Use of Random Building Height Model for Fixed Wireless Access Systems in Urban and Suburban Environments

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***Abstract*— This paper investigates stochastic propagation models appropriate for Fixed Wireless Access (FWA) Systems. A widely used model for the prediction of path loss in mobile systems for urban environments is the Walfisch-Bertoni model [1]. It was later modified to accommodate FWA systems in [2] and is known as the Random Building Height (RBH) model. In this paper, a comparison is made between the predictions made by the RBH model and a comprehensive set of propagation measurements taken in urban and suburban environments around Cambridge, UK. The comparisons show that at receiver antenna height of 10 m the model has a mean prediction error of 5.6 dB of in an urban environments and 11.8 dB in a suburban environments.**

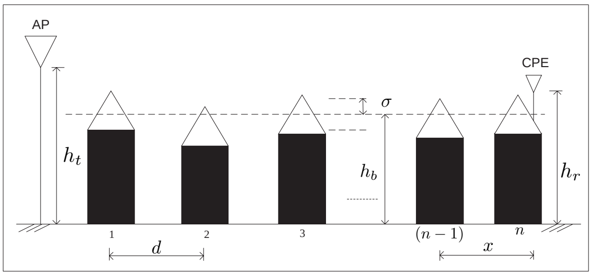

Fig. 1. Definition of parameters used in the RBH model

I. INTRODUCTION

Among the many propagation models available, the stochastic (or probabilistic) models are attractive owing to their limited reliance on very detailed data of the environment. One of the more popular models of this kind is the Walfisch-Bertoni model [1], which is used to predict the average path loss for mobile systems in urban areas. It assumes that propagation takes place over rows of buildings having equal height and spacing and calculates the path loss as a sum of two components; the first owing to multiple diffraction over rows of building rooftops and the second caused by diffraction from the final building rooftop down to the Customer Premises Equipment (CPE) antenna. This model is widely used for mobile cellular networks and has been verified by measurements, for example in [3], and has been adopted as the basis for the COST-231 model [4]. However, the Walfisch-Bertoni model when used for planning FWA networks has two main limitations. Firstly, the assumption that the environment consists of buildings of equal height and equal separation arranged in perfect grids is not very realistic, especially in European cities. The first limitation has been addressed to some extent by introducing variation of building heights into the model for example in [5], [6]. Secondly, the assumption made in the Walfisch-Bertoni model that the CPE antenna is placed well below the building rooftop height is also not appropriate for the FWA scenario. Whitteker [7] has expanded the analytic solution of Xia [8] to allow the field amplitude to be calculated at a range of heights above and below rooftop level. Due to computational limits, the expression can only be practically evaluated for up to 15 buildings (modelled as half-screens). However, an approximating expression is presented which is valid over a much greater range. The Random Building Height (RBH) model, which was introduced in [2] and is also described in [9] addresses both of the limitations mentioned previously.

Figure 1 shows the parameters required for the RBH model. The transmitted signal from the Access point (AP) antenna is assumed to be propagated over n rooftops to reach the CPE antenna. The AP antenna height, h_t is defined above the ground level. To improve the accuracy of the model predictions, all height data has been referenced to a linear Least Squares (LS) approximation of terrain profile. This gives rise to the concept of an effective AP antenna height and has removed the error resulting from the average terrain slope on the field predicted at rooftop level [10]. The CPE antenna height above ground level, h_r may be similarly referenced to this datum. This process is further explained in section III.

The mean building separation, d and the mean building height, h_b are estimated based on the environment between the AP and the CPE. In addition, the Standard Deviation (SD) of the building heights, σ also needs to be estimated and input to the model. Note that the distance from the final diffraction rooftop to the CPE antenna, x is assumed to be equal to d. This is a reasonable assumption given that the CPE antennas in FWA networks are usually fixed on the roof as shown in Figure 1.

The RBH model is an approximating expression for the Excess Path Loss, X (i.e., the loss in excess of that owing to free space loss) determined from a large number of numerical simulations. The simulations are evaluated in the propagation environment shown in Figure 1 for a large number of different geometries and with all buildings modelled as perfectly absorbing half-screens. For an elevated CPE antenna (i.e., $h_r \geq h_b$) the median value of X was found to be closely predicted by;

$$X = 20\log_{10} Q + A(h_r) \tag{1}$$

where, Q is the value of the settled field at average rooftop level, modified to include the effects of building height variation [2], [9];

