# File Broadcast in the Land-Mobile Satellite Channel 

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#### Abstract

Satellite radio for vehicles is hampered by the problem of the shadowing of the line of sight between satellite and receiver, which can be often modelled as a Gilbert channel, where in the shadowed state no successful reception is possible. In this paper, use is made of an alternative audio radio concept, described in [1], where instead of transmitting a number of continuous radio streams, individual audio and multimedia files are broadcasted to the vehicle. The received files are then stored in a large cache located in the receiver of each user. By using dynamically generated playlists for these files, a service similar to traditional radio/entertainment programs is regenerated. This change in concept has an impact on the way such a system behaves and the critical points, which need to be analyzed.

In the paper, the time until a file is correctly received is derived, under the assumption of using forward error correction techniques at the transport layer. Based on this result, different further statistics can then be computed.


## I. Introduction

In this section a short outline of the satellite system is given, which forms a background for the following analysis. Nevertheless, the analysis is not limited to this specific case, and is based also on the more general work of communication over satellite to mobile users, which has been analyzed in the EU-project FIFTH and especially the work inside SATNEX on land-mobile satellite communication.

The main idea of the proposed satellite system, as described more in detail in [1], is to reuse existing satellite capacity in the Ku-band. Combined with the usage of an unobtrusive musicCD sized antenna on the vehicle itself, it allows to achieve an overall data rate in the range of $1 \mathrm{Mbit} / \mathrm{s}$ per transponder [2].

Since in principal the space-infrastructure of such a system is already in place, no major upfront costs for the system would be necessary. On the other hand, such a system does not have the high transmit power of specialized radio satellites in the S-Band and therefore no high link margin or satellite diversity scheme is possible in this case.

## A. Land-Mobile Satellite (LMS) Channel at Ku-Band

The main effect in an LMS-channel is signal shadowing, experienced when no clear Line-Of-Sight (LOS) between the satellite and the user terminal is present. A relative robust model for the description of the land-mobile satellite channel is the two-state Lutz model [3], which focus on this effect. In our case in the Ku-band, one can even make the assumption


Fig. 1. File based Radio Concept
that only the line of sight (LOS) state can be used for transmission purpose, whereas the shadowed state always results in erroneous reception.

The simplified Lutz Model is presented in figure 2, where $D_{g}$ and $D_{b}$ are the mean durations of the good (LOS) and of the bad (NLOS) state respectively, and $p_{\mathrm{xx}}$ denotes the state transition probabilities. The probability $P_{\text {Stay }}$ to still be in the same state after a time $T$ decreases exponentially:

$$
\begin{equation*}
P_{\text {Stay }}(T)=\mathrm{e}^{-\frac{T}{D_{g}}} \tag{1}
\end{equation*}
$$

The average duration of this shadowing is a key parameter for any such system. For this reason, a measurement campaign was performed in 2002 and the recorded data was analysed for four different environment types: Highway, Rural, Sub-Urban and Urban (for further information see [4]).


Fig. 2. Two-State Narrowband Channel Model [3].

TABLE I
LUTZ MODEL PARAMETERS

| Environment | $D_{g}$ | $D_{b}$ | $L O S$ |
| :--- | :---: | :---: | :---: |
| Highway | 18 s | 2 s | $90 \%$ |
| Rural | 16 s | 4 s | $80 \%$ |
| Suburban | 8 s | 2 s | $80 \%$ |
| Urban | 22 s | 15 s | $60 \%$ |

where $D_{g}$ and $D_{b}$ are the mean durations of staying in the good (LOS) or respective the bad (shadowed) state.
One can see that there will be relative long outages in the range of some ten seconds, which make concepts which rely on an uninterrupted reception difficult.

## B. Using FEC Redundancy for Improving Reliability

A first approach to this problem is to increase the reliability of the system by using mechanisms which try to overcome the time-variant on-off structure of the channel. In the proposed system, one option is the usage of long interleaving schemes in the range of up to 1 minute at physical layer. Alternatively, forward error correction (FEC) is also used at the transport layer to increase the robustness of long file transfers. It is based on the methods developed by the IETF Reliable Multicast Group. In RFC 3450 "Asynchronous Layered Coding Protocol" (ALC), the necessary Internet protocols for it are specified. Similar approaches are also under discussion for DVB-H ('Handheld') under the name of MPE-FEC and by the Consultative Committee for Space Data Systems (CCSDS) under the name 'Long Erasure Codes'. Further information on such schemes can be found e.g. in [5], [6] or [7].

