Filter Bank Based Frequency Domain Equalizer in Single Carrier Modulation

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***Abstract*—This paper is concerned with the use of complex-modulated oversampled filter bank for frequency domain equalization in single carrier systems. A novel equalizer structure consisting of a low order linear phase FIR and an all-pass filter is applied in subband-wise compensation of channel amplitude and phase distortions separately. The performance of the proposed filter bank based frequency domain equalizer is studied and compared against a fractionally spaced FFT-based equalizer. It is demonstrated that a filter bank based equalizer system with modest number of subbands and without guard-interval overhead is able to provide similar or better performance than an cyclic-prefix and FFT-based equalizer.**

I. Introduction

The frequency domain equalization (FDE) with the use of a single carrier (SC) scheme, referred to as SC-FDE, has been considered as a low-complexity scheme for broadband wireless system [1]–[3]. These systems are known to be simply derived from multicarrier (MC) transmission systems, such as orthogonal frequency-division multiplexing (OFDM), by shifting the inverse discrete Fourier transform (IDFT) from transmitter to the receiver end. In [1], [3], it has been demonstrated that SC-FDE systems have performance advantage, more diversity benefits compared with MC system.

Furthermore, SC-FDE radio modems can advantageously overcome some of the key problems inherent to MC systems. For instance, single carrier scheme provides an RF signal that shows considerably less envelope fluctuations and can thus better tolerate non-linear power amplifier (PA) characteristics, showing less spectral spreading and in-band distortion or requires several dB less power back-off than a comparable MC system. In practice this means that a given cell coverage requirement can be achieved with less expensive power amplifier. Alternatively, this translates to higher cell coverage for a fixed average power consumption or longer battery life.

The function of the FDE is to minimize the inter-symbol interference (ISI), i.e., the same as that of a time domain equalizer. Because FDE processes the data sequences in blocks, therefore it is known to be simpler to implement and easier to initialize than the time-domain equalizer [4], especially when channels have severe delay spread. The basic structure of SC-FDE system is illustrated in the Figure 1. The operations of the blocks basically include forward transform from time to frequency domain, channel inversion and reverse transform from frequency to time domain. Typically, DFT and IDFT are used in these transforms. A cyclic prefix is usually applied as a guard-interval in front of a transmitted symbol block. If the length of the cyclic prefix exeeds the channel delay spread, optimum frequency-domain linear equalization can be performed through a complix multiplier for each of the frequency bins.

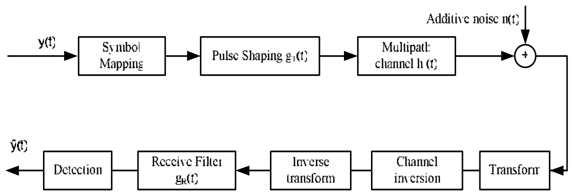

Fig. 1. General Model for Single Carrier Frequency Domain Equalization.

In this paper, an alternative SC-FDE implementation based on complex-modulated filter bank (FB) in analysis-synthesis configuration, instead of DFT-IDFT banks, is proposed. The use of FBs is motivated by the better spectral containment. Complex modulated FBs split the entire signal spectrum into a number of narrow frequency bins, such that only the transition bands of adjacent frequency bins are overlapping. The subchannel signals can be compensated independently and effectively. After equalization, the subchannel signals are then combined together through the synthesis FB. This scheme is a SC extension of Amplitude-Phase ASCET (Adaptive sine-modulated/cosine-modulated filter bank equalizers for TMUXs), introduced in the papers [5]–[7]. We implement two equalization cases of ASCET in single carrier system with MSE criterion. Performance of such a FB-based equalizer (FBEQ) system in frequency selective quasi-static wireless channel is presented with comparison to the reference FFT-based fractionally spaced equalizer (FSE) [8], [9].

The contents of this paper are organized as follows: Section II overviews a complex-modulated oversampled FB. Subchannel equalization principles are presented in Section III. Analytic performance evaluation and numerical simulation results are described in Section IV. Section V gives performance comparison between FBEQ and FFT-based FSE. Conclusions are drawn in Section VI.

