A Simple Upper Bound on Mutual Information for Ricean-Fading MIMO Channel

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***Abstract*— This paper deals with the problem of MIMO link capacity assessment. Unlike previous analysis model whose transfer function $\boldsymbol{H}$ is preassigned as circularly-symmetric complex-valued Gaussian random matrix, we consider a directional MIMO channel model in the presence of Ricean fading and physical scattering. Spatial parameter of the model is first separated into directional mean and its associating standard deviation, the *nominal direction* and the *angular spread*, respectively. An upper bound on the ergodic capacity mean is then proposed by using such a separable parameterization. Shown as a deterministic function of Rice factor, nominal direction, and angular spread, the proposed upper bound provides, fortunately, an insight into Ricean fading and physical scattering. Since it does not require any eigenvalue decomposition, the bound computation is thus very simple. Numerical examples are conducted to demonstrate not only the bound tightness compared with sample mean capacity, but also relationships of the bound to various parameters.**

I. Introduction

The use of array antennas at both receiver and transmitter has emerged a prominent role in wireless communication systems. The cause of this is due to the possibility of significant gains. Advantage of the above exploitation can be verified by information theoretic results. Regarding fading behavior, Rayleigh channel is reasonable for a lot of environments. As encountered in the situation where LOS (*L*ine-*O*f-*S*ight) portion is significant, however, the Ricean channel is, in turn, anticipated to be a more suitable candidate. Additionally, the channel might be considered, in particular, as a generalized approach with respect to the Rayleigh channel.

The era of MIMO began indeed with modeling all entries of NLOS (*N*on-*L*ine-*O*f-*S*ight) channel matrix as identically-independent complex-valued Gaussian random variables with zero mean and unit variance (see *e.g.*, [1], [2], [3]). Previous *unstructured statistics* paved the praiseworthy way for a foundation of stochastic MIMO channel analysis. In certain situations, this seemed, however, insufficient to reflect the actual impact of spatial parameters in the matrix $\boldsymbol{H}$. This is because the ignorance of physical scattering will cause the mismatch in capacity assessment. On behalf of *Kronecker structure* [4], a disagreement where the unstructured channel did not render multipath effects accurately, indicated that the Kronecker factorization leads to systematic prediction errors. Based on actual field measurements, therefore, there exists a lot of physical MIMO models that takes the geometrical scattering into account (see *e.g.*, [5], [6], [7], [8]).

In the vicinity of the mobile sources, the Rayleigh fading channel in the presence of spatial scattering leads to angular spread of the signals impinging on the array. Such a spatial channel model has been validated against experimental data [9] whereby a transmitter in the field experiments has been placed in urban areas approximately 1 km from the receiving array. Furthermore, a lot of measurements have shown that local scattering in the vicinity of a mobile is a non-negligible phenomenon [10]. These independent investigations coincide quite well with the so-called *distributed source MIMO* model [11]. Accounting for LOS portion, a significant result [11] reports that when the transmitter-to-receiver distance is short and the antenna spacing is large, the assumption of distributed source is more accurate than the point source model.

Regarding the capacity assessment of MIMO link, it is well-known that the ergodic mean of capacity, *i.e.*, statistical expectation of the stochastic channel capacity, will, in fact, converge to the ergodic capacity mean when the number of independent runs tends to infinity. However, it requires numerous computational burdens to ascertain all realizations of the stochastic channel. Moreover, no straight insight is available from this manipulation due to the lack of explicit relationships to considered parameters. One way to satisfy both desires is to find a deterministic function of such stochastic quantities. Some analytic derivations have already been dedicated to the unstructured model [12] and [13] for dual antenna systems (either two transmit or two receive antennas). To avoid the lengthy Monte Carlo simulations, upper bounds for MIMO capacity assessment were proposed according to various assumptions [14], [15] and [16].

