Inter-Frame, Fine Frequency/Phase Synchronization in Space Time – OFDM Receivers

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Abstract—**This paper proposes a Fine Frequency Synchronization (FFS) scheme for a 2x2 Multiple-Input, Multiple-Output (MIMO) OFDM system. It is based on the fact that the Residual Frequency Offset (RFO), which is defined as the resulting error from the frequency synchronization process, is static for at least one transmission frame. The proposed scheme is evaluated for a 2x2, Space-Time (ST), OFDM scheme, which utilizes the Alamouti code. However, it can be employed with any other ST scheme. The proposed scheme, which also includes a channel estimation module (i.e., the channel is not perfectly known), assumes a realistic scenario where both Phase Noise (PHN) and RFO are present. It is shown through simulations that the proposed scheme can effectively cope with PHN and RFO and that the overall system, including the channel estimation part, provides significant performance gain with insignificant complexity increase.**

Index Terms— **Frequency offset, OFDM, phase noise, channel estimation**

I. Introduction

OFDM is a multi-carrier transmission scheme whose performance degrades significantly whenever deep spectral fades are present; thus, diversity schemes have been proposed to alleviate the fading effect. Such a ST scheme is the Alamouti code [1], originally devised for single-carrier systems. However, its adaptation to OFDM is straightforward by means of per-sub-carrier ST coding. In [2] it is shown that the performance of this type of ST-OFDM schemes degrades drastically in the joint presence of PHN/RFO and channel estimation errors, the latter also affected negatively by composite phase impairments.

For MIMO it is inappropriate to employ conventional SISO approaches, and variations have been proposed for its channel estimation ([3]-[5]). These typically ignore the presence of phase impairments, thus resulting in degraded performance. In this paper the effect of phase impairments on the method of [3] is described and the corresponding performance loss is depicted through simulations, substantiating the claim that they degrade significantly the quality of the channel estimates and, consequently, system performance.

The typical frequency synchronization algorithms proposed [6] are correlation-based, performed in the time domain. They are designed for AWGN channels, i.e., in the absence of other impairments (such as PHN). The herein proposed inter-frame FFS scheme is coupled with a PHN/RFO compensation scheme similar to [7], which addresses PHN/RFO estimation/compensation under the assumption of perfect channel estimates. It is demonstrated through simulations that the proposed scheme achieves performance close to one in the absence of phase impairments, even for the simple algorithm of [3].

In Section II, the MIMO system model in the presence of PHN and RFO is presented, along with a description of the Alamouti ST for OFDM. In Section III, the effect of the phase impairments on the channel estimation method is described and its effect demonstrated via simulations. In Section IV the proposed scheme is described and further evaluated in the simulations Section VI.

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II. MIMO System Model plus Alamouti ST Code in the Presence of PHN&RFO

In this work, frame-based transmission is assumed. Known preamble symbols are transmitted at the beginning of each frame, and used at the Rx side for synchronization and channel estimation. Since synchronization tasks (timing and frequency) take place before channel estimation, any synchronization error will affect the quality of the channel estimates. Here, perfect timing synchronization is assumed, whereas frequency synchronization is imperfect, yielding a fixed but unknown RFO. For a typical MIMO-OFDM system, PHN originates in both Tx and Rx oscillators, whereas the RFO is modeled as an additional linear (in time) phase rotation at the Rx side. When the phase process of the Tx oscillator is fairly static over the length of the channel impulse response, the post-DFT (length-*N*) received observable by antenna i, on the k-th sub-carrier of the m-th transmitted OFDM symbol, can be approximated by [7]

$$Y_{m,k}^{(i)} \approx U_{m,0} \sum_{l} X_{m,k}^{(l)} H_{m,k}^{(l,i)} + n_{ICI,k}^{(i)} + w_k^{(i)} \quad (1)$$

where

$$n_{ICI,k}^{(i)} = \sum_{\substack{s=0 \\ s\neq k}}^{N-1} U_{m,s-k} \sum_{l} X_{m,s}^{(l)} H_{m,s}^{(l,i)} \tag{2}$$

is the Inter-Carrier-Interference (ICI) term. The $U_{m,q}$ term is given by:

