Effects of multi-rate mechanisms in ARQ and Hybrid ARQ systems using Downlink beamforming

Tal Kaitz

Abstract—**This paper considers the effects of ARQ and Hybrid ARQ in conjunction with multi-rate mechanisms, for the special case of Downlink beamforming. When applied in a cellular system, beam-formed users experience temporal variations in the received signal to interference and noise ratio (CINR). These variations can be mitigated by the use of ARQ and Hybrid ARQ techniques. In particular, when a multi-rate scheme is used, the transmission rates can be optimized to increase the system throughput.**

We present here a simplified model for the DL beamforming channel, in an interference limited case, and use the Shannon capacity formula to relate the transmission failures to the instantaneous CINR values. Next, we derive optimization criteria for the transmission rates. The optimal rates are used to compute the throughput of the system as a function number of antennas. The results indicate the significant benefits associated with beamforming in conjunction with ARQ and Hybrid-ARQ mechanisms.

Index Terms—**ARQ, H-ARQ, Downlink Beamforming, interference mitigation, multi-rate.**

I. INTRODUCTION

HYBRID Automatic Repeat reQuest (Hybrid ARQ, or HARQ) is a long recognized technique to combat temporal variations in channel quality. HARQ operates by retransmitting versions of the original message in such a way that the decoder can utilize the energy embedded in previous re-transmissions. There are several major types of HARQ (see [1] [2] and [3]). In type I ARQ scheme the message is sent together with error correction and error detection codes. If, after applying the error correction code, errors are detected, the entire message is retransmitted. In type II HARQ, only additional redundancy bits are sent in retransmissions. The receiver uses the redundancy bits from the first transmission and from subsequent retransmissions to decode the message. This scheme is also known as incremental redundancy (IR) HARQ. Another scheme is based on Chase combining [4]. In this scheme the exact same message is retransmitted over and over again. The receiver combines the received message with the previously received replicas, until the message is successfully decoded. In yet another scheme, known as partial IR, each retransmission contains the message and partially different redundancy bits. In both the chase combing and partial IR schemes, each retransmission is independently decodable. These two schemes are also known as type III HARQ. HARQ techniques are now used in most advanced cellular systems, such as High Speed Downlink Packet Access, (HSDPA) and IEEE 802.16e standard for metropolitan area networks.

For efficient operation the ARQ or HARQ schemes must be incorporated with the rate adaptation scheme. The rate adaptation selects constellation size and the coding for each transmission. This has direct impact on the performance of the link when ARQ or HARQ are used.

Here we consider the rate adaptation mechanism in conjunction with HARQ for a special time varying channel: the beamforming channel (cf. [5]). To see that the beamforming channel is time varying, let us consider the following simplified scenario. We consider the downlink of cellular system, such as depicted in Figure 1. We consider only two opposing sectors. The base station in each sector is equipped with an adaptive antenna array, and uses beamforming towards each of the users. Let us observe user a connected to BST A. In BST B there are two users, b_1 and b_2. When BST B forms a beam towards b_1, user a is not illuminated, and the CINR of a is high. When BST B forms a beam towards b_2, user a is illuminated and its CINR is low. Thus the CINR varies from transmission to transmission even when the users are completely stationary.

The variability in the CINR of A can be utilized by ARQ and H-ARQ techniques, by transmitting at a high rate, and retransmitting in case of decoding error. Retransmission can occur at a lower rate, thereby insuring that the message will be decoded correctly.

This paper is organized as follows. Section II provides the channel and link models to be considered. Section III analyzes the case of ARQ for the beamforming channel, with optimal rate selection. Section IV discusses the same when Hybrid-ARQ is employed.

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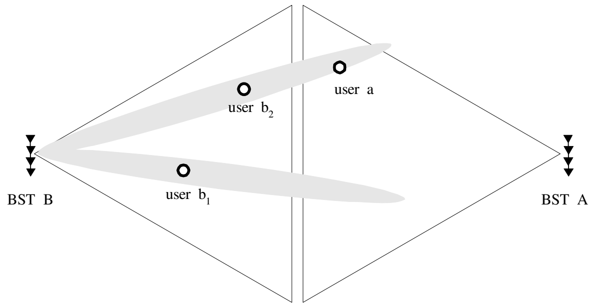

Figure 1 Illustration of interference in a beamforming system

II. SYSTEM MODEL

A. Channel and interference models

Let us consider a cellular deployment scenario and consider the DL performance of a given sector within a cell. The BST is equipped with an antenna array of N_T transmitting antennas. Users within the sector are illuminated by a single opposing co- channel sector, of a BST with the same parameters. The antenna array is used to perform BF towards each of the users. DL transmission occurs in bursts. In each burst, both the desired and the interfering BST form beams towards their respective users. We assume that the system capable of adapting the transmission rate from burst to burst.