Contributions of this paper are twofold. For either receiver or transmitter, we capture the random scattering directions into two associated terms of *directional statistic* in angular cluster. By means of population mean and its corresponding standard deviation (see *e.g.*, [17] and [18]), it is, in general, called the nominal direction and angular spread. We formulate a parametric framework in order to analyze the MIMO channel, which indeed concerns the scattering characterization due to realistic propagation. Incorporating the LOS portion into the random channel, we offer an *upper bound on the ergodic mean of channel capacity* for the Ricean-fading MIMO system where uniform power allocation is assumed and the perfect channel knowledge at the receiver is available. It appeared that the proposed upper bound not only is computationally simple

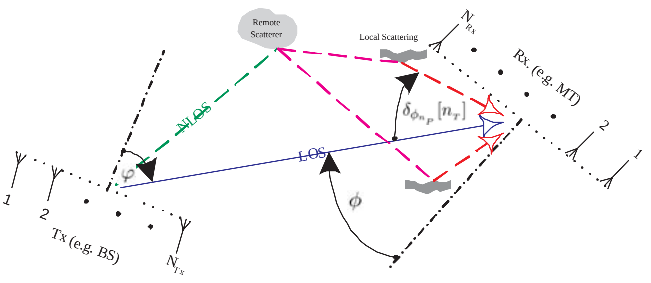

Fig. 1. Dispersive MIMO channel.

but also provides an insight into Ricean fading and scattering.

For terminology conciseness, we designate $\boldsymbol{A}$, $\boldsymbol{a}$ and a as matrix, column vector and scalar, respectively. The multidimensional sets $\mathbb{C}$ and $\mathbb{R}$ stand for complex and real quantities. The operators $(\cdot)^*$ and $(\cdot)^\mathsf{T}$ denote element-wise conjugate and matrix transpose, whereas $(\cdot)^\mathsf{H}$ signifies either transpose or conjugate. The notation $\boldsymbol{x} \sim \mathcal{N}_c(\boldsymbol{\mu}_x; \boldsymbol{\Sigma}_{xx}, \boldsymbol{\Gamma}_{xx})$ is called the complex-valued Gaussian random vector $\boldsymbol{x}$ that holds population mean $\boldsymbol{\mu}_x = \mathcal{E}\langle \boldsymbol{x} \rangle$, covariance $\boldsymbol{\Sigma}_{xx} = \mathcal{E}\left\langle (\boldsymbol{x}-\boldsymbol{\mu}_x)(\boldsymbol{x}-\boldsymbol{\mu}_x)^\mathsf{H} \right\rangle$ and complementary covariance $\boldsymbol{\Gamma}_{xx} = \mathcal{E}\left\langle (\boldsymbol{x}-\boldsymbol{\mu}_x)(\boldsymbol{x}-\boldsymbol{\mu}_x)^\mathsf{T} \right\rangle$, where $\mathcal{E}\langle \cdot \rangle$, or exactly $\mathcal{E}\langle \cdot \rangle_x$, designates the statistical expectation with respect to x.

II. Ricean-Fading MIMO Channel Model

Restrict our attention to a number of signals transmitting through a dispersive channel and then impinging on the sensor array antenna. With phase reference at the first element, the array response vector $\boldsymbol{a}(\psi) : [-90^\circ, 90^\circ] \mapsto \mathbb{C}^{N_E \times 1}$ of both ends can be written ideally as

$$\boldsymbol{a}(\psi) \triangleq \begin{bmatrix} 1 & \mathrm{e}^{\mathrm{i}kd_E \sin(\psi)} & \cdots & \mathrm{e}^{\mathrm{i}kd_E (N_E-1)\sin(\psi)} \end{bmatrix}^\mathsf{T}, \quad (1)$$

where $k = \frac{2\pi}{\lambda}$ designates the wave number, d_E denotes equi-distance between two adjacent elements, $\psi \in \{\phi, \varphi\}$ and $N_E \in \{N_{Rx}, N_{Tx}\}$ signify the azimuth angle and the number of antenna elements for the receiver or transmitter respectively. The physical model can be illustrated corresponding to Fig. 1.

In previous developments, many scattering models assume that the nominal angles ϕ and φ of the receiver or transmitter, direct path gain α, Doppler frequency f_D and phase shift ν are deterministic while angular deviation $\delta_\psi \in \{\delta_\phi, \delta_\varphi\}$ and associating random path gain γ can be considered as stochastic quantities during the data burst. At a continuous time $t \in \mathbb{R}_+^{1\times 1}$, the array receiver output $\boldsymbol{x}(t) \in \mathbb{C}^{N_{Rx}\times 1}$ can be characterized such that (see *e.g.*, [19] and [20])

$$\boldsymbol{x}(t) = \boldsymbol{H}(t)\boldsymbol{s}(t) + \boldsymbol{n}(t), \quad (2)$$

where $\boldsymbol{s}(t) \in \mathbb{C}^{N_{Tx}\times 1}$ designates baseband signal vector due to the flat-fading channel, $\boldsymbol{n}(t) \in \mathbb{C}^{N_{Rx}\times 1}$ denotes additive noise imposed at receiver array and $\boldsymbol{H}(t) \in \mathbb{C}^{N_{Rx}\times N_{Tx}}$ is transfer function matrix. In Ricean-fading channel, the transfer function might be expressed as (see *e.g.*, [12])