In the following, the second, transport layer based approach, is assumed. The forward error correction at the physical layer is only used, to ensure that all packets received in LOS conditions are correct and that all incorrect packets are reliably detected as erroneous and thrown away. We extend our view therefore to a two layer model, where we can think of an upper transport layer and a lower physical layer coding. The file is divided into $k$ data packets. Then $h$ redundancy packets are added. The individual packets are then transferred to the physical layer, which adds independent channel coding to each packet. The principle is visualized in figure 3.

This results in the before mentioned double structure. The normal channel coding combats the effect of fast fading and


Fig. 3. Transport Layer FEC
white Gaussian noise for an individual packet. On top of it, $h$ redundancy packets are added to the $k$ normal data packets, resulting in the transmission of $n=k+h$ packets. They allow combating the slow fading events, where typically whole packets, or series of packets, are destroyed. The coding rate at transport layer $R$ is then equal to $k / n$.

Thus, packets from the channel affected by low noise levels can be corrected by the physical layer and passed to the transport layer as correct with assumed $100 \%$ probability. If the noise level exceeds the correcting capability of the physical layer, the whole packet is declared erased, and we must rely on transport layer coding to recover from these erasures. Therefore the transport layer "sees" either correct packets or erasures, which is the definition of an Packet Erasure Channel (PEC).

Using Maximum Distance Separable (MDS) codes, like the Reed-Solomon (RS), allows a receiver to reconstruct the original information if at least $k$ out of $n$ packets of a packet group are received. Therefore, the receiver can cope with erasures, as long as they result in a total loss not exceeding $h$ packets, independently, where this erasures did happen.

This does even mean, that at any time instance, when $k$ packets have been received, the decoder can reconstruct the file and does not need to know, if any further packets are being sent or not.
If we now assume that the number of packets per file is very large (which can be assumed for multimedia file transfers consisting typically of more than 500 IP packets) and the duration of a transmission of a single packet is very small compared to the channel (in our case around 10 ms ), one can extend the analysis from the individual packet level to a time continuous approach. A dedicated mathematical analysis, including the effect of the packetisation, resulted in an effect in the range of the time distance between two consecutive packet starts. But it complicated the resulting formulas, without giving in our case an improved insight in the overall behaviour. An analysis regarding this effect for different packet sizes and sending combinations can be found in [8]. In the following this effect is neglected.

The time for the transmission of a file is now defined as


Fig. 4. Transport Layer FEC
$T$. It does not only need to depend on the file size, code and transmission rate, but one can artificially extend the file by multiplexing different file transfers in one stream, so that only every $j$ th packet is used for a specific file transfer.

Included in the file transfer time $T$ is a redundancy part with a duration of $(1-R) \cdot T$. Therefore, the reception of the file is successful as soon as the sum of the time spend in the good states is larger than $T_{G m i n}=R \cdot T$.

## II. Number of Transitions during File Decoding

When a file transfer takes a time $T$, we note that more than one transition from the good state to the bad state can occur until the file is decoded. As previously stated the file transfer is successful, as soon as the total time spent in good states is larger than the needed time $T_{G m i n}$.

Therefore, the time needed to decode a file is

$$
\begin{equation*}
\tau=T_{G m i n}+\sum_{i=0}^{n_{d}} T_{B_{i}}+T_{B_{n d+1}} \tag{2}
\end{equation*}
$$

where $T_{B_{n d+1}}$ is the time after the start until the first good state has been encountered. It is zero, if the reception started in the good state. This equation motivates us to analyze in this section the number of transmissions which occur, until the file could theoretically be successfully decoded.

The probability of having a number of transitions $n_{d}$ less than or equal to $N$, until decoding a file, is the probability that the sum of the time spent in the different intervals in which the channel is in the good state is greater than the time necessary to decode a file $T_{G m i n}$.

$$
\begin{equation*}
P\left(n_{d} \leq N\right)=P\left(T_{G_{1}}+T_{G_{2}}+\ldots+T_{G_{N+1}} \geq T_{G \min }\right) \tag{3}
\end{equation*}
$$

The probability of having exactly $n_{d}$ transitions is then

$$
\begin{equation*}
P\left(n_{d}\right)=P\left(n \leq n_{d}\right)-P\left(n \leq n_{d}-1\right) \tag{4}
\end{equation*}
$$

