Improved Phase Impairment Compensation for Frequency Selective OFDM Systems

Konstantinos Nikitopoulos, Georgios Tsaramiadis and Andreas Polydoros

***Abstract*—This paper extends the Inter-Carrier Interference (ICI) self-cancellation scheme of [1] for coherent demodulation and transmission via frequency-selective fading channels to scenarios with significant composite PHase Noise (PHN) and Residual Frequency Offset (RFO) effects. The proposed scheme is matched to the (assumed known) spectral shape of the channel and therefore outperforms simpler variations that do not take that shape into account. Both (matched and unmatched) versions are evaluated and are shown to be superior performance-wise to standard OFDM (i.e., without ICI compensation), at the cost of smaller channel utilization versus the latter.**

***Index Terms*—ICI self-cancellation, frequency offset, phase noise, OFDM.**

I. Introduction

OFDM is a multi-carrier transmission scheme suitable for high spectral utilization. It is known, however, that sub-carrier orthogonality is violated by the joint presence of PHN and RFO. The latter phase impairments produce a Common Error (CE) to all sub-carriers of the same OFDM symbol, plus ICI ([2]-[4]). To alleviate the effect of this composite (PHN and RFO) phase impairment, two different approaches have been proposed in the literature: the first ([4]-[8]) addresses the CE by itself, while the second targets joint CE and ICI compensation at the cost of either increased complexity or reduced throughput. The "increased complexity" scheme employs direct estimation and equalization of the most significant ICI terms ([9]), while the "reduced throughput" schemes employ data repetition on adjacent sub-carriers, thus providing observables with suppressed ICI ([1], [10]). This ICI cancellation approach has been proposed for flat fading channels in the presence of RFO only (i.e., no PHN). CE and channel (gain) estimation/equalization have thus been avoided via this type of differential encoding. Our method herein is proposed for frequency selective channels and coherent demodulation.

This paper is organized as follows: In Section II the OFDM system model is described. Sections III and IV describe the proposed scheme along with its design approaches. Finally, in Section V simulations substantiate the adopted claims and evaluate the performance of the proposed scheme.

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II. System Model

In OFDM, the incoming QAM or PSK symbols are serial-to-parallel converted before channelization via an N-point Inverse Discrete Fourier Transform (IDFT). A cyclic prefix of length ν is added, larger than the Channel Impulse Response (CIR), assumed static for the duration of at least one OFDM symbol. The resulting output is parallel-to-serial and digital-to-analog converted and then transmitted. At the receiver side, the inverse process takes place. The final observables after the receiver's Discrete Fourier Transform (DFT) are given by ([6]):

$$Y_m(k) = X_m(k)H_m(k)U_m(0) + I_m(k) + N_m(k) \tag{1}$$

where

$$I_m(k) = \sum_{\substack{l=0 \\ l\neq k}}^{N-1} X_m(l)H_m(l)U_m(l-k) \tag{2}$$

represents the ICI noise term. Here, $X_m(k)$ denotes the transmitted QAM symbol of the *k*-th sub-carrier of the *m*-th OFDM symbol, $H_m(k)$ is the *k*-th tap of the frequency-domain channel shape, $U_m(0)$ is the CE, and $N_m(k)$ represents the discrete additive white noise. The $U_m(k)$ term is given by

$$U_m(k) = \frac{1}{N}\sum_{p=0}^{N-1} u_m(p) e^{j\frac{2\pi}{N}pk} \tag{3}$$

where $u_m(p) = \exp\left\{ j\left(\phi_{m,PHN}(p) + \phi_{m,RFO}(p)\right)\right\}$ is a complex term due to PHN and RFO.

Two approaches for modelling PHN are described in the literature. The first ([2]-[3]) models it as a discrete-time Wiener process, equivalent to the sampled version of the continuous-time process with zero-mean Gaussian independent increments of variance σ^2_{PHN}. The second model [7] considers PHN as a stationary random process, characterized by a power spectral density that is measured by a Phase Locked Loop (PLL). The proposed methods are independent of the PHN model used. Furthermore, since the subsequent analysis is independent of the OFDM symbol sequence, the subscript m is discarded.