$$\boldsymbol{H}(t) = \bar{\boldsymbol{H}}(t) + \tilde{\boldsymbol{H}}(t), \quad (3)$$

where $\bar{\boldsymbol{H}}(t)$ and $\tilde{\boldsymbol{H}}(t)$ denote the LOS and NLOS portions, respectively. The LOS component might be described as a time-varying term $\alpha\mathrm{e}^{\mathrm{i}(2\pi f_D t+\nu)}$ [21, pp. 34–35]. Together with specular part [22], the channel matrix $\bar{\boldsymbol{H}}(t)$ of directional MIMO model can be represented as

$$\bar{\boldsymbol{H}}(t) = \alpha\mathrm{e}^{\mathrm{i}(2\pi f_D t+\nu)}\boldsymbol{a}(\phi)\boldsymbol{a}^\mathsf{H}(\varphi). \quad (4)$$

Let T_s be a sampling period in which the condition $t = n_T T_\mathrm{s}$ satisfies the Nyquist's rate. One can easily transform such a deterministic portion into discrete-time framework as

$$\bar{\boldsymbol{H}}[n_T] = \alpha\mathrm{e}^{\mathrm{i}(2\pi f_D T_\mathrm{s} n_T+\nu)}\boldsymbol{a}(\phi)\boldsymbol{a}^\mathsf{H}(\varphi), \quad (5)$$

where $n_T \in \mathbb{N}^{1\times 1}$ denotes the sampling instant. For simplicity, let the Rayleigh portion $\tilde{\boldsymbol{H}}[n_T]$ hold zero mean, i.e. $\mathcal{E}\langle \boldsymbol{H}[n_T] \rangle = \bar{\boldsymbol{H}}[n_T]$.

Let each NLOS path be i.i.d. (*i*dentical and *i*ndependent *d*istribution). The superposition of all N_P paths turns out to (see *e.g.*, [6], [7] and [23])

$$\tilde{\boldsymbol{H}}[n_T] \triangleq \sum_{n_P=1}^{N_P} \gamma_{n_P}[n_T]\boldsymbol{a}(\tilde{\phi}_{n_P}[n_T])\boldsymbol{a}^\mathsf{H}(\tilde{\varphi}_{n_P}[n_T]), \quad (6)$$

where $\tilde{\phi}_{n_P}[n_T] \triangleq \phi + \delta_{\phi_{n_P}}[n_T]$ and $\tilde{\varphi}_{n_P}[n_T] \triangleq \varphi + \delta_{\varphi_{n_P}}[n_T]$ denote the arrival and departure directions respectively.

For a certain incoming ray, we employ the central limit theorem in such a way that $\gamma_{n_P}[n_T] \sim \mathcal{N}_c\left(0; \sigma_\gamma^2, 0\right)$, where the second zero stems from circularly-symmetric property.

Remark 1: Due to the i.i.d. assumption, the path gain statistic of directional model with $\sigma_\gamma^2 = \frac{1}{N_P}$ equals to that given by the unstructured model (see *e.g.*, [3, Def. 1]), where the NLOS portion is defined as $\left[\tilde{\boldsymbol{H}}[n_T]\right]_{[n_{Rx}, n_{Tx}]} \overset{\text{i.i.d.}}{\sim} \mathcal{N}_c(0; 1, 0)$; $\forall n_{Rx}, n_{Tx}$.

III. Link Capacity of MIMO System

Let $\check{n}_E = \min(N_{Rx}, N_{Tx})$ be the minimum number of antenna elements. In this paper, we assume that the channel state is available at the receiver but not at the transmitter. When the additive noise $\boldsymbol{n}(t)$ in (2) is spatially uncorrelated according to $\mathcal{E}\left\langle n_{n_E} n_{\acute{n}_E}^* \right\rangle = \sigma_n^2$; $\forall \acute{n}_E, n_E$, it is well-known that the the ergodic mean of channel capacity of the MIMO system is given by [2]

$$c = \log_2 |\boldsymbol{I}_{(\check{n}_E)} + \frac{\varrho}{N_{Tx}}\boldsymbol{\Upsilon}|, \quad (7)$$

where $|\cdot|$ signifies the matrix determinant, ϱ is the average SNR (signal-to-noise ratio) per received element and the Hermitian matrix $\boldsymbol{\Upsilon} \in \mathbb{C}^{\check{n}_E \times \check{n}_E}$ is classified into

$$\boldsymbol{\Upsilon} = \begin{cases} \boldsymbol{H}\boldsymbol{H}^\mathsf{H} & ; N_{Rx} \leq N_{Tx} \\ \boldsymbol{H}^\mathsf{H}\boldsymbol{H} & ; N_{Rx} > N_{Tx}. \end{cases} \quad (8)$$