To obtain this probability we can define: $T_{G}$ is the sum of the time spent in the different intervals in which the channel is in the good state :

$$
\begin{equation*}
T_{G}=\sum_{i=1}^{n^{\prime}} T_{G i} \tag{5}
\end{equation*}
$$

where $n^{\prime}$ is the number of the intervals in the good state.
Since all of these random variables are independent from each other, the probability density function of $T_{G}$ is a convolution of all the pdfs of $T_{G_{i}}$ [9].

$$
\begin{equation*}
f_{T_{G}}=f_{T_{G 1}} * f_{T_{G 2}} \ldots * f_{T_{G n^{\prime}}} \tag{6}
\end{equation*}
$$

where $T_{G i}$ is a exponentially distributed random variable and $n^{\prime}$ is the number of times the channel is in good state. It
exceeds by one the number of transitions from good state to bad state $n_{d}$, therefore $n^{\prime}=n_{d}+1$. To obtain the pdf of (6) we now change into the Laplace domain [10].

$$
\begin{equation*}
F_{T_{G}}(s)=\frac{\left(\frac{1}{D_{g}}\right)^{n^{\prime}}}{\left(s-\left(-\frac{1}{D_{g}}\right)\right)^{n^{\prime}}} \tag{7}
\end{equation*}
$$

Then the density function in time domain can be obtained applying the inverse Laplace transform to $F_{T_{G}}(s)$ function :

$$
\begin{equation*}
f_{T_{G}}(t)=\frac{\left(\frac{1}{D_{g}}\right)^{n^{\prime}}}{\left(n^{\prime}-1\right)!} \cdot t^{n^{\prime}-1} \cdot e^{-\frac{t}{D_{g}}} \tag{8}
\end{equation*}
$$

To obtain probability (3), first the probability of $T_{G}$ being greater than $T_{G \min }$ must be found.

$$
\begin{align*}
& P\left(T_{G}>T_{G \min }\right)=\int_{T_{G \min }}^{\infty} f_{T_{G}} d t \\
& =\left(\frac{1}{D g}\right)^{n^{\prime}} \frac{1}{\left(n^{\prime}-1\right)!}\left(-D g \cdot T_{G \min }^{n^{\prime}-1} \cdot e^{-\frac{T_{G \min }}{D g}}\right. \\
& \quad+D g\left(n^{\prime}-1\right) \cdot\left(-D g \cdot T_{G \min }^{n^{\prime}-2} e^{-\frac{T_{G \min }}{D}}\right. \\
& \left.\left.\quad+D g \cdot\left(n^{\prime}-2\right)(\ldots .)\right)\right) \tag{9}
\end{align*}
$$

This formula can be expressed by using a recursive approach as shown in expression (10):
$P\left(T_{G}>T_{G \min }\right)=\left(\frac{1}{D g}\right)^{n^{\prime}} \cdot \frac{1}{\left(n^{\prime}-1\right)!} \cdot D g \cdot e^{-\frac{T_{G \min }}{D g}} \cdot G_{n^{\prime}}$

$$
G_{n^{\prime}}=\left\{\begin{array}{cc}
1 & n^{\prime}=1  \tag{10}\\
T_{G \min }^{n^{\prime}-1}+G_{n^{\prime}-1} \cdot\left(n^{\prime}-1\right) \cdot D g & n^{\prime}>1
\end{array}\right.
$$

Once probability (10) is obtained, the expression of the probability of having $n_{d}$ transitions according to (4) is :

$$
\begin{align*}
P\left(n_{d}\right) & =\left(\frac{1}{D g}\right)^{n_{d}+1} \cdot \frac{1}{\left(n_{d}+1-1\right)!} \cdot D g \cdot e^{-\frac{T_{G \text { min }}}{D g}} \cdot G_{n_{d}+1} \\
& -\left(\frac{1}{D g}\right)^{\left(n_{d}\right)^{\prime}} \cdot \frac{1}{\left(n_{d}-1\right)!} \cdot D g \cdot e^{-\frac{T_{G \text { min }}}{D g}} \cdot G_{n_{d}} \tag{11}
\end{align*}
$$

In figure 5 this probability of having $n$ transitions for different values of T between 0 and 300 s is shown.
It can be seen, that if $T$ is small, the file can be decoded without any transition from the good state to the bad state because the probability of having one or more transitions is nearly zero. If $T$ increases, the probability of being in the good state during the full decoding time goes exponentially to zero and the probability of having more than one transitions becomes greater.

