Effect of Pulse Selection on the Capacity of Multi-User UWB Systems

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Abstract— This paper focuses on the effect of the pulses selection on the channel capacity of M-ary pulse position modulation (PPM) ultra wideband (UWB) communication systems over multiple-access and additive white Gaussian noise (AWGN) channel. Based on Gaussian approximation for the multiple access interference, an expression of the signal-to-noise ratio is derived for the UWB system using higher order derivatives Gaussian pulses. The effect of derivative order on the UWB capacity is investigated in AWGN multiuser channel. The information theoretic capacity of the UWB system is expressed as a function of system parameters. Analytical and simulation results show that the capacity of UWB system is not sensitive for high derivative Gaussian pulses, but very sensitive to the pulse shaping factor. A new recursive equation is introduced to generate high order derivative Gaussian pulses.

Index Terms—**Ultra wideband communications (UWB), M-ary-PPM, Information Capacity, Multiple-Access, FCC, High derivative Gaussian Pulses.**

I. Introduction

Ultra wideband (UWB) systems were recently proposed as one of the possible solutions for short range wireless networks [1]. The new technology has the potential to deliver high data rates with very low power densities. The original UWB systems are characterized by the transmission of series of sub-nanosecond pulses (monocycles) that spread the energy of the signal from near DC to a few GHz. The current spectrum allocated by US Federal Communications Commissions (FCC) is from 3.1GHz to 10.6 GHz. The pulse train is transmitted without any modulation with sinusoidal carrier. This is one of the major advantages of UWB since it has high immunity against multipath fading effect as experienced in other wireless systems. In addition, high processing gain and very low power density ensure minimal mutual interference between the UWB and other wireless systems. Pulse position modulation (PPM) has been proposed as a modulation scheme suitable for the UWB communications [1]. With PPM, the data modulates the position of the transmitted pulse within an assigned window in time. UWB radio is the generic term describing radio systems having very large bandwidths; "bandwidths greater than 20% of the center frequency measured at the –10dB points," and "RF bandwidth greater than 500 MHz," are the two of the definitions under consideration by FCC [2].

In this paper, Time Hopping (TH) is used in the UWB system as the multiple-access method. The PPM scheme is used in TH-mode with pulse transmission instants defined by a pseudo random code. One data bit is spread over multiple pulses to achieve a processing gain due to the pulse repetition. The processing gain is increased by the low duty cycle. The multiple-access interference (MAI) may be the dominant factor on the bit error rate (BER) performance. In this paper we compute the information theoretic capacity of an UWB system with respect to different wave forms and multiuser M-ary PPM modulation. Some published works considered the information capacity for UWB system with rectangular pulse shape [5]. The correlation properties and the frequency spectra of UWB pulses are very crucial. For example, the effect of Hermite pulses on the BER performance is presented in [7]. A detailed study of PPM capacity in Gaussian and Webb channels is considered in [8]. In [9], the capacity of M-ary PPM-UWB system is considered for rectangular, Rayleigh and 2nd derivative Gaussian pulses. Our paper is different from previous studies by considering higher derivative order Gaussian pulses. That is, the effect of derivative order of the Gaussian pulses on the information capacity of multiuser UWB system is under investigation.

The remainder of this paper is organized as follows. Section 2 introduces the signal, channel, and receiver models. Section 3 carries out the system analysis for high order derivative Gaussian pulses. Section 4 presents the capacity analysis of the UWB M-ary PPM in multi-user and Gaussian channel. Section 5 presents the numerical results obtained. Section 6 concludes this study.

II. UWB System Model

The time-hopping M-ary PPM system model examined in this paper is shown in Fig. 1. The v-th user's transmitted signal has the form [1]:

$$S^{v}(t)=\sum_{j=-\infty}^{\infty} A^{v} P\left(t-jT_{f}-C_{j}^{v}T_{C}-d_{j}^{v}\right) \qquad (1)$$

where $P(t)$ is the UWB pulse of duration T_p. The pulse repetition interval, referred to as frame, is T_f, $A^v=\sqrt{E_p^v}$, C_j^v, and d_j^v are respectively, the amplitude, user dependent time-hopping code and data modulation for user v, where E_p is the

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energy per pulse. The PPM time shift is $d_j^v \in \{\delta_1,...,\delta_M\}$, Fig. 2. For a fixed T_f, the symbol rate $R_s = 1/(N_p T_f)$ where N_p is the number of pulses that form one symbol. The symbol duration is $T_s = N_p T_f$ and, the spreading ratio is defined by $\beta = T_f / T_p$.