Note that when $\varrho = 0$, this leads to $c = 0$ for whatever $\check{n}_E$, N_{Tx} and $\boldsymbol{\Upsilon}$. If the channel matrix $\boldsymbol{H}$ is random, the associating ergodic capacity becomes [1, Th. 1]

$$c_e = \mathcal{E}\langle c \rangle = \mathcal{E}\left\langle \log_2 |\boldsymbol{I}_{(\check{n}_E)} + \frac{\varrho}{N_{Tx}}\boldsymbol{\Upsilon}| \right\rangle. \quad (9)$$

Next we shall consider the capacity assessment based on spatial fading correlation. Using Jensen's inequality, there exists an upper bound $\bar{c}$ according to write [24]

$$\bar{c} = \log_2 \left| \boldsymbol{I}_{(\acute{n}_E)} + \frac{\varrho}{N_{Tx}} \mathcal{E}\langle \boldsymbol{\Upsilon} \rangle \right|. \quad (10)$$

IV. Separable Parameterizations

In this section, the proposed upper bound on the ergodic mean of channel capacity will be derived from the explicit form of $\mathcal{E}\langle \boldsymbol{H}[n_T]\boldsymbol{H}^{\mathsf{H}}[n_T]\rangle$.

A. Approximating Spatial Fading Correlation

Assuming that $N_{Rx} \leq N_{Tx}$, it allows us to

$$\mathcal{E}\langle \boldsymbol{H}[n_T]\boldsymbol{H}^{\mathsf{H}}[n_T]\rangle = \bar{\boldsymbol{H}}[n_T]\bar{\boldsymbol{H}}^{\mathsf{H}}[n_T] + \mathcal{E}\langle \tilde{\boldsymbol{H}}[n_T]\tilde{\boldsymbol{H}}^{\mathsf{H}}[n_T]\rangle. \quad (11)$$

Straightforward calculating $\bar{\boldsymbol{H}}[n_T]\bar{\boldsymbol{H}}^{\mathsf{H}}[n_T]$, we arrive at

$$\bar{\boldsymbol{H}}[n_T]\bar{\boldsymbol{H}}^{\mathsf{H}}[n_T] = \alpha^2 N_{Tx} \boldsymbol{a}(\phi)\boldsymbol{a}^{\mathsf{H}}(\phi). \quad (12)$$

Taking an incoherently distributed channel into account, the second-order statistic is characterized by [25]

$$\mathcal{E}\langle \gamma_{n_P}[n_T]\gamma^*_{\acute{n}_P}[\acute{n}_T]\rangle = \sigma^2_\gamma \delta_{n_P,\acute{n}_P}\delta_{n_T,\acute{n}_T}, \quad (13)$$

where $\delta_{\cdot,\cdot}$ signifies the Kronecker delta function and σ^2_γ designates the power of the n_P-th path. We proceed afterwards on

$$\begin{aligned}
&\mathcal{E}\langle \tilde{\boldsymbol{H}}[n_T]\tilde{\boldsymbol{H}}^{\mathsf{H}}[n_T]\rangle \\
&= \mathcal{E}\Bigg\langle \left(\sum_{n_P=1}^{N_P} \gamma_{n_P}[n_T]\boldsymbol{a}(\tilde{\phi}_{n_P}[n_T])\boldsymbol{a}^{\mathsf{H}}(\tilde{\varphi}_{n_P}[n_T])\right) \\
&\quad \left(\sum_{\acute{n}_P=1}^{N_P} \gamma^*_{\acute{n}_P}[n_T]\boldsymbol{a}(\tilde{\varphi}_{\acute{n}_P}[n_T])\boldsymbol{a}^{\mathsf{H}}(\tilde{\phi}_{\acute{n}_P}[n_T])\right)\Bigg\rangle \\
&= N_{Tx}\sum_{n_P=1}^{N_P} \mathcal{E}\langle |\gamma_{n_P}[n_T]|^2 \\
&\qquad \boldsymbol{a}(\tilde{\phi}_{n_P}[n_T])\boldsymbol{a}^{\mathsf{H}}(\tilde{\phi}_{n_P}[n_T])\rangle_{\gamma_{n_P}[n_T],\delta_{\phi_{n_P}}[n_T]} \\
&= \sigma^2_\gamma N_{Tx}\sum_{n_P=1}^{N_P} \mathcal{E}\langle \boldsymbol{a}(\tilde{\phi}_{n_P}[n_T])\boldsymbol{a}^{\mathsf{H}}(\tilde{\phi}_{n_P}[n_T])\rangle_{\delta_{\phi_{n_P}}[n_T]}.
\end{aligned} \quad (14)$$