III. The Proposed Scheme

The employed transmitter differential code, as per [1], is $X(k+1) = -X(k)$, for $k = 0,2,\ldots,N-2$. The proposed scheme is shown in Fig. 1 and works as follows: The received observables $Y(k)$ feed the "combiner" module, which results to $Y^{eq}(k)$ ($k = 0,2,\ldots,N-2$). The "Equivalent Channel (EC) equalizer" compensates for the channel variations post-combination, yielding the observables $R^{eq}(k)$, which then feed the final decision device after "CE estimation" and "CE compensation".

A. *Combiner*

The general combination rule can be written as

$$Y^{eq}(k) = G(k)Y(k) - G(k+1)Y(k+1) \tag{4}$$

where $k = 0,2,\ldots,N-2$

The pair $\{G(k),\ G(k+1)\}$ of complex gains are design parameters. The resulting observables can be written as

$$Y^{eq}(k) = Q(k)X(k) + I^{eq}(k) + N^{eq}(k) \tag{5}$$

$k = 0,2,\ldots,N-2$

with

$$\begin{aligned} Q(k) = {} & G(k)\left[U(0)H(k) - U(1)H(k+1)\right] \\ & + G(k+1)\left[U(0)H(k+1) - U(-1)H(k)\right] \end{aligned} \tag{6}$$

The $I^{eq}(k)$ is the equivalent ICI term

$$\begin{aligned} I^{eq}(k) = {} & \sum_{\substack{l=0,l\neq k\\ l:even}}^{N-2} \Big\{X(l)G(k)\big[H(l)U(l-k) - \\ & -H(l+1)U(l-k+1)\big]\Big\} - \\ & - \sum_{\substack{l=0,l\neq k\\ l:even}}^{N-2} \Big\{X(l)G(k+1)\big[H(l)U(l-k-1) - \\ & - H(l+1)U(l-k)\big]\Big\} \end{aligned} \tag{7}$$

and

$$N^{eq}(k) = G(k)N(k) - G(k+1)N(k+1) \tag{8}$$

is the corresponding additive noise. Since $|U(1)|^2$ and $|U(-1)|^2$ are much smaller than $|U(0)|^2$ ([10]), and because $H(k)$ and $H(k+1)$ are of the same order of magnitude even for severe frequency selectivity, $Q(k)$ can be approximated as

$$Q(k) \approx U(0)\left[G(k)H(k) + G(k+1)H(k+1)\right] \tag{9}$$

(a fact also verified via extensive simulations). Thus, (5) becomes

$$Y^{eq}(k) \approx U(0)H^{eq}(k)X(k) + I^{eq}(k) + N^{eq}(k) \tag{10}$$

with $H^{eq}(k)$ describing the EC

$$H^{eq}(k) = G(k)H(k) + G(k+1)H(k+1) \tag{11}$$

Because $U(k) \approx U(k+1)$ (for $k, k+1 \neq 0$) ([10]), the Signal to (ICI) Interference Ratio (SIR) of the proposed scheme is naturally expected to be much better than that of a typical uncompensated OFDM system, a fact also easily substantiated through simulations. Since the observable $Y^{eq}(k)$ is of the same form as (1), typical channel estimation and CE compensation methods can be employed ([6]) before final decisions.

B. *EC equalization*

Zero-forcing equalization (i.e., one-tap division) is employed here to counter the EC, which is known to the receiver since the {$G(k)$ and $G(k+1)$} gains are known and the original channel taps are estimated ([11]- [13]). Since we assume perfect channel knowledge, the resulting $R^{eq}(k)$ observables are

$$R^{eq}(k) \approx U(0)X(k) + W(k) \tag{12}$$

where $W(k)$ stands for the total (ICI plus thermal) equalized noise samples.

C. *CE estimation & compensation*

Pilot-Symbol-Assisted-Modulation (PSAM) is employed for $U(0)$ estimation and compensation: A set of adjacent pairs of (pilot) sub-carriers Ω is modulated with known pilot symbols ($P(k)$ with $k \in \Omega$), which are also frequency encoded. Least-Squares CE estimation is employed, thus

$$\hat{U}(0) = \frac{\sum_{k\in\Omega} P^*(k)R^{eq}(k)}{\sum_{k\in\Omega} |P(k)|^2} \tag{13}$$

Compensation can thus be achieved by simple division (zero-forcing).