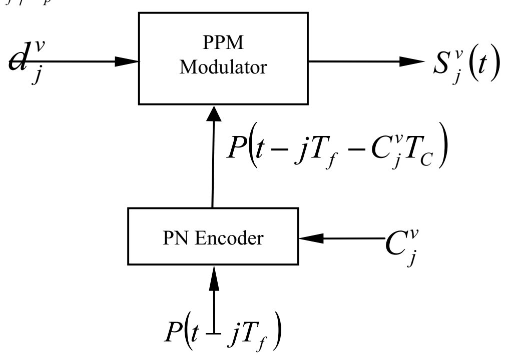

Fig. 1. TH-PPM UWB modulator.

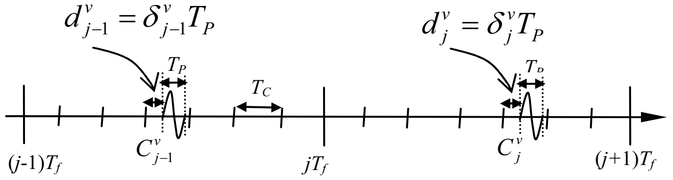

Fig. 2. Example of TH-PPM asynchronous format.

Each user's signal propagates over a single path channel with attenuation factor α and propagation delay τ. The received signal $R(t)$ from all users is given by:

$$R(t) = \sum_{v=1}^{N_u} \alpha_v S^v(t - \tau^v) + \eta(t) \qquad (2)$$

where α_v and τ^v are the channel attenuation and time delay associated with user v out of total number of N_u users, respectively, and $n(t)$ is zero-mean AWGN with power spectral density $N_0/2$.

III. UWB SYSTEM ANALYSIS

Without loss of generality, we assume the desired user is v=1. The single-user optimal receiver is M-ary pulse correlation receiver followed by a detector. We also assume that the receiver is perfectly synchronized to user 1, i.e., τ^1 is known. Furthermore, the time hopping sequence C_j^1 is known at the receiver. The M-ary correlation receiver for user 1 consists of M filters matched to the basis function $\phi_i^1(t)$ defined as:

$$\phi_i^1(t) = P(t - d_i^1 - \tau^1) \qquad i = 1..M \quad . \qquad (3)$$

Fig. 3 depicts the detector selecting the Max i-th symbols of M possible outputs.

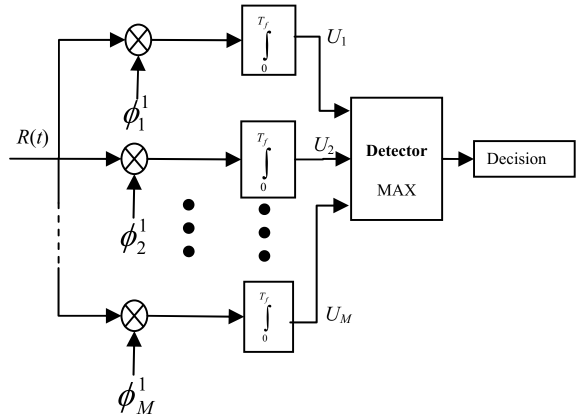

Fig. 3. M-ary PPM UWB receiver for the 1st user.

The decision variables at time sample (t=jT_f), are now given by:

$$U_i = \int_{(j-1)T_f}^{jT_f} R(t)\phi_j^1(t - jT_f - C_j^1 T_c)dt \qquad (4)$$

by substituting for $R(t)$ from (2), the parameter U_i can be represented as $U_i = d + MAI + N$, where d denotes the desired part of the received signal, MAI the multiple access interference from other users and is the noise component at the output of the receiver all respectively expressed by:

$$d = \int_{(j-1)T_f}^{jT_f} \alpha_1 S^1(t - \tau^1)\phi_i^1(t - jT_f - C_j^1 T_c)dt \ , \qquad (5)$$

$$MAI = \int_{(j-1)T_f}^{jT_f} \sum_{v=2}^{N_u} \alpha_v S^v(t - \tau^v)\phi_i^1(t - jT_f - C_j^1 T_c)dt \ , \qquad (6)$$

and

$$N = \int_{(j-1)T_f}^{jT_f} \eta(t)\phi_i^1(t - jT_f - C_j^1 T_c)dt \ . \qquad (7)$$

