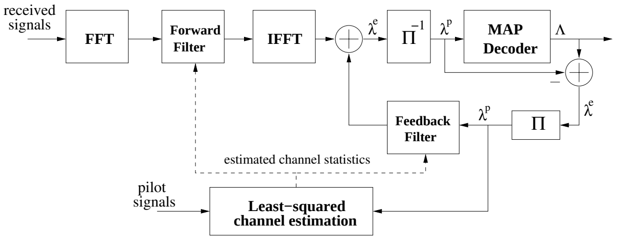

Fig. 4. Iterative FDE-TDDF-CE Equalizer

In the following, it is assumed that one training block (contains only pilot signals) is used for each data block. Fig. 5 shows the packet structure including the data symbols and pilot symbols, In this packet structure, the last N_{cp} pilot symbols from the training block (which has length M) are used as the cyclic prefix. In the beginning, only the training block is utilized for channel estimation and in the subsequent iterations, the entire block (training + data + CP, with a total block size of $N + N_{cp} + M$) is utilized for channel estimation. It should be noted that the proposed channel estimation can be applied to any other packet structures. Now, let $h_t = [h_t(0), h_t(1), \cdots, h_t(N_{cp}-1)]^T$ be the time domain channel response vector. Then, except for the first iteration, the channel estimates (which can be alternatively interpreted as the mean values of the random channel coefficients) can be expressed as,

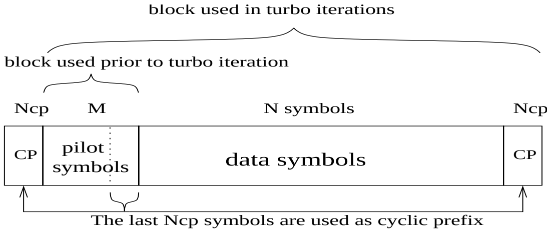

Fig. 5. Packet structure with training and data block

$$\hat{\mathbf{h}}_t = (\mathbf{\Omega}^H\mathbf{\Omega})^{-1}\mathbf{\Omega}^H\mathbf{y} \tag{9}$$

where $\mathbf{\Omega}$ is a $(N + N_{cp} + M) \times N_{cp}$ toeplitz matrix with the first row given by the pilot symbols $[p(0), p(N_{cp} - 1), \cdots, p(1)]$ and the first column given by

$$[p(0), p(1), \cdots, p(M-1), \hat{x}_0, \hat{x}_1, \cdots, \hat{x}_{N-1}, p(M - N_{cp}), \\ p(M - N_{cp} + 1), \cdots, p(M-1)]^T \tag{10}$$

where $\hat{x}_j, j = 0, \cdots, N-1$ are the soft decisions of the data symbols obtained from the outputs of the decoder during the last iteration. And the estimated channel coefficients covariance matrix can be approximated as,

$$(\mathbf{h}_t - \hat{\mathbf{h}}_t)(\mathbf{h}_t - \hat{\mathbf{h}}_t)^H = \sigma^2(\mathbf{\Omega}^H\mathbf{\Omega})^{-1} \tag{11}$$

In this FDE-TDDF-CE scheme, the filter coefficients of the FDE-TDDF are updated in accordance with the feedback signals and their reliability (ρ), as well as the newly estimated channel statistics (the mean and the variance). See [4] for the derivation of the filter coefficients when channel estimation becomes part of the iterative process.

VI. SIMULATIONS RESULTS AND DISCUSSIONS

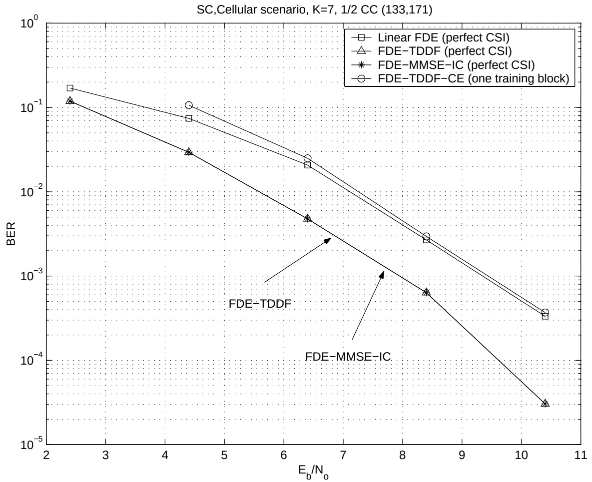

Fig. 6. Urban Macro Channel

First, we consider the scenarios assuming that the perfect CSI is available and no channel estimation is needed. In Fig. 6, QPSK with convolutional code is used. It is