Let us focus on a single desired user in the desired cell. At any given burst, the signal to the desired user is due to the formed beam from the desired BST. The noise at this user is composed of the thermal noise and the illumination from the interfering BST pointing to its user. Let $\boldsymbol{c_d}(k)$ denote the N_Tx1 channel vector from each of the desired BST antennas to the user under investigation, at burst k. Let $\boldsymbol{c_u}(k)$ $\boldsymbol{c_i}(k)$ denote channel vectors from the interfering BST to its user and to the desired user, respectively. Let σ^2_n denote the power of the thermal noise at the desired user.

The desired BST uses the set of weights, $\boldsymbol{w_d}$, given by :

$$\mathbf{w_d}(k) = \sqrt{P_T}\exp(j \cdot \angle \mathbf{c_d}(k)), \quad (1)$$

where P_T is the transmitting power per antenna and $\angle \boldsymbol{c}$ is a vector of angles of $\boldsymbol{c}$. Likewise:

$$\mathbf{w}_u(k) = \sqrt{P_T}\exp(j \cdot \angle \mathbf{c}_u(k)) \quad (2)$$

The CINR at burst k, is denoted by γ_k and is given by :

$$\gamma_k = \frac{P_T\left|w_d^H \boldsymbol{c_d}(k)\right|^2}{P_T\left|w_u^H \boldsymbol{c}_i(k)\right|^2 + \sigma_n^2}. \quad (3)$$

Now we assume that all channel vectors $\boldsymbol{c_d}$(k) $\boldsymbol{c_i}$(k) and $\boldsymbol{c_u}$(k) are independent complex circular random variables with zero mean and covariance matrix $\sigma_d^2 \boldsymbol{I}_{N_T x N_T}, \sigma_i^2 \boldsymbol{I}_{N_T x N_T}, \sigma_u^2 \boldsymbol{I}_{N_T x N_T}$ respectively. This corresponds to the case of rich scattering, causing independent Rayleigh fading. We assume that channel realizations are independent from burst to burst. Under these conditions (3) can be written as:

$$\gamma_k = \frac{\left|\sum_{l=1}^{N_T} \left|v_{d,l}\right|\right|^2}{\frac{\sigma_i^2}{\sigma_d^2} N_T \left|v_d\right|^2 + \frac{\sigma_n^2}{\sigma_d^2}} \quad (4)$$

where $\boldsymbol{v_d}$ and v_i, are independent complex circular random vectors with zero mean and covariance matrix $\boldsymbol{I}_{N_T x N_T}$ and 1, respectively. In (4) we assumed P_T=1. Using (4) we can define CDF of γ_k to be:

$$P_\gamma(\Gamma) = \Pr(\gamma \le \Gamma) \quad (5)$$

For the rest of the paper we shall consider the case of σ^2_n=0, thereby concentrating on the interference limited case.

B. Link model

We assume that the system utilizes a coded-modulation scheme. To ease the analysis and to make it independent of the actual coding scheme, we shall model the link as follows: A burst at rate R shall be received correctly if the CINR of this burst exceeds some threshold CINR, Γ_{th}, and received incorrectly if the CINR is below this threshold. Here R should be interpreted as the number of bits per channel use, or alternatively as the number of bits/sec/Hz. The threshold CINR Γ_{th} is related to the rate R using Shannon's capacity formula

$$R = \log_2(1 + \Gamma_{th}/\Gamma_{Gap}), \quad (7)$$

where Γ_{Gap} is the SNR gap used to compensate for the use of error correction codes which are not capacity achieving. If a burst is received with CINR of γ and rate R, it will be successful if and only if:

$$\log_2(1 + \gamma/\Gamma_{Gap}) > R. \quad (8)$$

To validate the correctness of this model, we evaluated (7) with a value of Γ_{Gap}=6dB, and compared the results with the performance of the rate=1/2 K=7 convolutional code, defined in [6], for a bit error rate of 10^{-6}, and for various modulations and coding rates. The results are shown in Figure 2.