Note that the random amplitude $|\gamma_{n_P}[n_T]|^2$ is Rayleigh distributed since $\gamma_{n_P}[n_T]$ is Gaussian.

The compact form shown above depends on the stochastic variable $\tilde{\phi}_{n_P}[n_T]$. Let us consider a heuristic approximation of such parameterization.

Heuristic 1: Over the spatial continuum of interest, a large number of incoming paths allows us to

$$\begin{aligned}
&\mathcal{E}\langle \tilde{\boldsymbol{H}}[n_T]\tilde{\boldsymbol{H}}^{\mathsf{H}}[n_T]\rangle \\
&\approx \rho N_{Tx}\int_{-\pi}^{\pi} f(\delta_\phi|0;\sigma^2_\phi)\boldsymbol{a}(\phi+\delta_\phi)\boldsymbol{a}^{\mathsf{H}}(\phi+\delta_\phi)d\delta_\phi,
\end{aligned} \quad (15)$$

where $\rho \triangleq N_P\sigma^2_\gamma$ is the power due to random paths and $f(\delta_\phi|0;\sigma^2_\phi)$ denotes a conditional PDF for random deviation δ_ϕ given a priori knowledge of the angular spread σ_ϕ.

B. Spatial Frequency Parameterization

Rather than the physical angles ϕ and σ_ϕ, spatial frequency response is preferable due to the better accuracy of approximating the first-order Taylor series around the array broadside [25]. Indeed, the spatial frequency ω and its associating standard deviation σ_ω are provided by

$$\omega(\psi) = kd_E\sin(\psi) \quad (16a)$$

$$\sigma_\omega(\psi,\sigma_\psi) = kd_E\cos(\psi)\sigma_\psi. \quad (16b)$$

For small angular spreads, the so-called *spatial frequency* approximation results in a separable form as

$$\begin{aligned}
\mathcal{E}\langle \tilde{\boldsymbol{H}}[n_T]\tilde{\boldsymbol{H}}^{\mathsf{H}}[n_T]\rangle &\simeq \rho N_{Tx}\boldsymbol{a}(\phi)\boldsymbol{a}^{\mathsf{H}}(\phi)\odot\tilde{\boldsymbol{B}}(\sigma_{\omega_\phi}) \\
&= \rho N_{Tx}\boldsymbol{D}_a(\omega_\phi)\tilde{\boldsymbol{B}}(\sigma_{\omega_\phi})\boldsymbol{D}^{\mathsf{H}}_a(\omega_\phi),
\end{aligned} \quad (17)$$

where the diagonal and unitary matrix $\boldsymbol{D}_a(\omega) : [-kd_E, kd_E] \mapsto \mathbb{C}^{N_E\times N_E}$ and the symmetric Toeplitz matrix $\tilde{\boldsymbol{B}}(\sigma_\omega) : \mathbb{R}^{1\times1}_+ \mapsto \mathbb{R}^{N_E\times N_E}$ are parameterized by nominal angle and angular spread, respectively. The $(n_E,\acute{n}_E)$-th elements of both can be given from [25, p. 22]

$$[\boldsymbol{D}_a(\omega)]_{[n_E,\acute{n}_E]} = \mathrm{e}^{\mathrm{i}(n_E-1)\omega}\delta_{n_E,\acute{n}_E} \quad (18a)$$

$$[\tilde{\boldsymbol{B}}(\sigma_\omega)]_{[n_E,\acute{n}_E]} = \Psi((n_E-\acute{n}_E)\sigma_\omega|0,1), \quad (18b)$$

where the characteristic function $\Psi(t|,0,1) \triangleq \mathcal{F}(f(\delta_\omega|,0,1))$ of the governed PDF is given for a random variable with zero-mean and unit variance. In a certain situation, the $(n_E,\acute{n}_E)$-th element in $\tilde{\boldsymbol{B}}(\sigma_\omega)$ can be expressed as