With $P(t)$ being normalized and orthogonal pulses, the desired part of the signal is $d = \alpha_1 A^1 \delta(i - j)$. The MAI part can be written as $MAI = \sum_{v=2}^{N_u} \alpha_v A^v \int_0^{T_f} P(t)P(t - \Delta)dt$, where Δ is the time difference between the different users expressed as $\Delta = (C_j^1 - C_j^v) - (d_i^1 - d_j^v) - (\tau^1 - \tau^v)$.

If the correlation function of the pulse $P(t)$ is defined by:

$$h(\Delta) = \int_0^{T_f} P(t)P(t - \Delta)dt \qquad (8)$$

then the expression for MAI in can be written as:

$$MAI = \sum_{v=2}^{N_u} \alpha_v A^v h(\Delta) \ . \qquad (9)$$

It is assumed that all the time-hopping code elements j are random and independent, uniformly distributed over a frame interval T_f for all users and frames. Each user has a uniformly distributed data source. The time delays are also assumed random, i.i.d. uniformly distributed over the frame interval. Under the assumptions listed above, and noting that MAI pulses of interest fall within the same UWB frame, the time difference Δ is a uniformly distributed random variable over the interval [-T_f, T_f].

Various waveforms with complex mathematical formats have been proposed for impulse radio including Gaussian pulse, Gaussian monocycle [1], and Rayleigh monocycle [7]. All of these waveforms reflect the high-pass-filtering impact of the transmitter and receiver antennas. In [9] three types of waveforms; namely, Rectangular, Rayleigh and the 2nd derivative Gaussian pulses were considered. The analysis in this paper is focused on higher order derivative Gaussian pulses that satisfy the FCC mask. The first 8 derivative Gaussian pulses are presented in Fig. 4.

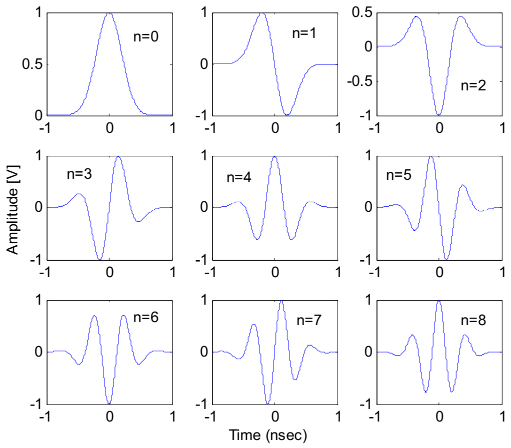

Fig. 4. The first 8 derivative Gaussian pulses.

The *Gaussian pulse* is expressed as:

$$P(t) = e^{-2\pi\left(\frac{t}{\varepsilon}\right)^2} \tag{10}$$

where ε is a time scale factor and its relation to pulse width T_P is $T_P = 7\varepsilon$ which contains 99.99% of the total energy. The nth derivative Gaussian pulses $P^{(n)}(t,\varepsilon) = \frac{d^n}{dt^n}P(t)$ can be obtained using the following recursive equations, with $\alpha = \dfrac{2\pi}{\varepsilon^2}$,

$$P(t) = e^{-\alpha t^2} \tag{11-a}$$

$$P^{(1)}(t) = -2\alpha t P(t) \tag{11-b}$$

$$P^{(n)}(t) = -2\alpha t P^{(n-1)}(t) - 2(n-1)\alpha P^{(n-2)}(t) \tag{11-c}$$

Using (11), we can generate any higher order derivative Gaussian pulses using lower order pulses. By repeated substitution of (11), one can obtain a single formula that related the nth derivative Gaussian pulse to the Gaussian pulse. Using Laplace transform properties, we found the autocorrelation function of the nth derivative Gaussian pulses as;

$$h(n,\Delta) = \frac{d^{2n}}{d\Delta^{2n}}\left[e^{-\pi\left(\frac{\Delta}{\varepsilon}\right)^2}\right] \tag{12}$$