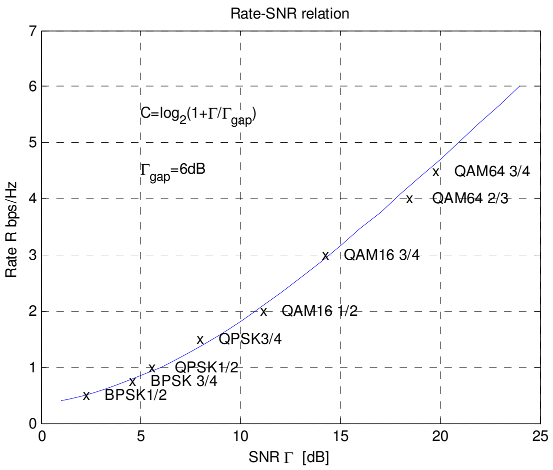

Figure 2 Rate - SNR relation for the convolutional code of [6]

When a burst is re-transmitted using different coding rates and optimal combining is used (as in the case of Hybrid-ARQ), we model the receiver as follows: A burst will be received successfully if and only if the average rate is lower than the average channel capacity. If $R_0 \ldots R_{m-1}$, and $\gamma_0 \ldots \gamma_{m-1}$ denote the rates and instantaneous CINRs, respectively, in each of the *m* retransmission attempts, then a burst will be received successfully if and only if:

$$\sum_{j=0}^{m-1} R_j^{-1} \log_2 (1+\gamma_j / \Gamma_{Gap}) > 1 . \quad (9)$$

Using (4) and (8) we can compute the probability that a burst of rate R will fail. We denote this probability as $P_F(R)$. We evaluated, using Monte Carlo techniques $P_F(R)$ for the case of $\sigma_n^2=0$ and several values of σ_d^2/σ_i^2. This is shown in Figure 3.

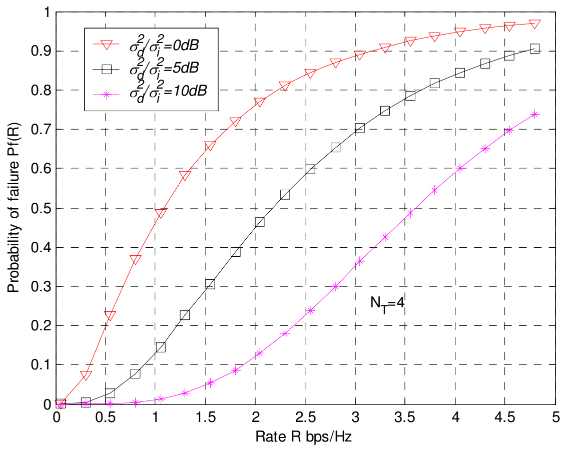

Figure 3 Probability of Failure $P_F(R)$

III. EFFECT OF ARQ

In this section we evaluate the effect of ARQ schemes, and the associated rate adaptation mechanism. We assume that there exists a noiseless feedback channel. The user tries to decode the incoming burst. If a reception failure occurs, the user transmits a not-acknowledged (NACK) message on the feedback channel. The BST then re-transmits the message, possibly at a lower rate.

We consider here the case where the system is delay sensitive and the message must be transmitted successfully within a prescribed number of transmissions. Let *M* denote that value. Let us define a drop event as the event in which the message was not transmitted successfully within the specified *M* transmissions. Let $\{R_m\}_{m=0}^{M-1}$ denote the set of rates for each of the transmissions, and compute the average transmission time per bit, T_{avg}. We have exactly one transmission with probability $1-P_F(R_0)$. The first transmission takes R_0^{-1} channel usage per bit. The probability of having exactly two retransmissions is $P_F(R_0)(1-P_F(R_1))$ and this case take $R_0^{-1}+R_1^{-1}$ channel usage per bit. Following this line of analysis, it is easy to show that:

$$T_{avg} = R_0^{-1} + P_F(R_o)R_1^{-1} + P_F(R_o)P_F(R_1)R_2^{-1} + \ldots$$
$$+ R_{M-2}^{-1} \prod_{m=0}^{M-2} P_F(R_m) - \prod_{m=0}^{M-1} P_F(R_m) \sum_{m=0}^{M-1} R_m^{-1}$$

(10)

The drop probability is given by

$$P_{drop} = \prod_{m=0}^{M-1} P_F(R_m) \qquad (11)$$

The problem can be reformulated as minimizing (10) subject to $P_{drop} \leq P_{drop,max.}$ When the minimum is achieved the constraint is met with equality and one degree of freedom can be reduced from the minimization process.