$$U_{m,q} = \frac{1}{N}\sum_{p=0}^{N-1} u_m(p) e^{j\frac{2\pi}{N}pq} \tag{3}$$

where $H_{m,k}^{(l,i)}$ is the k-th tap of the DFT of the corresponding CIR from Tx antenna *l* to Rx antenna *i*, $X_{m,k}^{(l)}$ denotes the corresponding transmitted symbol by antenna *l* and $w_k^{(i)}$ represents the corresponding discrete additive white noise. The term

$$u_m(p) = \exp\left\{ j\left(\phi_{m,PHN}^{(Tx)}(p) + \phi_{m,PHN}^{(Rx)}(p) + \phi_{m,RFO}(p)\right)\right\} \tag{4}$$

denotes the complex phase modulation term due to the PHN processes of the Tx and the Rx plus the rotation due to the RFO.

Two approaches for modelling PHN are described in the literature ([7]-[9]). According to the first, it can be modelled as a discrete-time Wiener process which is equivalent to the sampled version of the continuous-time process, and it has zero-mean Gaussian independent increments with variance σ_{PHN}^2. The second model considers PHN as a stationary random process, characterized by its power spectral density, which is obtained through measurements of a real tuner using a Phase Locked Loop (PLL). The proposed methods are independent of the PHN model used.

For the Alamouti STC scheme (which assumes a static channel for two consequent OFDM symbols), two different OFDM symbols are transmitted simultaneously from each of the Tx antennas in each symbol period, denoted by $X_{m,k}^{(0)} = S_{j0,k}$ and $X_{m,k}^{(1)} = S_{j1,k}$, respectively. During the subsequent OFDM symbol period, $X_{m+1,k}^{(0)} = -S_{j1,k}^{*}$ and $X_{m+1,k}^{(1)} = S_{j0,k}^{*}$ are transmitted. Here, j indexes the corresponding transmitted pair (e.g., $j=1$ denotes the first pair of transmitted OFDM symbols). The corresponding received symbols during symbol times (*m*, *m*+1) are combined for Maximum-Likelihood (ML) detection through the rule:

$$\begin{cases} \tilde{S}_{j0,k} = H_{j,k}^{(0,0)*} Y_{m,k}^{(0)} + H_{j,k}^{(1,0)} Y_{m+1,k}^{(0)*} + \\ +H_{j,k}^{(0,1)*} Y_{m,k}^{(1)} + H_{j,k}^{(1,1)} Y_{m+1,k}^{(1)*} \\ \tilde{S}_{j1,k} = H_{j,k}^{(1,0)*} Y_{m,k}^{(0)} - H_{j,k}^{(0,0)} Y_{m+1,k}^{(0)*} + \\ +H_{j,k}^{(1,1)*} Y_{m,k}^{(1)} - H_{j,k}^{(0,1)} Y_{m+1,k}^{(1)*} \end{cases} \tag{5}$$

III. PHN/RFO Effect On The Channel Estimation Algorithm

Following [3], the channel estimation process takes place at the start of the frame via known (training) OFDM symbols, with $C_{w,k}^{(l)}$ the known (QAM or PSK) symbol loaded on the k-th sub-carrier of the w-th training symbol, transmitted by antenna l. Two training symbols are employed with $C_{0,k}^{(0)} = -C_{1,k}^{(0)} = C_{0,k}^{(1)} = C_{1,k}^{(1)}$. Then, the frequency domain channel estimator is

$$\hat{H}_{0,k}^{(i,l)} = \frac{1}{2C_{0,k}^{(0)}}\left[Y_{0,k}^{(l)} + (-1)^{i+1} Y_{1,k}^{(l)}\right]; \quad i,l = 0,1 \tag{6}$$

By use of (1), it can be shown that the effect of PHN and RFO on the resulting channel estimates is manifested as

$$\begin{cases} \hat{H}_{0,k}^{(0,l)} \approx \dfrac{\left(U_{0,0}+U_{1,0}\right)H_{0,k}^{(0l)} + \left(U_{0,0}-U_{1,0}\right)H_{0,k}^{(1,l)}}{2} + n_k^{(0,l)} \\ \hat{H}_{0,k}^{(1,l)} \approx \dfrac{\left(U_{0,0}+U_{1,0}\right)H_{0,k}^{(1,l)} + \left(U_{0,0}-U_{1,0}\right)H_{0,k}^{(0,l)}}{2} + n_k^{(1,l)} \end{cases} \tag{7}$$

with the $n_k^{(i,l)}$ terms denoting the thermal plus the ICI induced noise. Eq. (7) proves that PHN and RFO impair the orthogonality of the channel estimates.