$$[\tilde{\boldsymbol{B}}(\sigma_\omega)]_{[n_E,\acute{n}_E]} = \begin{cases} \frac{\sin((n_E-\acute{n}_E)\sqrt{3}\sigma_\omega)}{(n_E-\acute{n}_E)\sqrt{3}\sigma_\omega} & \text{; uniform} \\ \mathrm{e}^{-\frac{1}{2}(n_E-\acute{n}_E)^2\sigma^2_\omega} & \text{; Gaussian} \\ \frac{1}{1+\frac{1}{2}(n_E-\acute{n}_E)^2\sigma^2_\omega} & \text{; Laplacian.} \end{cases} \quad (19)$$

It is worthwhile to note that if the incoming ray is not random *i.e.*, with zero variance, the LOS enables each element in $\bar{\boldsymbol{B}} \triangleq \tilde{\boldsymbol{B}}(0) = \boldsymbol{1}_{(N_E\times N_E)}$ to be unity. In this paper, we commonly define the Ricean factor as [21, p. 40] $\mu \triangleq \frac{\alpha^2}{\rho}$. Let $\boldsymbol{B}(\sigma_{\omega_\phi},\mu) : \mathbb{R}^{2\times1} \mapsto \mathbb{R}^{N_{Rx}\times N_{Rx}}$ be

$$\boldsymbol{B}(\sigma_{\omega_\phi},\mu) \triangleq \mu\bar{\boldsymbol{B}} + \tilde{\boldsymbol{B}}(\sigma_{\omega_\phi}). \quad (20)$$

Then, it readily follows that

$$\mathcal{E}\langle \boldsymbol{H}[n_T]\boldsymbol{H}^{\mathsf{H}}[n_T]\rangle = \rho N_{Tx}\boldsymbol{D}_a(\omega_\phi)\boldsymbol{B}(\sigma_{\omega_\phi},\mu)\boldsymbol{D}^{\mathsf{H}}_a(\omega_\phi). \quad (21)$$

V. Proposed Upper Bound on the Mean of ergodic Capacity

Throughout this section, algebraic manipulations are devoted to express the upper bound in such a way that rather than stochastic expression in (9), it should be written as a deterministic function. Together with the heuristic (15), the upper bound indicated in (10) becomes

$$\bar{c} = \log_2 |\boldsymbol{I}_{(N_{Rx})} + \rho\varrho\boldsymbol{D}_a(\omega_\phi)\boldsymbol{B}(\sigma_{\omega_\phi},\mu)\boldsymbol{D}^{\mathsf{H}}_a(\omega_\phi)|. \quad (22)$$

Notice that $\boldsymbol{I}_{(N_{Rx})} + \rho\varrho\boldsymbol{D}_a(\omega_\phi)\boldsymbol{B}(\sigma_{\omega_\phi},\mu)\boldsymbol{D}^{\mathsf{H}}_a(\omega_\phi)$ can be seen as $\boldsymbol{D}_a(\omega_\phi)\big(\boldsymbol{I}_{(N_{Rx})} + \rho\varrho\boldsymbol{B}(\sigma_{\omega_\phi},\mu)\big)\boldsymbol{D}^{\mathsf{H}}_a(\omega_\phi)$. Invoking

$|\mathbf{ABC}| = |\mathbf{A}||\mathbf{B}||\mathbf{C}|$ and $|\boldsymbol{D}_a^{\mathsf{H}}(\omega)| = \frac{1}{|\boldsymbol{D}_a(\omega)|}$, the upper bound is then

$$\bar{c} = \log_2 |\boldsymbol{I}_{(N_{Rx})} + \rho\varrho \boldsymbol{B}(\sigma_{\omega_\phi}, \mu)|. \tag{23}$$