II. Complex-Modulated Filter Bank

In wireless communications, it is desirable to use a complex I/Q baseband system model because of good spectral efficiency. Complex valued signals for subband signal processing

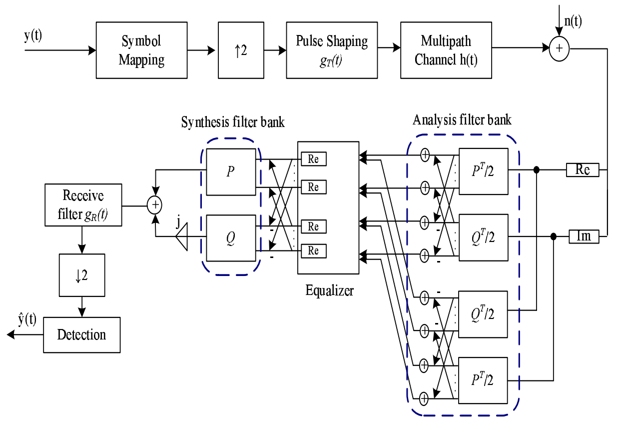

Fig. 2. Complex-modulated FB and AP-ASCET equalizer.

systems can be obtained by applying complex modulated FBs consisting of cosine- and sine-modulated FBs. The CMFB's and SMFB's subfilters are generated from a real low-pass prototype filter $h_p(n)$ by multiplying with cosine and sine sequences. Their amplitude responses divide the whole frequency range $[-\pi \ \ ,\pi]$ into equally wide passbands. The prototype filter $h_p(n)$ can be optimized in such a manner that the FB system satisfies the perfect reconstruction (PR) condition. This means that if there is no additional signal processing between analysis and synthesis stages, the output of the overall FB system is just a delayed version of the input signal. In paraunitary FBs the PR condition is fulfilled by choosing the impulse responses of the analysis and synthesis subband filters to be complex-conjugated and time-reversed versions of each other. [10], [11].

The impulse response of *i*th CMFB synthesis filters is

$$f_i^c(n) = \sqrt{\frac{2}{M}} h_p(n) \cos((n + \frac{M+1}{2})(i + \frac{1}{2})\frac{\pi}{M}), \quad (1)$$

where time index *n*=0,1, ..., *L-1* and subfilter index *i*=0,1,...,*M-1*. The overlapping factor *K* controls the subfilter length *L=2KM*. The overlapping factor *K*, together with the roll-off parameter ρ determine the achievable stopband attenuation of subband filters. Commonly, ρ=1 is used, meaning that the overall subband bandwidth between stopband edges is twice the channel spacing.

The CMFB is a paraunitary FB and the *i*th CMFB analysis filter is equal to the time-reversed synthesis filter

$$h_i^c(n) = f_i^c(N-1-n). \quad (2)$$

The SMFB is defined in a similar manner, only sine modulation is applied instead of cosine modulation. The *i*th synthesis filter of SMFB is thus

$$f_i^s(n) = \sqrt{\frac{2}{M}} h_p(n) \sin((n + \frac{M+1}{2})(i + \frac{1}{2})\frac{\pi}{M}). \quad (3)$$

The FBEQ system utilizing the complex modulated oversampled analysis bank is illustrated in Figure 2. Channel equalization is performed between the analysis and synthesis FBs, and the synthesis bank is critically sampled. P^T denotes the real transform matrix of the analysis CMFB, and Q^T denotes the transform matrix of the analysis SMFB. Then *P* and *Q* represent the corresponding transform matrices of the synthesis FBs, respectively.

$$[P]_{ni} = f_i^c(n) \quad [Q]_{ni} = f_i^s(n) \quad (4)$$

The dimension of matrix P^T is $L \times M$ and its columns are the basis functions of the transform. The *k*th basis function is the *k*th CMFB subfilter response. For every *M* input samples, the P^T and Q^T transform *L* time samples into *M* frequency domain coefficients.

The oversampled analysis part contains two CMFBs and two SMFBs, which decompose a complex-valued high-rate data sequence into *2M* complex-valued low-rate subband data sequences. Equalization is carried out on each subband at a low rate. Only the real parts of the subband signals are considered after the equalizer and the synthesis bank is realized in the critically sampled form, consisting only of one CMFB and one SMFB block [7].