IV. The Proposed Scheme

The proposed scheme assumes that the RFO is fairly invariant for two consequent frames (just as the channel); thus, channel estimation can be performed at the start and no adaptive tracking is needed. The scheme, sketched in Fig. 1, works as follows: The received observables after the Rx DFT, $Y_{m,k}^{(i)}$, from all the antennas are combined in order to symbol-by-symbol estimate and compensate for the phase impairments in the "PHN/RFO estimation" and "PHN/RFO compensation" modules, respectively. The "PHN/RFO estimation" module utilizes the channel estimates $\hat{H}_{0,k}^{(l,i)}$, which have been obtained from the channel estimation process at the start of the frame. The outputs of the "PHN/RFO estimation" module $\hat{U}_{m,0}$ are utilized by the "PHN/RFO compensation" module, which performs zero-forcing compensation (division by the $U_{m,0}$ term) on the received observables, yielding the symbols $R_{m,k}^{(i)} = Y_{m,k}^{(i)} \big/ \hat{U}_{m,0}$. Additionally, the $\hat{U}_{m,0}$ terms feed the "FFS" module, which produces a refined estimate of the frequency offset $\Delta\hat{f}$. This is then used in the next frame instead of any other typical frequency synchronization algorithm in the time domain (which has, however, been employed for the synchronization needs of the first frame). The initial RFO for the first frame results from a typical frequency synchronization process, as described in [6].

A. The PHN/RFO Estimation and Compensation Algorithms

Pilot-Symbol-Assisted-Modulation (PSAM) can be employed for $U_{m,0}$ estimation and compensation: A set of (pilot) sub-carriers Ω is modulated with known pilot symbols ($P_{m,k}^{(l)}$ with $k \in \Omega$) and is used for symbol-by-symbol estimation:

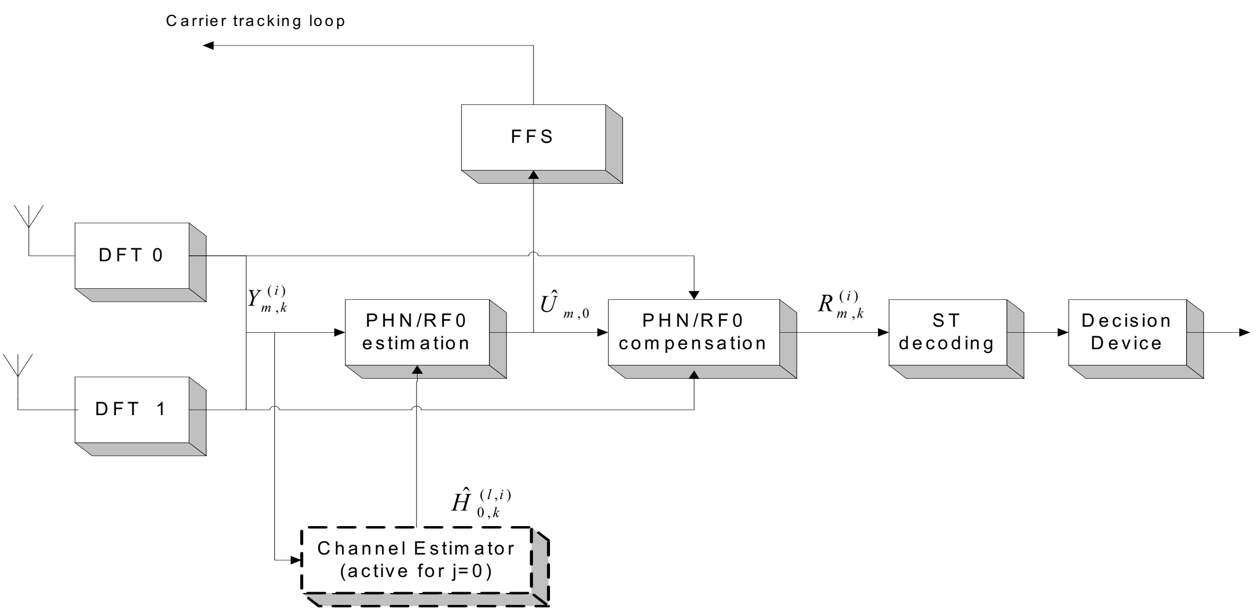

Fig. 1. Block diagram of the proposed scheme.

$$\hat{U}_{m+g,0} = \frac{\sum_i \sum_{k\in\Omega} Y^{(i)}_{m+g,k} \sum_l \hat{H}^{(li)*}_{0,k} P^{(l)*}_{m+g,k}}{\sum_i \sum_{k\in\Omega} \left| \sum_l \hat{H}^{(li)}_{0,k} P^{(l)}_{m+g,k} \right|^2}; \quad g = 0,1 \tag{8}$$

which is in fact the Maximum Likelihood estimator when the ICI noise is negligible and the channel estimates $\hat{H}^{(li)}_{j,k}$ are perfect. Compensation is achieved by division, as mentioned above. When orthogonal pilots are employed (i.e., for any of the pilot sub-carriers only one of the Tx antennas is loaded with symbols), the algorithm coincides with the ones used for the Single-Input, Single-Output (SIS0) case [8].

B. The Fine Frequency Synchronization (FFS) scheme

When the PHN process is slow enough so that the corresponding rotation can be assumed negligible for τ OFDM symbols, then

$$U_{m+\tau,0} = U_{m,0} \exp\left[j2\pi\Delta f(\mathrm{N}+v)\tau\right] \tag{9}$$

where v is the cyclic prefix length and Δf is the RFO. Thus, the effect of the PHN/RFO for different OFDM symbol indexes can be modeled as

$$\hat{U}_{m+\tau,0} = \hat{U}_{m,0} \exp\left[j2\pi\Delta f(\mathrm{N}+v)\tau\right] + n_{err,m,\tau} \tag{10}$$

where $n_{err,m,\tau}$ is a function of the estimation errors at times m and $m+\tau$. By minimizing the cost function

$$\sum_{m=m_0}^{m_0+\tau'} \left| \hat{U}_{m+\tau,0} - \hat{U}_{m,0} \exp\left[j2\pi\Delta f(\mathrm{N}+v)\tau\right] \right|^2 \tag{11}$$

a refined RFO estimate $\Delta\hat{f}$ can be produced:

$$\Delta\hat{f} = -\frac{1}{2\pi(N+\nu)\tau} \arg\left\{ \sum_{m=m_0}^{m_0+\tau'} \hat{U}_{m,0} \hat{U}^*_{m+\tau,0} \right\} \tag{12}$$

where $\tau' < \tau$ and m_0, $m_0+\tau'$ can take values up to the number of the OFDM symbols per frame.

V. SIMULATIONS

The assumptions are 64-QAM modulation, N=256 OFDM, and cyclic prefix of $\nu = 33$ (longer than the CIR). The link includes PHN sources (oscillators) at both the Tx and Rx sides, namely random processes modeled as in [9], with characteristic values -78.8 dBc and -95.5 dBc at 10 kHz for the RF and IF parts, respectively. The assumed RFO equals 1% of the sub-carrier spacing. The channel is static (during a frame) and it is based on the modification of the SUI-4 model ([10]) for MIMO channels. The set Ω consists of 8 equally-spaced, BPSK-modulated pilot sub-carriers, boosted by 2.5 dB, transmitted orthogonally by the two antennas. The training symbols are also BPSK modulated, boosted by 3 dB. The parameters τ and τ' are $\tau'+1 = \tau = 10$. The simulations depict the Symbol Error Rate (SER) performance of the system with and without the FFP scheme, when actual and perfect channel estimates are assumed. The performance of the first frame is ignored. In Fig. 2, it is shown that, for perfect channel estimates, the PHN/RFO compensation scheme performs very well. However, the use of FFS provides a further gain, since this fine frequency estimate results in reduced ICI. On the other hand, it is shown in Fig. 3 that the presence of the phase impairments degrades highly the performance of the system, since they affect the channel estimates. In this case, FFS usage provides a significant gain of about 4 dB for a SER of 10^{-3}.