(a) Urban environment

(c) Rural environment

(b) Suburban environment

(d) Highway environment

Fig. 5. Probability of having 10 transitions during the file decoding time, for the four environments and $r=2 / 3$

## III. Duration of File Decoding

The general expression of the time spent to decode a file is:

$$
\begin{equation*}
\tau_{n_{d}}=\left(T_{G \min }+\sum_{i=0}^{n_{d}} T_{B i}\right) \cdot P_{G}+\left(T_{G \min }+\sum_{i=0}^{n_{d}+1} T_{B i}\right) \cdot P_{B} \tag{12}
\end{equation*}
$$

Where $n_{d}$ is the number of transitions from the good to the bad state and $P_{G}$ and $P_{B}$ the probability of starting in the good or in the bad state. As all $T_{B i}$ are again exponentially distributed random variables, the pdf of the sum of all the times in which we are in the bad state is:

$$
\begin{equation*}
f_{T_{B}}\left(t \mid n_{d}\right)=f_{T_{B 1}}(t) * f_{T_{B 2}}(t) * \ldots * f_{T_{B n_{d}}} \tag{13}
\end{equation*}
$$

The density function in the time domain is then:

$$
\begin{equation*}
f_{T_{B}}\left(t \mid n_{d}\right)=\frac{\left(\frac{1}{D_{b}}\right)^{n_{d}}}{\left(n_{d}-1\right)!} \cdot t^{n_{d}-1} \cdot e^{-\frac{t}{D_{b}}} \tag{14}
\end{equation*}
$$

The total time spent in the good state is always $T_{\text {Gmin }}$, so its density function $f_{T_{G}}\left(t \mid n_{d}\right)$ is a delta function, whose integral is the step function $u(t)$ :

$$
\begin{equation*}
f_{T_{G}}\left(t \mid n_{d}\right)=\delta\left(t-T_{G m i n}\right) \tag{15}
\end{equation*}
$$

Since we already know the expression of $f_{T_{B}}\left(t \mid n_{d}\right)$, it is possible to obtain the density function of $\tau_{n_{d}}$, applying the convolution between $f_{T_{B}}\left(t \mid n_{d}\right)$ and $f_{T_{G}}\left(t \mid n_{d}\right)$ once for the case in which the initial channel state is the good state, and
once, if it is the bad one:

$$
f_{\tau}\left(t \mid n_{d}\right)=\left\{\begin{array}{l}
n_{d}=0 \\
{\left[P_{G}+\frac{1}{D_{b}} \cdot e^{-\frac{t}{D_{b}}} \cdot P_{B}\right] * \delta\left(t-T_{G \min }\right)} \\
n_{d} \geq 1 \\
{\left[\frac{\left(\frac{1}{D_{b}}\right)^{n_{d}\left(t-T_{G m i n}\right)^{n_{d}-1}}}{\left(n_{d}-1\right)!}\right.} \\
\left.\cdot e^{-\frac{\left(t-T_{G \min }\right)}{D_{b}}} u\left(t-T_{G \min }\right)\right] \cdot P_{G} \\
+\left[\frac{\left(\frac{1}{D b}\right)^{\left(n_{d}+1\right)}\left(t-T_{G \min }\right)^{n_{d}}}{n_{d}!}\right. \\
\left.e^{-\frac{\left(t-T_{G \min }\right)}{D_{b}}} u\left(t-T_{G \min }\right)\right] \cdot P_{B}
\end{array}\right.
$$

## A. Probability of Non Successful File Decoding

The next step is finding the probability that the time $\tau$ is greater than a time $\tilde{T}$, which is the real transmission time of the file.
To obtain this probability we must solve the following integral:

$$
\begin{align*}
I_{n_{d}} & =\int_{\tilde{T}}^{\infty} f_{T_{B}}(t) d t \\
& =\left(\frac{1}{D_{b}}\right)^{n_{d}} \cdot \frac{1}{\left(n_{d}-1\right)!}\left(-D_{b}\left(\tilde{T}-T_{G \min }\right)^{n_{d}-1} \cdot e^{-\frac{\left(\tilde{T}-T_{G \min }\right)}{D_{b}}}\right. \\
& +D_{b}\left(n_{d}-1\right)\left[-D_{b}\left(\tilde{T}-T_{G \min }\right)^{n_{d}-2} \cdot e^{-\frac{\left(\tilde{T}-T_{G \min }\right)}{D_{b}}}\right. \\
& \left.\left.+D_{b}\left(n_{d}-2\right) \cdot(\ldots)\right]\right) \tag{16}
\end{align*}
$$