Comparing (12) with (10) and (11), one can write the autocorrelation function for the nth derivative Gaussian plus as a function of the $2n$-th derivative Gaussian pulse. Again this is important property of the Gaussian pulse that simplifies the analysis. Therefore the autocorrelation function is,

$$h(n,\Delta) = P^{2n}\left(\frac{\Delta}{\sqrt{2}},\varepsilon\right) \tag{13}$$

The normalized nth derivative Gaussian pulses can be expressed as,

$$P^n_{Normalized}(t,\varepsilon) = \frac{1}{\sqrt{\left|h(n,\Delta)\right|_{\Delta=0}}}P^n(t) \tag{14}$$

And the corresponding normalized autocorrelation function is written as,

$$h_{Normalized}(n,\Delta) = \frac{1}{\left|h(n,\Delta)\right|_{\Delta=0}}h(n,\Delta) \tag{15}$$

The autocorrelation formula for the 2nd derivative Gaussian pulse presented by [9] can be generated using the above method. Fig. 5 shows the autocorrelation functions some selected Gaussian waveforms, n = 0, 1, 2, and 3.

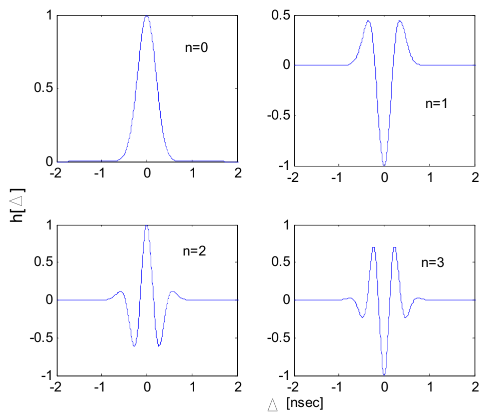

Fig.5. Autocorrelation of the first 3 derivatives of Gaussian pulses.

IV. Capacity Of Multiuser TH/PPM-UWB

Without loss of generality, let the signals are transmitted in the l-th time slot. With perfect synchronization, both the channel delay of τ^1 and time hopping sequence of C_j^1 are known. We also consider that the multiple-access interference is non-Gaussian distributed, yet the number of users in the system is large enough to justify the Gaussian assumption for MAI by invoking the central limit theorem.

In AWGN, the channel attenuation factor can be assumed as unity, $\alpha = 1$, and average SNIR per symbol at the output of the correlation receiver is given by:

$$\begin{aligned}\gamma_{PPM-UWB} &= \frac{(A^1)^2}{\sigma^2_{MAI} + Var(N)} \\ &= \frac{(A^1)^2}{\sum_{v=2}^{N_u}(A^v)^2\left[E\left[h^2(n,\Delta)\right] - E^2\left[h(n,\Delta)\right]\right] + \frac{N_0}{2}}\end{aligned} \tag{16}$$

With perfect power control, $A^1 = A^v = A = \sqrt{E_P}$, γ becomes:

$$\gamma = \frac{\gamma_0}{(N_u-1)\left[E\left[h^2(n,\Delta)\right]-E^2\left[h(n,\Delta)\right]\right]\gamma_0+1} \tag{17}$$

where $\gamma_0 = \frac{2E_P}{N_0}$ is the pulse energy to noise ratio, $E[h(\Delta)]$ and $E[h^2(\Delta)]$ the first and second moments of the correlation function of the selected pulse waveform, respectively.

The mean value of the correlation function $h(n,\Delta)$, $E[h(n,\Delta)]$, can be calculated as shown bellow:

$$E[h(n,\Delta)] = \frac{1}{\left|h(n,\Delta)\right|_{\Delta=0}\Big|}\int_{-T_p}^{T_p} h(n,\Delta)\frac{1}{2T_f}\,d\Delta \quad . \tag{18}$$

Similarly, the second moments of $h(n,\Delta)$, $E\left[h^2(n,\Delta)\right]$ can be calculated as s:

$$E\left[h^2(n,\Delta)\right] = \frac{2}{\left|h(n,\Delta)\right|_{\Delta=0}\Big|^2}\int_{0}^{T_p}\left|h(n,\Delta)\right|\frac{1^2}{2T_f}\,d\Delta \tag{19}$$