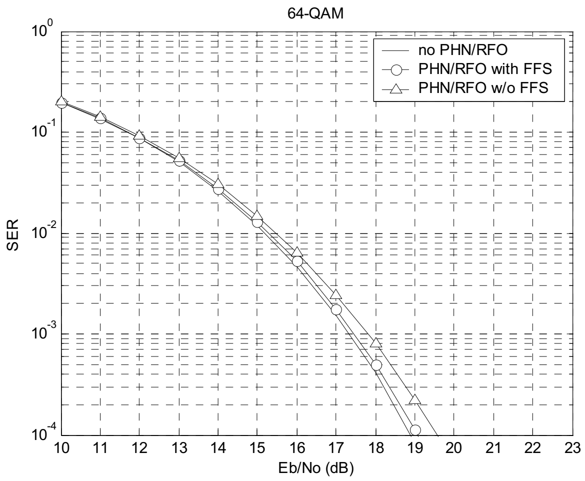

Fig. 2. Performance of the proposed schemes for perfect channel knowledge.

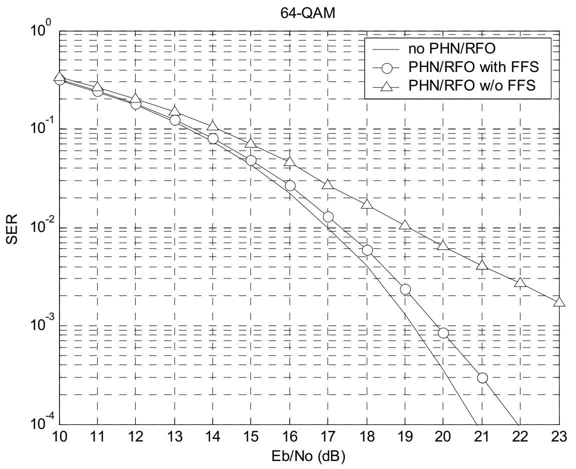

Fig. 3. Performance of the proposed schemes for actual (imperfect) channel estimates.

VI. Conclusions

It is shown that the PHN/RFO affects significantly the simple, frequency-domain, MIMO channel estimator of [3]. Thus, an inner-frame fine frequency estimator process is proposed, which utilizes the outcomes of the PHN/RFO estimator and improves significantly the quality of the channel estimates. Although the method has been evaluated for the Alamouti ST code and the specific channel estimation method, it is adaptable to any ST code and any MIMO channel estimation scheme, since the PHN/RFO estimation/ compensation method is ST-independent and frequency synchronization precedes channel estimation. Since the proposed frequency-domain estimator is also correlation-based, exactly as typical initial frequency synchronizers in the time domain, the same modules can be utilized.

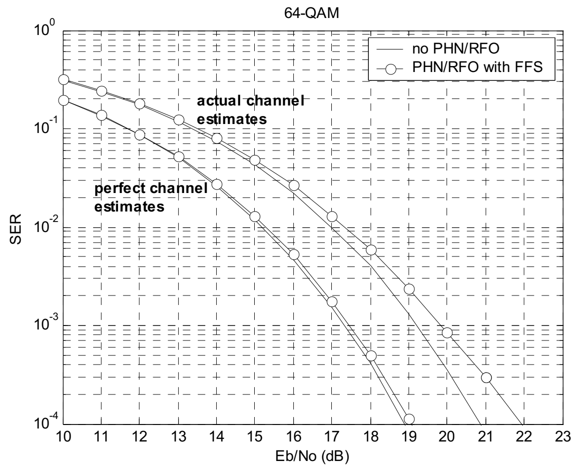

Fig. 4. Performance of the proposed schemes for perfect and actual (imperfect) channel estimates.

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