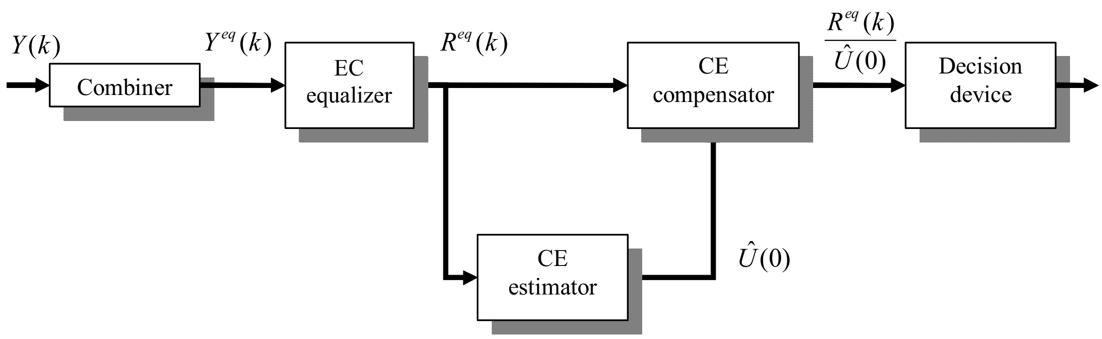

Fig. 1. Block diagram of the proposed schemes at the Rx side

IV. $G(k)$ AND $G(k+1)$ SELECTION

Two different versions of $G(k), G(k+1)$ are examined: For the first, $G(k) = H^*(k)$ and $G(k+1) = H^*(k+1)$ are selected (*Option 1 - O_1*), so as to match channel variations. In reality, the $H(k)$ terms will be replaced by the corresponding estimates $\hat{H}(k)$. The second option (*Option 2 - O_2*) simply lets $G(k) = G(k+1) = 1$. The chosen values of the gains modify accordingly the "combiner" and the CE modules of (4) and (11).

V. SIMULATIONS

We fix 64-QAM modulation, $N = 256$, $v = 33$ (longer than the CIR). The set Ω consists of 8 equally-spaced pairs of pilot symbols for the proposed scheme and 8 single symbols for the typical OFDM system ([6], [14]) with CE correction. Two static channels are modeled, namely different realizations of a Non-Line-Of-Sight channel for a fixed wireless access system in a small urban scenario at 5.8 GHz characterized by severe (Type-A channel) and mild (Type-B channel) frequency selectivity, shown in Fig. 2. The assumed PHN-RFO process corresponds to $\sigma_{PHN} = 0.01$ and RFO of 2% of the sub-carrier spacing.

In Fig, 3, the $Q(k) \approx U(0)H^{eq}(k)$ approximation is verified, by plotting the ratio

$$\rho_1(k) = \frac{E\left\{|Q(k)|^2\right\} - E\left\{|U(0)H^{eq}|^2\right\}}{E\left\{|Q(k)|^2\right\}} \tag{14}$$

for both versions and the Type-A channel. In Fig. 4 the same assumptions are employed and the ratio $\rho_2(k) = SIR^{eq}(k) \big/ SIR(k)$ is shown, with $SIR(k) = E\left\{|X(k)H(k)U(0)|^2\right\} \big/ E\left\{|\mathrm{I}(k)|^2\right\}$ and $SIR^{eq}(k) = E\left\{|X(k)H^{eq}(k)U(0)|^2\right\} \big/ E\left\{|\mathrm{I}^{eq}(k)|^2\right\}$. It is clear that the proposed method results to improved SIR. The Symbol Error Rate (SER) performance of the proposed schemes, also compared to standard OFDM for Type-A and Type-B channels is shown in Figs. 5 and 6. As expected, the two options provide identical performance for Type-B channels, while significant performance difference appears in severe frequency selectivity. We note that all schemes are normalized to the same transmitted energy per symbol.

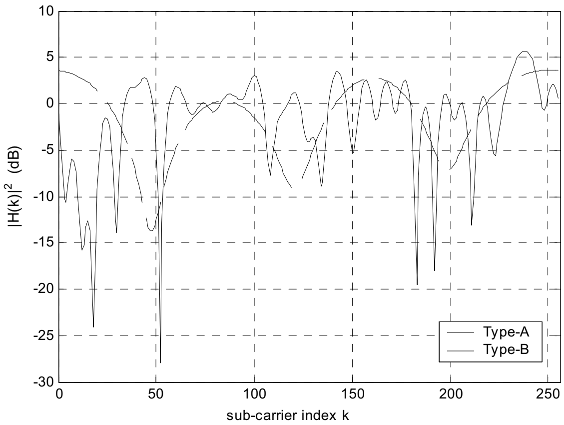

Fig. 2. Frequency response for Type-A and Type-B channels.