The minimization of (10) under (11) was preformed for the case of N_T=4 , *M*=3 and $P_{drop,max}=10^{-3}$. The resulting rates and the capacities are shown in Figure 4.

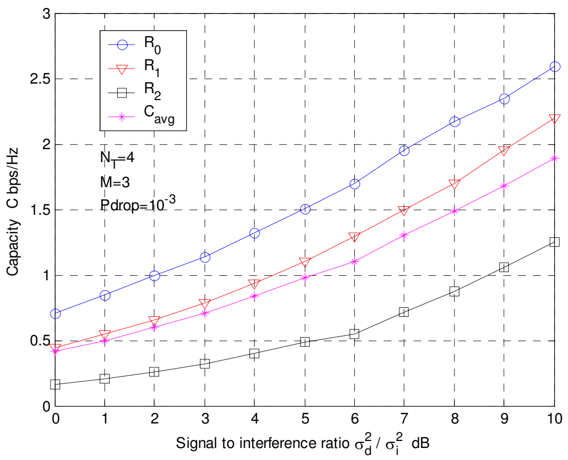

Figure 4 Retransmission rates and capacity

The first transmission rate high, but subsequent retransmissions use lower and lower rates, to insure that the drop probability is below the required threshold. Next, in Figure 5, we show the capacity for various values of N$_T$.

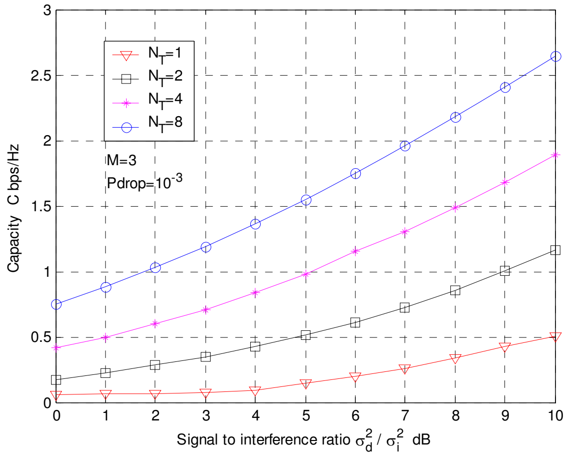

Figure 5 Capacity as a function of number of antennas

Note that the performance improvement due to the use of beamforming is from 0.04bps/Hz (N$_T$=1) to 0.76bps/Hz (N$_T$=8).

IV. EFFECT OF HYBRID-ARQ

When using Hybrid-ARQ (H-ARQ) the decoder saves the soft copies of the received bursts that were not decoded correctly, and tries to combine them to provide error free reception. We shall consider two cases:

a. Upon an incorrect reception the receiver sends on the feedback channel a NACK message. The transmitter resends at a possibly different rate. This is referred to as ‘hard feedback’
b. Upon an incorrect reception the receiver sends on the feedback channel a NACK message accompanied by a report of received SNR of the last burst. The transmitter makes use of this information to optimize the rate in which the subsequent burst is sent. Alternatively, the receiver sends the optimal rate for the next transmission. This is referred to as ‘soft feedback’

A. *Hard feedback*

In this case the analysis can be performed as in the pervious section, but with the following differences:

- The probability of failure $P_F(R_m)$ in the m’th retransmission is replaced by $P_F(R_m \mid R_0, R_1, \ldots R_{m-1})$. The latter expression denotes the probability that the $m+1$’th transmission failed given that the previous m transmissions, using the rate set $\{R_0, R_1, \ldots R_{m-1}\}$ have failed.
- Equation (9) is used to link the instantaneous SNR values $\gamma_0 \ldots \gamma_{m-1}$ and the transmission rates $R_0 \ldots R_{m-1}$, and with the probability of failure.