$$Q = 2.35 g_p^{0.9} \tag{2}$$

where,

$$g_p = \sqrt{\frac{d}{\lambda}} \frac{\alpha}{(1 + 4.88\gamma + 2.88\gamma^2)^{0.556}} \tag{3}$$

and,

$$\gamma = \frac{\sigma^2}{\lambda d}\,. \tag{4}$$

The parameter λ of (4) is the wavelength of the propagating wave, α is the incidence angle of the transmitted wave at average rooftop height and σ is the standard deviation of the building heights. $A(h_r)$ is the height gain of the subscriber antenna. Various formulations can be used for this factor (e.g., [5] considers the standard the ITU-R height gain model and the Okumura-Hata correction factor). In the model to be presented here an approximate expression derived from numerical simulations will be used,

$$A(h_r) = -20\log_{10}(1 + a_1 p_a + a_2 p_a^2) \tag{5}$$

where,

$$\begin{aligned} a_1 &= 0.9 + 1.3e^{-0.866\sigma} \\ a_2 &= 1.0 - 0.5e^{-1.823\sigma} \\ p_a &= \frac{h_r - h_b}{(\lambda d)^{0.45}}\,. \end{aligned} \tag{6}$$

II. MEASUREMENT PROGRAM

A comprehensive set of propagation measurements have been collected during a project funded by Ofcom, UK. As part of the project a commercial-scale FWA network was deployed in Cambridge, UK beginning in May 2003. The network consists of 5 Base Station (BS) sites and CPEs installed at over 65 subscriber premises. Four AP antennas, each having a 90^0 horizontal beamwidth, are located at each BS site to achieve 360^0 coverage. The propagation measurements were taken using a van having a pneumatically operated mast to which a CPE antenna was mounted. Measurements gathered in urban and suburban environments were compared with the predictions made by the RBH model. To this end, the areas covered by two BS sites were selected, namely Addenbrooke's Hospital (ADD) and the Anglia Polytechnic Institute (APU). The AP antenna heights at these BS sites were 16 m and 36 m, respectively.

Understandably, this model is only appropriate for built-up environments. To this end, measurements from three AP sectors have been selected for the model validation process, namely the South and East APs of the APU BS site and the North AP of the ADD BS site. The former two are classified as urban whereas the last one is suburban. For the East AP of the APU BS site, the measurement locations lie between 344 m from the BS site to 1.48 km away from the BS site. Measurements were taken in every street in between. Further measurements were taken for the South AP of the APU BS site. In this case, the measurement locations lie between 135 m from the BS site to 950 m from the BS site.

For the ADD North AP, measurements were taken in two different regions owing to their rather different propagation profiles. The first region runs from 590 m from BS site to just beyond Mill Road and is some 2.80 km away from the BS site. The other region chosen for measurements was in Chesterton, which lies to the Northeast of the City centre. These measurement locations lie in a range from 4.67 km to 6.34 km away from the ADD BS site. The measurement locations in these two regions are denoted as 'Mill Rd.' and 'Chest.' Both these regions show characteristics of a suburban environment.

The model requires statistics for both Mean Building height, and the Standard Deviation (SD) of the building height, which have to be estimated. To improve the accuracy of these statistics, the 3D terrain and building database for the Cambridge area has been used.

III. ESTIMATION OF BUILDING STATISTICS

The RBH model requires 3 parameters, h_b, d and σ, as shown in Figure 1. For each of the measurement sectors, these values are calculated by processing building height and location data from a digital clutter database of the Cambridge area.

For each measurement sector, the building height statistics are calculated using the following method. Nine 'sample' ray paths are constructed from the AP to the edge of each sector as shown in Figure 2. The angular separation of the ray paths is constant and the orientation of the two outer rays is such that they enclose the measurement sector and hence all measurement locations. The length of each ray path is constant and equal to the maximum AP to CPE measurement location range. The clutter database is then searched to determine the path profile along each of these rays. This is performed using a computer program that checks each building structure in the database for an intercept with a given ray path. An