To coincide with other previous studies, the Rice factor is usually preferred to be normalized (see *e.g.,* [14] and [22]). Then, it follows from $\alpha^2 + \rho = 1$ that

$$\alpha = \sqrt{\frac{\mu}{\mu+1}} \tag{24a}$$

$$\rho = \frac{1}{\mu+1}. \tag{24b}$$

Proposition 1: For the ergodic capacity of Ricean channel generated by (3), (5) and (6), an explicit form of the upper bound on its mean can be written as

$$\bar{c} = \log_2 \left| \boldsymbol{I}_{(N_{Rx})} + \varrho \left(\frac{\mu}{\mu+1} \bar{\boldsymbol{B}} + \frac{1}{\mu+1} \tilde{\boldsymbol{B}}(\sigma_{\omega_\phi}) \right) \right|, \tag{25}$$

where

$$\bar{c} \geq \mathcal{E} \langle c \rangle . \tag{26}$$

Proof: The upper bound (25) of ergodic capacity mean in MIMO link is given by putting the ρ in (24b) into (23) and then algebraically manipulating the result in conjunction with (20). As stated before, the expression (26) holds from the concavity employed in $\log_2 \mathcal{E} \langle a \rangle \geq \mathcal{E} \langle \log_2 a \rangle \ ; a > 0$. ∎

Remark 2: Although the proposed upper bound in (25) is not shown explicitly as a function of nominal direction, the relationships between $\bar{c}$ and ϕ can be seen from (16b).

Remark 3: Apart from the average SNR and the number of antenna elements, it can be pointed out that the proposed upper bound $\bar{c}$ in (25) also depends on other channel parameters, such as, Rice factor μ, and angular spread σ_ϕ.

Remark 4: Let $\lceil \cdot \rfloor$ be the trace operator of matrix . Recall the derivative for $\frac{\partial}{\partial x} \ln |\mathbf{A}(x)| = \lceil \mathbf{A}^{-1}(x) \dot{\mathbf{A}}(x) \rfloor$, where $\dot{\mathbf{A}}(x) \triangleq \frac{\partial}{\partial x} \mathbf{A}(x)$. With $\log_a(x) = \frac{\ln(x)}{\ln(a)}$, we obtain

$$\dot{\bar{c}}(\phi) = \dot{\omega}_\phi(\phi) \dot{\bar{c}}(\sigma_{\omega_\phi}) = -\frac{\varrho k d_E \sin(\phi) \sigma_\phi}{(1+\mu)\ln(2)} \lceil \boldsymbol{C}^{-1} \dot{\tilde{\boldsymbol{B}}}(\sigma_{\omega_\phi}) \rfloor, \tag{27}$$

where $\boldsymbol{C} \triangleq \boldsymbol{I}_{(N_{Rx})} + \varrho \left(\frac{\mu}{\mu+1} \bar{\boldsymbol{B}} + \frac{1}{\mu+1} \tilde{\boldsymbol{B}}(\sigma_{\omega_\phi}) \right)$. At the critical point $\dot{\bar{c}}(\phi) = 0$, the result of $\sin(\phi) = 0$ implies $\phi = 0$. Then, $\phi = 0$ is a critical point of the upper bound.

VI. Numerical Simulations

Customarily, we employ the ULA with half-wavelength separation for both receiver and transmitter. All significant parameters are set up as indicated in each figure. The sample mean capacity is to average all N_R independent calculations in (7). Indeed the proposed upper bound is computed from (25).

In Fig. 2, simulation is conducted by varying the average SNR ϱ. As expected, the increase of SNR and/or antenna elements allows the MIMO channel to more affordable capacity. At low SNR, the proposed upper bound agrees well with sample mean capacity. When both SNR and antenna elements are high, however, the difference between mean capacity and the upper bound is noticeable. This is due to the effect of directional approximation in (15).

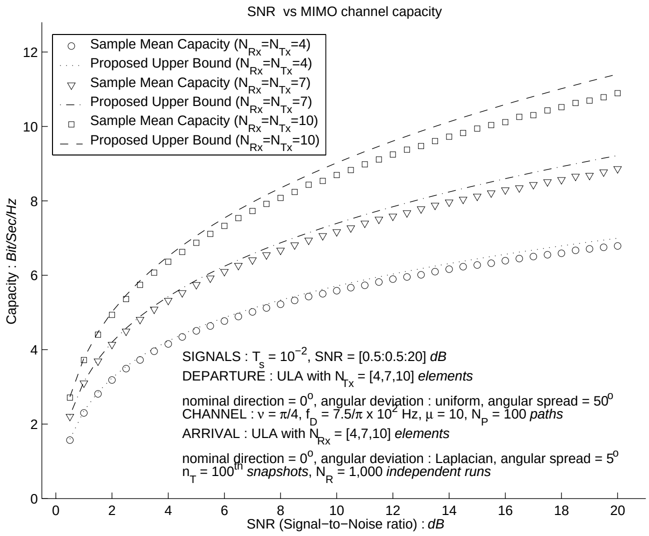

Fig. 2. MIMO link capacity as a function of average SNR ϱ for several values of antenna elements.