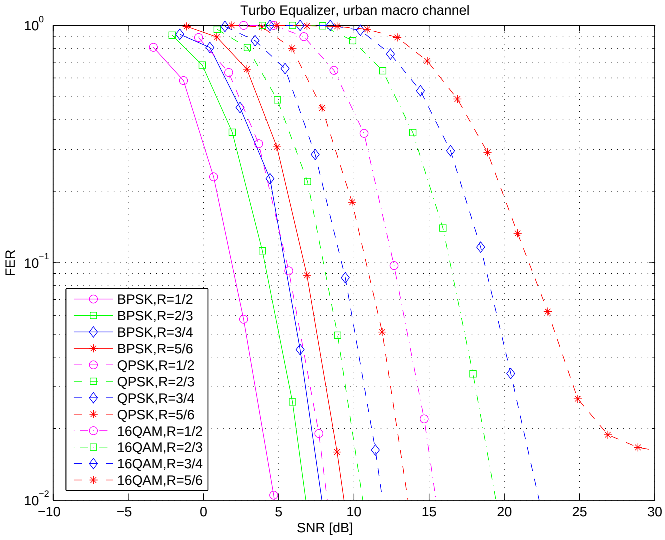

Fig. 7. Turbo MMSE FDE.

clear that, from this figure, the FDE-TDDF and FDE-MMSE-IC essentially yield the same BER performance (with four turbo iterations). They both improve the performance by about 1.5dB compared with the linear FDE case (which is identical to the iterative schemes when the number of iteration is equal to one). Fig. 7 shows how the turbo gain (using the FDE-MMSE-IC) over the linear equalizer in Fig. 1 can be realized either by higher transmission reliability or higher throughput by the standard utilization of adaptive modulation and coding (AMC) techniques. With the turbo equalizer, all of the tested modes up to 5/6-rate 16-QAM are usable. In summary we can show that the link throughput can be increased by the simple combination of turbo methods and adaptive coding and modulation. By using turbo methods higher throughput modes can be used than with the simple linear equalizer

Finally, we revisit Fig. 6 in which the simulation results for the proposed FDE-TDDF-CE scheme (no perfect CSI is assumed) are shown. It is clear that its performance is excellent (with 6 iterations) and it approaches the performance of a liner FDE with perfect CSI. It is only 1-2 dB off from the FDE-TDDF scheme with perfect CSI. The only apparent drawback is the occurrence of an error-floor-like performance at high SNR (due to the channel mismatch using the LS channel estimation). The remarkable performance of the FDE-TDDF-CE can be explained by the fact that the filter coefficients of the FDE-TDDF are updated according to the estimated channel statistics provided by the channel estimator. Hence, it will take into consideration the possible channel error level predicted by the estimator. This is in contrast with conventional approaches in which the estimated channel coefficients are assumed to be the true deterministic channel values.

VII. SUMMARY

In summary, a powerful iterative technique, namely the turbo frequency domain equalization (FDE), is proposed in this paper as a promising receiver algorithm for the detection of single-carrier modulated signals. The performance of various types of turbo frequency domain equalizers has been evaluated for the suggested channel scenario and for different combinations of channel codes and modulation schemes. With only three to four iterations, a performance gain of 1-3 dB over the linear FDE can be generally observed for all the scenarios. The consistent turbo gain in all coding and modulation modes, together with the fact that the highest modes are usable only with turbo equalization, suggests that adaptive modulation and coding is a very attractive approach to be used with the iterative technique. Furthermore, in a situation when pilot-assisted channel estimation is employed, it is shown that the turbo FDE can incorporate the iterative channel estimation process to yield a remarkable performance, which is shown to be very close to the performance of a linear FDE with ideal channel state information. Finally, it should be remarked that the turbo FDE has the advantage of smaller complexity compared with its time-domain equalization counterpart in severely frequency-selective channels. This, coupled with the advantage of small PAPR associated with the single-carrier modulated signals, makes the turbo FDE a suitable receiver technology for the uplink transmission employing single-carrier modulated signals.

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