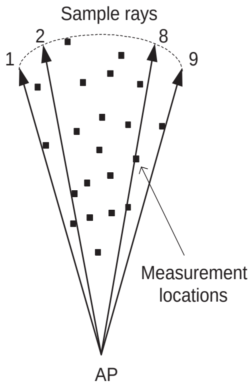

Fig. 2. Ray sampling method used to determine the building statistics

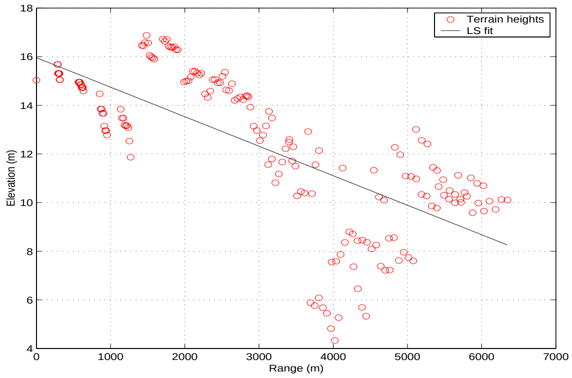

Fig. 3. Terrain data for the (linear) LS fit

intersection could result from either a building wall or roof-ridge. The distance (measured from the AP) and the height of each intercept along the ray path are stored. In this way a path profile, consisting of an array of distance and height values, for each ray path is computed. The path profiles are post-processed to ensure that a given building had only one wall or roof-ridge layer intersection point. The building statistics are then obtained from the analysis of these nine profiles.

To improve prediction accuracy, both the recorded CPE antenna heights and the AP heights are normalised based on terrain height. The terrain height along all nine ray paths was processed to produce a single linear LS approximation of the terrain which was then applied to the entire sector. The height of the CPE and the AP antennas are then referenced relative this datum. These values are called the effective CPE antenna height, h_r and the effective AP antenna height, h_t, respectively.

Similarly, it is also necessary to correct all the building heights (which are also measured relative to the sea-level) before calculating any building height statistics. The process is illustrated further in Figures 3 and 4. Figure 3 shows the terrain heights taken from all nine ray paths in the Chesterton region. Also shown is the resulting LS approximation for the sector. Figure 4 shows the combined building height data extracted from the database for the same nine ray paths. The LS terrain height approximation is then subtracted from all the building height data to produce the corrected building height values shown in the Figure 4. These values are then processed to determine the mean, h_b and SD, σ of the building heights. The results obtained for effective BS height, h_t, mean building spacing, d, mean building height, h_b and SD of building height, σ in the four regions under consideration are shown in Table I.

The area around the APU is highly built-up, consisting mostly of two storey terraced houses and buildings. The consistent building topology is reflected by the low SD for building heights obtained from the estimation process, as shown in Table I. The streets are approximately 40 m - 60 m apart. Although both South and East sectors in the APU BS site are built-up, they differ in one important characteristic. The streets in the East sector are predominantly transverse to the direction of propagation from the East AP, whereas the streets in the South sector are predominantly parallel to the direction of propagation from the South AP. The measurements in both sectors are used to validate the model for built-up areas having different street orientations.

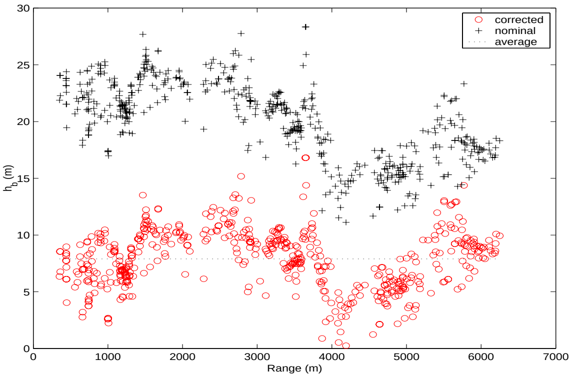

Fig. 4. Corrected building heights

The environment from the ADD BS site as far as Mill Road is suburban. However, it contains very high trees, especially around the area close to the BS site. The houses close to the BS site are detached but the area close to Mill Road contains terraced two-storey houses. The roads are wider apart in the

Param-eter	**APU E (m)**	**APU S (m)**	**Mill Rd. (m)**	**Chest. (m)**
h_t	11.5	12	36.1	35.0
d	58.5	47.8	92.4	95.9
h_b	9.3	8.9	6.5	7.9
σ	2.5	1.3	4.4	2.5

TABLE I

ESTIMATED PARAMETERS FOR THE FOUR REGIONS UNDER CONSIDERATION.