Using Gaussian approximation for multiuser interference, the expression for single–user capacity (bits/symbol) is defined as [8]:

$$C = \log_2 M - E_{\mathbf{U}|\mathbf{X}_1}\log_2\sum_{m=1}^{M}\exp\left(\sqrt{\gamma}\left(U_m-U_1\right)\right) \quad . \tag{20}$$

and the random variables U_m, m=1,..,M have the following distributions conditional on the transmitted signal $\mathbf{X}_1$, where $\mathbf{X}$ is interpreted as a collection of points in M-dimensional signal space with one point located on each coordinate axis :

$$\begin{aligned} &U_1 : N\left(\sqrt{\gamma},1\right) \\ &U_m : N(0,1) \quad , \quad m \neq 1 \end{aligned} \quad , \tag{21}$$

where $N(\mu,2\sigma^2)$ denotes the Gaussian distribution with mean μ and variance $2\sigma^2$. An analytical study of (20) for a binary PPM (M=2) is presented as a special case in [5]. We consider the channel capacity with respect to pulse waveform of M-ary PPM UWB communications over multiple-access and AWGN channel by using Monte-Carlo simulation of (20).

V. Numerical Results

Several figures were produced to obtain specified results and demonstrate the effect of the derivative order of Gaussian pulses on UWB. Fig. 6 presents the user capacity in bits per M-ary PPM symbol of UWB as a function of number of users for various number of modulation levels M using Rectangular, Rayleigh and 2[nd] order derivative Gaussian Pulses. The curves are obtained by Monte-Carlo simulation with spreading ratio β=10, and noise free channel [9]. From the figure it is clear that for low number of users, the UWB user capacity is approximately $\log_2(M)$. We clearly observe that the monocycle waveforms are beneficial in reducing the MAI. More explanation can be found in [9].

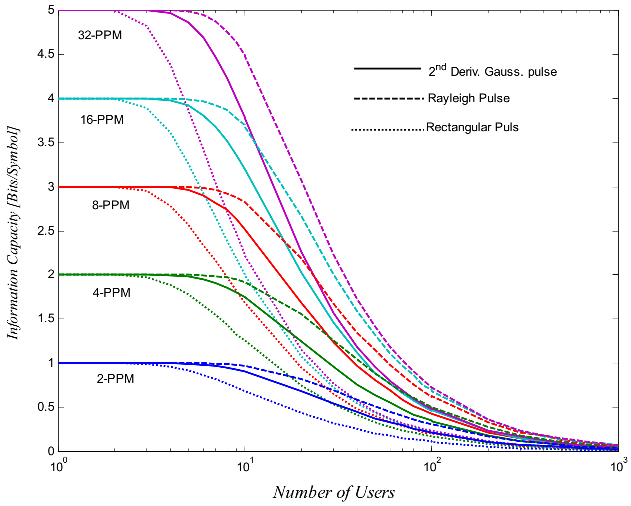

Fig6. User information capacity with respect to pulse selection for multiuser UWB, β=10, T_p=1ns, and noise free channel, $\gamma_0=\infty$.

Fig. 7 shows the effect of derivative order on the capacity of 8-ary PPM-UWB system in noise free environment. It is clear that higher order derivative Gaussian pulses have the same correlation properties that results to almost the same capacity for n>0 regardless of the spreading factor β. This is very important feature of higher order derivative Gaussian pulses that can be used for multiuser systems in mixed mode.

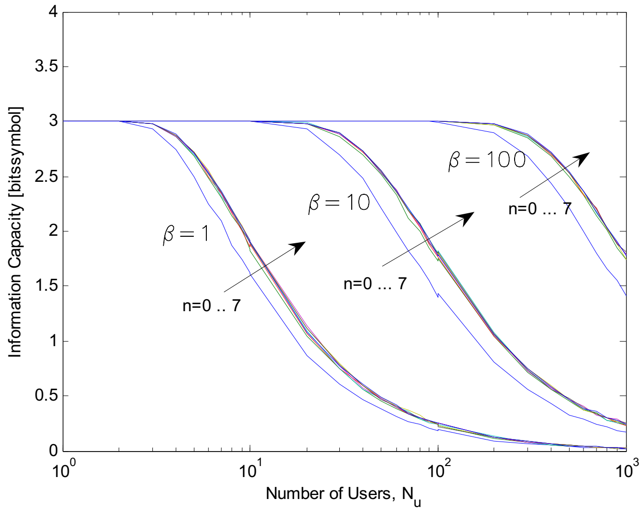

Fig. 7. User information capacity with respect to pulse selection for multiuser 8-ary PPM UWB, noise free channel, $\gamma_0=\infty$. T_p/ε=7, and T_p=1ns.