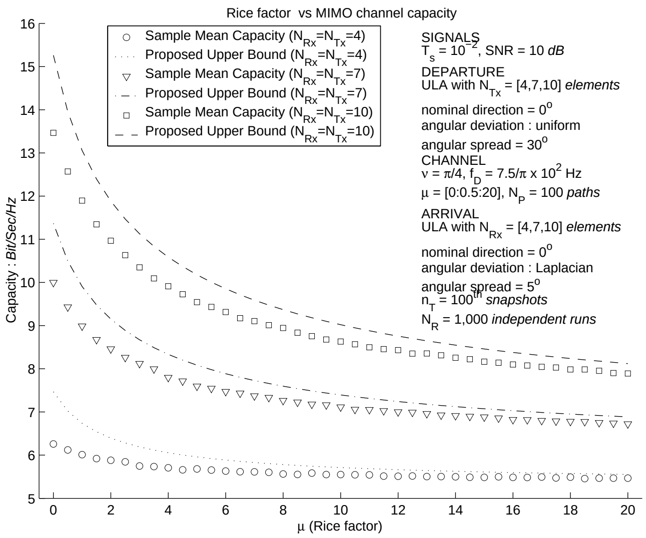

Fig. 3. MIMO link capacity as a function of Rice factor μ for several values of antenna elements.

Fig. 3 illustrates the tightness of the proposed upper bound in the aspect of Rice factor μ. When increasing the Rice factor, the spatial approximation in (15) is gradually insignificant. Then, the proposed upper bound tends to be tight in large Rice factor. It is noteworthy that the Rayleigh channel ($\mu = 0$) provides more capacity than the Ricean channel ($\mu > 0$). Fortunately, this coincides with the results performed in unstructured model (see *e.g.,* [12] and [26]).

The pictorial impact of nominal direction and angular spread in MIMO channel capacity is shown in Fig. 4. Note that the more the angular spread, the higher the available mean capacity. This is due to spatial diversity implicit in NLOS portion. For a considerable angular spread, the channel capacity is

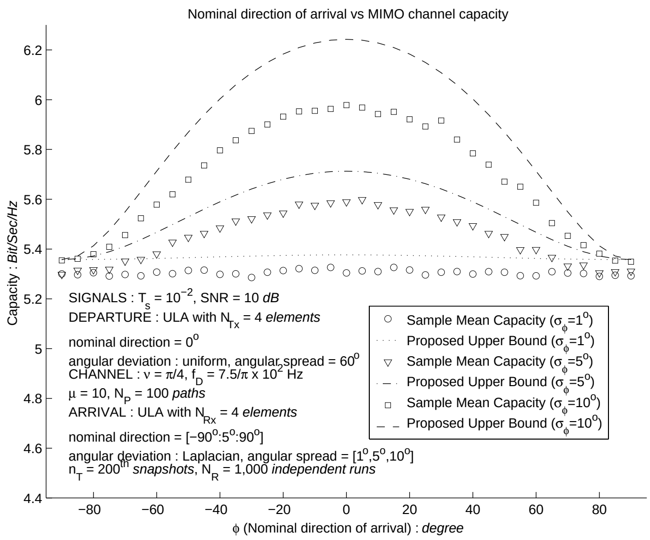

Fig. 4. MIMO link capacity as a function of nominal direction of arrival for several values of angular spreads.

maximal along broadside direction ($\phi = 0°$). In addition, the error effect of the first-order Taylor series approximation in (17) is quite accurate for small angular spread.

VII. Conclusion

We have developed a parametric framework for assessing the ergodic mean capacity of MIMO channel model in the presence of both Ricean fading and physical scattering. The objective of capacity assessment in this way is to investigate the realistic propagation model rather than making analysis on the unstructured statistic of Ricean channel matrix $\boldsymbol{H}$. To avoid the lengthy Monte Carlo simulation, we have also provided an upper bound on the ergodic mean of channel capacity based on the so-called separable parameterization. A rigorous advantage of the proposed upper bound is to reflect MIMO link capacity as an analytic function of Rice factor and directional parameters, such as, nominal direction, angular spread. Regarding computational and accuracy viewpoints, it holds not only simple calculation, but also asymptotic tightness in several situations, for instance, in small average SNR, small angular spread and large Rice factor. In addition, a rather distinctive study is shown herein that the Ricean channel leads to higher channel capacity when angular spread is larger.

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