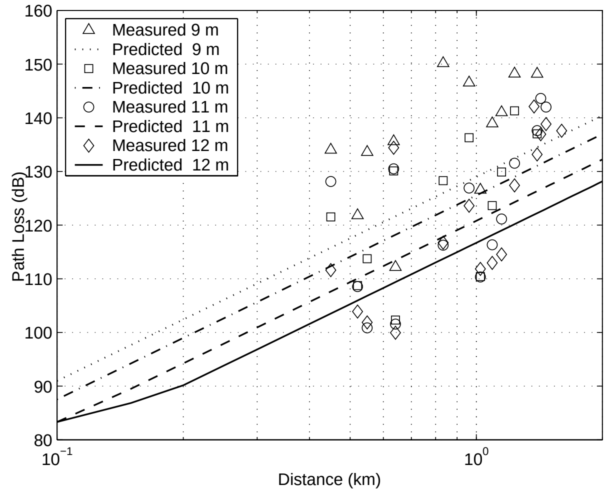

Fig. 5. Comparison of RBH model predictions and measurements in the East AP of the APU BS site with h_r from 12 m down to 9 m.

suburban area close to the BS site at approximately 90 m - 100 m. The varied building height seen in the area is reflected by the high SD for the building heights shown in Table I. The Chesterton measurement region contains some one-storey houses although two-storey houses predominate. Again, the area is not as built-up as that around APU BS site, with the streets being spaced more widely apart.

The Mill Road region of the North AP of the Addenbrooke's BS site is predominantly suburban but contains both suburban and urban areas. Hence, the measurements gathered in this region can be used to validate the model against regions having both urban and suburban characteristics. The Chesterton region is more than 4 km away from the BS site and is essentially suburban. The measurements in this region can be used to validate the model for greater AP to CPE separations.

IV. COMPARISON WITH MEASUREMENTS

The measurements gathered around the APU BS site were compared against the model predictions made using the parameters in Table I. The comparison between predictions and measurements for the East and the South sectors of the APU BS site are shown in Figure 5 and Figure 6, respectively. The corresponding error statistics, namely the mean prediction error, μ_e and the SD of the error, σ_e are presented in Table II. Note that the RBH model is based upon the Walfisch model simplification, which assumes that the houses are aligned in a generally transverse orientation to the direction of propagation. It can be seen that the mean prediction error is in general lower in the APU East sector than that in the APU South sector. This is probably because of the transverse street alignment which is characteristic of the APU East sector.

The comparison between predictions and measurements for the Mill Road and Chesterton regions within the Addenbrooke's North AP sector are shown in Figure 7 and Figure 8, respectively. The error statistics of the results are shown in Table III. The mean prediction errors are generally lower in the Chesterton region than those of the Mill Road region. This is because this environment is the closest fit to that assumed in the model.

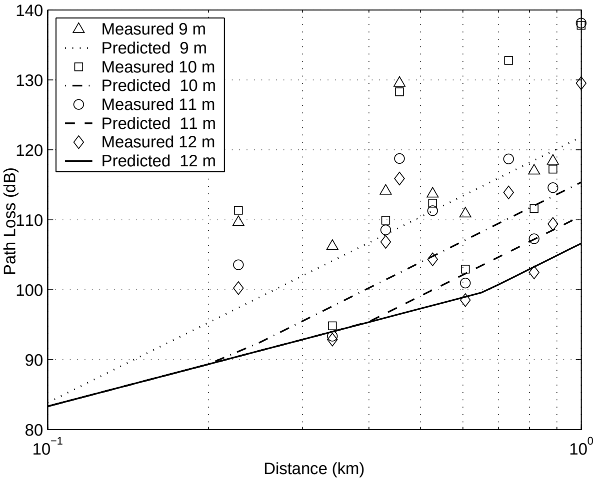

Fig. 6. Comparison of RBH model predictions and measurements in the South AP of the APU BS site with h_r from 12 m down to 9 m.

Effective CPE Ant. Height (m)	APU E (dB)		APU S (dB)	
	μ_e	σ_e	μ_e	σ_e
$12.5 \leq h_r < 11.5$	5.9	10.4	8.4	8.3
$11.5 \leq h_r < 10.5$	3.7	11.3	10.3	9.5
$10.5 \leq h_r < 9.5$	1.3	10.0	10.4	11.5
$9.5 \leq h_r < 8.5$	10.7	9.5	4.7	8.1

TABLE II
ERROR STATISTICS OF THE MODEL PREDICTIONS FOR THE EAST AND THE SOUTH AP SECTORS OF THE APU BS SITE

The positive mean prediction error observed at all sectors and CPE antenna heights may result from other propagation loss factors which are not accounted for by the RBH model. One factor is shadowing loss resulting from terrain height variation. Indeed the lowest mean errors are obtained in the East sector of the APU BS. This sector has the smallest peak-to-peak variation of terrain elevation and more closely matches the underlying assumptions made in the RBH model.