This formula can be also expressed via recursions. Then the formula can be written as:
$I_{n_{d}}=\left(\frac{1}{D_{b}}\right)^{n} \cdot \frac{1}{\left(n_{d}-1\right)!} \cdot D b \cdot e^{-\frac{\left(\tilde{T}-T_{G \text { min }}\right)}{D_{b}}} \cdot G_{n_{d}}$ $G_{n_{d}}=\left\{\begin{array}{cc}1 & n_{d}=1 \\ \left(\tilde{T}-T_{G m i n}\right)^{n_{d}-1}+G_{n_{d}-1} \cdot\left(n_{d}-1\right) \cdot D_{b} & n_{d}>1\end{array}\right.$
(17)

Finally using (17) an expression of $P(\tau>\tilde{T})$ can be obtained.:
$P\left(\tau>\tilde{T} \mid n_{d}\right)=\left\{\begin{array}{l}n_{d}=0 \\ P_{B} \cdot e^{-\frac{\left(\tilde{T}-T_{G \text { min }}\right)}{D_{b}}} \\ n_{d} \geq 1 \\ {\left[\frac{\left(\frac{1}{D_{b}}\right)^{n_{d}}}{\left(n_{d}-1\right)!} \cdot D_{b} \cdot e^{-\frac{\left(\tilde{T}-T_{G \text { min }}\right)}{D_{b}}} \cdot G_{n_{d}}\right] P_{G}+} \\ {\left[\frac{\left(\frac{1}{D_{b}}\right)^{n_{d}+1}}{n_{d}!} D_{b} e^{-\frac{\left(\tilde{T}-T_{G \text { min }}\right)}{D_{b}}} G_{n_{d}+1}\right] P_{B}}\end{array}\right.$

Thus the probability not to decode a file for a particular number of transitions is:

$$
\begin{equation*}
P\left(\overline{\text { decode file }} \mid n_{d}=i\right)=P\left(\tau>\tilde{T} \mid n_{d}\right) \tag{18}
\end{equation*}
$$

And finally the probability to decode a file is:
$P($ decode file $)=\sum_{\forall i}\left(1-P\left(\overline{\text { decode file }} \mid n_{d}=i\right)\right) \cdot P\left(n_{d}=i\right)$

## B. Success in decoding the first file received

As the pdf of the time spent to decode a file is now known, it is possible to obtain the probability of decoding correctly the very first file of a user. Let us denote $t_{0}$ the instant in which the user switches on the system. Unfortunately, this is at an arbitrary time during the file transmission, so only a certain time $\delta T=T-t_{0}$ of the file transmission time is available to decode correctly the file. Taking this effect into account and assuming that $t_{0}$ is uniformly spread over $T$, the probability to decode the very first file of a user is:

$$
\begin{align*}
P(\text { decode file }) & =\sum_{\forall i}\left[\int_{0}^{T} f_{T_{0}}\left(t_{0}\right)\right. \\
& \left.\cdot\left(1-P\left(\tau>\tilde{T}-t_{0} \mid n_{d}=i\right)\right) d t_{0}\right] \cdot P\left(n_{d}=i\right) \tag{20}
\end{align*}
$$

Finally one arrives for the proposed system and the assumed channel to results like shown in figure 6 for the urban case, with its $40 \%$ shadowing.


Fig. 6. Probability to decode the first file successfully for coding rates $\mathrm{R}=\mathrm{r}$ in the urban environment

Observing these figures, we can take different conclusions. When $T=0$ the probability of successful decoding is
$(1-r) \cdot P g$. This make sense, since if the initial state is the bad state and T is small, it is sure that the file will be not correctly decoded. If the initial state is the good one then the probability of decoding the first file must be the probability that the instant in which the user switches on, is smaller than $T-T_{G m i n}$. This probability is $1-r$.

Also very interesting is the behaviour of such a system in case of very long file transmission. If the LOS-share is smaller than the coderate, the probability to decode a file goes to zero. If the LOS share is larger than the code rate, the probability goes to the limit of $1-(R / L O S)$.

## IV. Conclusions

In the paper, a way was shown how to calculate the needed time until a file can be decoded in the case of a Gilbert channel. Based on the results, different statistics for this file reception can be derived, while still making use of relative manageable equations.

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