The results are shown in Figure 6. It can be seen that there is little improvement for the cases of N$_T$=1 and 2 relative to the ARQ case. However for N$_T$=4 and N$_T$=8, H-ARQ does provide performance improvement. For the case of $\sigma_d^2=\sigma_i^2$, N$_T$=8 the capacity is 1.10bps/Hz, a 30% improvement over the ARQ case.

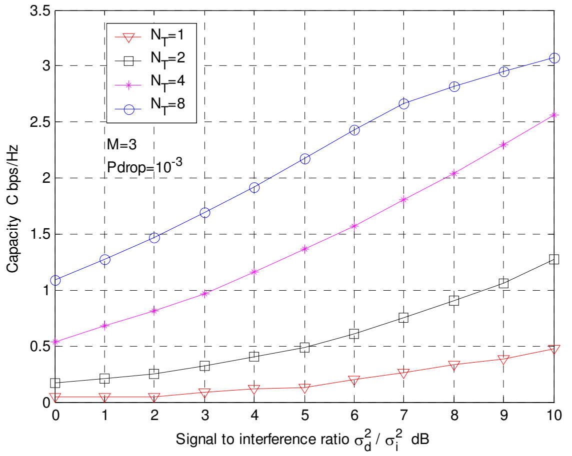

Figure 6 Capacity as a function of antenna number for the case of H-ARQ- Hard feedback.

B. *Soft feedback*

In this case the feedback information is used to select the optimal rate for the next transmission. The proposed approach is to select the rate that will achieve a predefined probability of failure. Let $\{P_T(m)\}_{m=0}^{M-1}$ denote the set of target probabilities of failure, at the m’th retransmission. We set $P_T(M-1)=P_{drop}$ to meet the drop probability requirement and optimize the rest of rates for optimal capacity.

The rate R_m for each transmission is set so:

$$P_F\left(R_m \middle| \{R_\ell\}_{\ell=0}^{m-1}, \{\gamma_\ell\}_{\ell=0}^{m-1}\right) = P_T(m), \quad (12)$$

where the RHS of (12) is the failure probability conditioned on past transmission rates and CINR values $\gamma_0,\ldots,\gamma_{m-1}$. Using (9) we have:

$$P_F\left(R_m \middle| \{R_m\},\{\gamma_m\}\right) =$$
$$P_F\left(R_m \cdot \left(1-\sum_{j=0}^{m-1} R_j^{-1}\log_2\left(1+\gamma_j/\Gamma_G\right)\right)\right). \quad (13)$$

The results are shown in Figure 7. For N_T=4 and N_T=8 there is an improvement of about 10% relative to hard feedback. For the case of $\sigma_d^2=\sigma_i^2$, N$_T$=8 the capacity is 1.22bps/Hz.

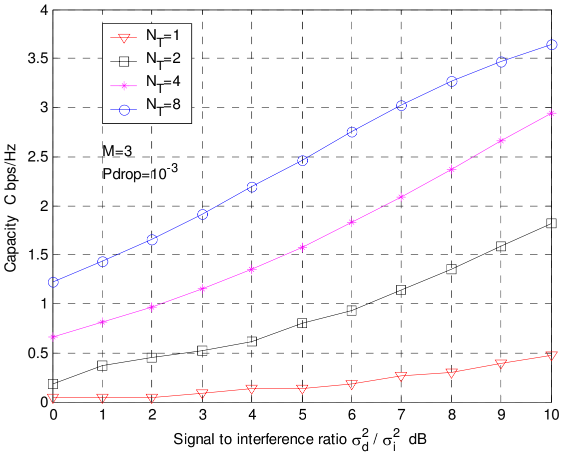

Figure 7 Capacity as a function of antenna number for the case of H-ARQ- Soft feedback.

V. CONCLUSION

We analyzed and simulated the effects of ARQ and HARQ when the system employs downlink beamforming and showed significant performance improvements over the single antenna case. For example, for equal mean signal and interference powers, the single antenna provided about 0.05bps/Hz while the ARQ system with 8 antennas provide 0.76bps/Hz. With HARQ the capacity of the 8-antenna system was about 1.10bps/Hz with hard feedback and 1.22 bps/Hz with soft feedback.

VI. ACKNOWLEDGEMENTS

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