Effective CPE Ant. Height (m)	Mill Rd. (dB)		Chest. (dB)	
	μ_e	σ_e	μ_e	σ_e
$12.5 \leq h_r < 11.5$	7.1	10.9	10.2	6.9
$11.5 \leq h_r < 10.5$	9.5	12.7	9.0	7.7
$10.5 \leq h_r < 9.5$	10.7	13.9	8.6	7.2
$9.5 \leq h_r < 8.5$	13.9	14.9	11.8	7.4

TABLE III
ERROR STATISTICS OF THE MODEL PREDICTIONS FOR THE TWO REGIONS WITHIN THE NORTH AP SECTOR OF THE ADDENBROOKE'S BS SITE

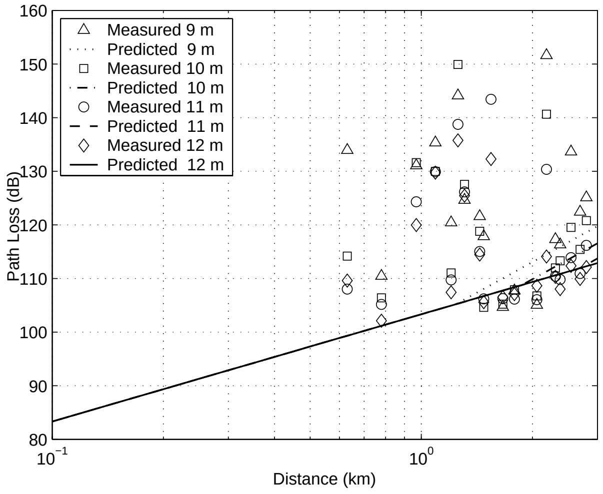

Fig. 7. Comparison of RBH model predictions and measurements taken as Mill Rd. in the North AP sector of the Addenbrooke's BS site with h_r from 12 m down to 9 m.

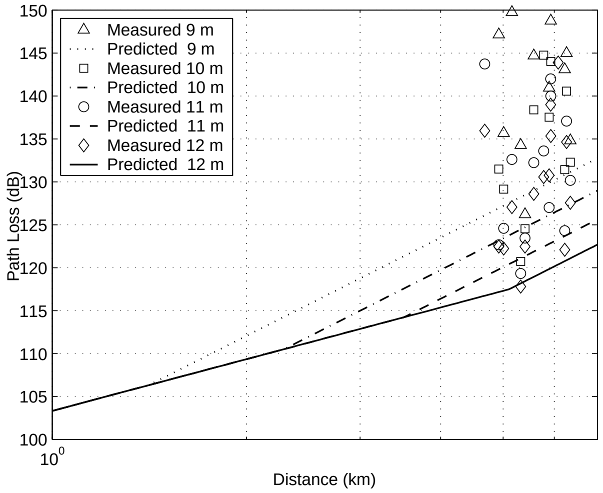

Fig. 8. Comparison of RBH model predictions and measurements taken in the Chesterton region of the Addenbrooke's BS site with h_r from 12 m down to 9 m.

V. CONCLUSION

The accuracy of the model predictions is clearly environment dependent. That is, the closer the environment matches that of the underlying model assumptions, the lower the error. So the APU East sector has the best fit while the Mill Road region of the Addenbrooke's North AP sector has the worst. For the Chesterton area, the propagation distances are large which reduces the errors due to assumptions made about successive rooftop diffraction since the number of rooftops is large. Also the effects of local shadowing are less pronounced in this area. Also note that the higher mean prediction errors and greater SD of the errors experienced in the ADD North, Mill Road region are probably due to heavy shadowing close to the BS site caused by the very tall trees and a chimney that lie in that area.

Terrain variations which are particularly evident in the APU South sector at around 1 km from the BS site. The Walfisch model assumes flat terrain and any departure from this ideal will introduce some error. In this study we have used a linear regression of the terrain height in order to estimate the effective AP and CPE antenna heights. Any departure of the terrain profile from this regression will be a source of error. The maximum variation of the terrain height from the linear estimate in meters for the measurement sectors of Mill Rd., Chest., APU E and APU S are 8, 8, 4 and 3, respectively. Hence, both regions in the ADD North sector exhibit quite large variations which cannot be accounted for by the RBH model and will contribute significantly to the prediction errors.

Some factors that influence the disagreement between predictions and measurements are, the accuracy of building height database, the accuracy of building height database, foliage effects which are neglected in the model and departures from model assumptions (i.e., propagation "down streets" instead of across rows as observed at the APU BS). Also the model is really only appropriate at longer ranges where the field will be multiply diffracted.

ACKNOWLEDGEMENTS

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