Fig. 8 presents the capacity in bits per PPM symbols of UWB as a function of symbol SNR for M=8 with processing gain β=10 and 10 users. From the figure it is clear that for higher SNR, the UWB user capacity is approximately 3 (=$\log_2(8)$). The difference between capacity performances of different pulse waveforms is less due to the higher processing gain that minimizes the MAI.

It is clear that the pulse shaping ratio is of importance that may influence the capacity performance. Fig. 9 depicts the effect of pulse shaping ratio on the performance of M-ary

PPM UWB system. The figure shows how strong is the effect of the pulse shaping factor on total system capacity. Due to practical limitations such a factor must be properly selected. We also realize the effect of the pulse shaping factor reduces with higher processing gain.

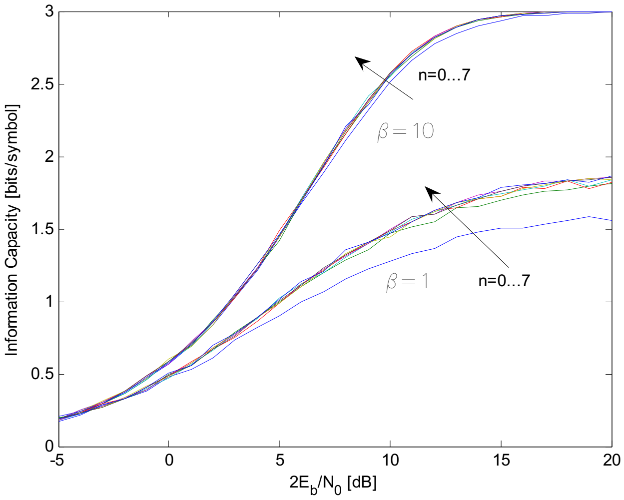

Fig. 8. User capacity versus γ_0=2E$_b$/N$_0$ for multiuser 8-ary PPM-UWB system, 10 users, T_p/ε=7, and T_p=1ns.

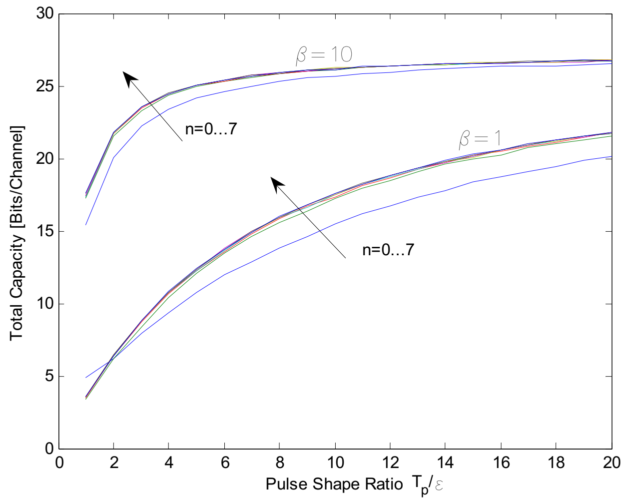

Fig. 9. Total capacity 8-ary PPM-UWB system versus T_p/ε ratio for pulse γ_0=10dB, and T_p=1ns.

VI. CONCLUSION

In this paper, we have investigated the effect of pulse waveform selection on the information theoretic capacity of the M-ary PPM UWB multiple-access system. Higher order derivative Gaussian pulses were selected to satisfy the FCC spectrum mask. We show that the information capacity is not sensitive to the derivative order but rather to the pulse shaping factor at low spreading factors. For multiuser environment, the use of mixed higher order derivative Gaussian pulses will have limited effect of the system capacity.

VII. REFERENCES

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