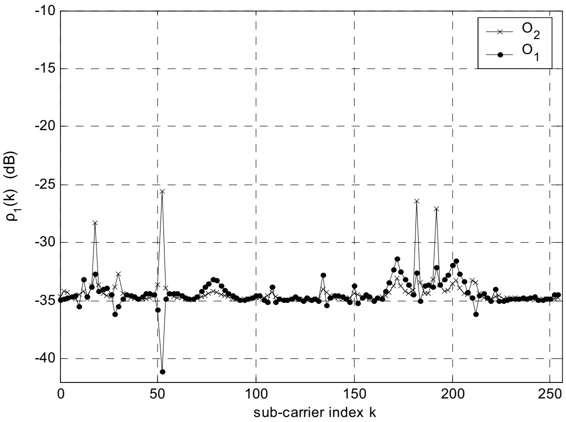

Fig. 3. $\rho_1(k)$ for both design assumptions and Type-A channel.

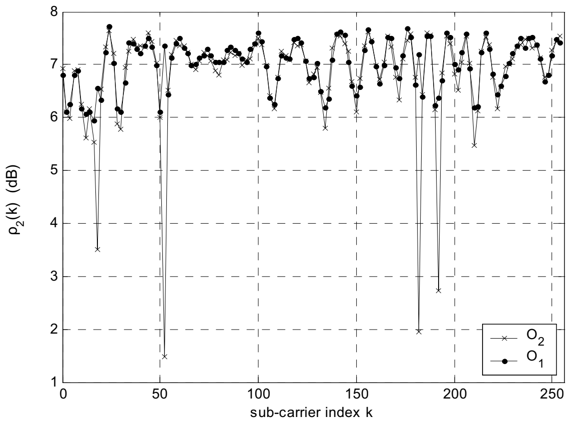

Fig. 4. $\rho_2(k)$ for both design assumptions and Type-A channel.

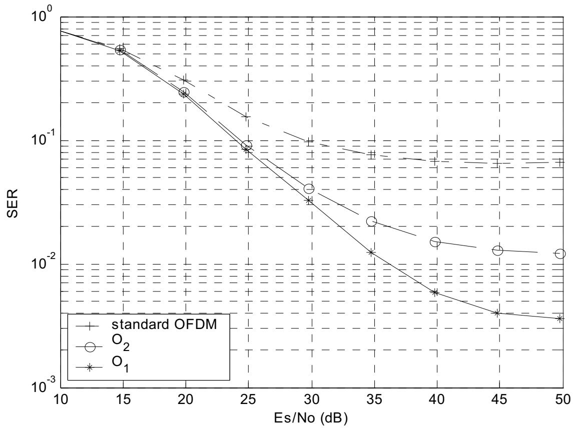

Fig. 5. Performance of the proposed scheme for both versions and Type-A channel.

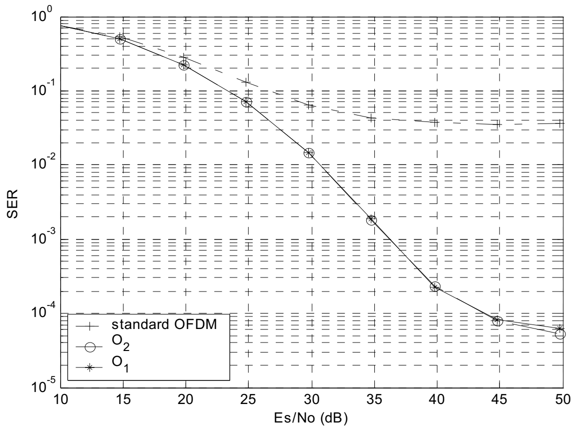

Fig. 6. Performance of the proposed scheme for both versions and Type-B channel.

VI. CONCLUSIONS

In this paper, the ICI self-cancellation scheme of [1] is extended to operate in fading channels. Two versions of the proposed scheme are discussed and evaluated. The proposed scheme can operate efficiently in adverse PHN/RFO environments with small complexity increase but reduced overall throughput.

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