III. Channel Equalization

The received signal is split into a number of subchannel signals by the analysis FB. Equalization is carried out independently on each subchannel in terms of separate amplitude and phase compensation. We consider the following two versions of the generic AP-ASCET [7] structure:

- Case 1: This model contains a complex equalizer coefficient q_k in each subchannel. The amplitude of equalizer $|q_k|$ is proportional to the inverse of the subchannel amplitude response $1/\left|C_k(e^{j\omega})\right|$, and the phase is the negative of the subchannel phase response $-\arg(C_k(e^{j\omega}))$.
- Case 2: The subchannel equalizer structure consists of a cascade of a first order complex allpass filter followed by a phase rotator and an operation of taking the real part of the signal. Finally, a symmetric linear-phase 3-tap FIR is applied for amplitude compensation, as depicted in Figure 3.
  The *k*th subchannel allpass filter is implemented in practical form as

$$H_k(z) = z^{-1}\frac{1 - jb_k z}{1 + jb_k z^{-1}}. \quad (5)$$

  Then the equalizer phase response excluding the effect of unit delay is given by:

$$\arg[H_k(e^{j\omega})] = \varphi_k + 2\arctan(\frac{-b_k\cos\omega}{1 + b_k\sin\omega}). \quad (6)$$

  The equalizer amplitude response for the k^{th} subchannel can be expressed as

$$\left|H_k(e^{j\omega})\right| = a_{0k} + 2a_{1k}\cos\omega \quad (7)$$

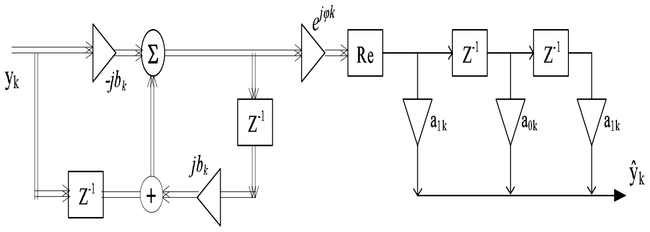

Fig. 3. Subchannel equalizer structure for Case 2 AP-ASCET.

Mean Squared Error (MSE) Criterion

In the case of zero forcing (ZF) criterion, the spectral components at the spectrum notches will be strongly affected by additive noise, inversion of the channel transfer function leads to very high noise enhancement. When the MSE criterion is applied, the output of FDE is a mixture of channel noise and intersymbol interference (ISI). In general, MSE criterion is a trade-off between ISI and noise enhancement. It provides an improved performance with a complexity comparable to that of the ZF.

When calculating the equalizer coefficients using MSE criterion, the target amplitude and phase responses are expressed as:

$$\begin{aligned} \epsilon_k &= \frac{\left|C(e^{j\omega_k})\right|}{\left|C(e^{j\omega_k})\right|^2 + N_0/\varepsilon_s} \\ \zeta_k &= -\arg(C(e^{j\omega_k})) \end{aligned} \tag{8}$$

where N_0/ε_s is noise-to-signal ratio (NSR) quantity. Then the equalizer coefficients of Case 2 can be derived as:

$$\begin{aligned} \varphi_k &= \frac{\zeta_{0k} + \zeta_{2k}}{2} & a_{0k} &= \frac{1}{2}(\epsilon_{0k} + \epsilon_{2k}) \\ b_k &= \pm\tan(\frac{\zeta_{2k} - \zeta_{0k}}{4}) & a_{2k} &= \pm\frac{1}{4}(\epsilon_{0k} - \epsilon_{2k}) \end{aligned} \tag{9}$$

where *k* is the subchannel index, and the + signs stand for odd subchannels and - signs for even ones. The phase responses ζ_{0k} and ζ_{2k} (amplitude response ϵ_{0k} and ϵ_{2k}) represent the lower and upper edge of *k*th subchannel passband resposes, respectively. In the Case 1, ϵ_{1k} and ζ_{1k} are calculated at the subband center frequency using the equation (8), and are used as amplitude and phase equalizer coefficients.

IV. SIMULATION RESULTS AND ANALYTIC PERFORMANCE

The system model, depicted in Figure 2, consists of a cascade of a data source, up-sampling, pulse shaping filter, multipath channel, analysis and synthesis FB transformation with subband equalizer, receiver filter, down-sampling and decision device. The pulse shaping filters both in the transmitter and receiver are the square root raised cosine filter with a roll-off factor $\alpha = 35\%$. The filter bank designs in the simulation use roll-off 100% and overlapping factor $K = 5$.

The performance of the proposed FBEQ system utilizing AP-ASCET subband equalizer was tested using the Vehicular A channel model of ITU-R with about 2.5μs delay spread and 10 *M* symbol/s data rate, leading to 13.5 *MHz* signal bandwidth. Here it is assumed that ideal channel estimates are available. The channel fading was modeled as quasi-static model, i.e., the channel frequency response is constant during the transmission of each block. 2000 independent channel realizations were used to obtain average performance. MSE criterion was applied to solve the equalizer coefficients. Uncoded bit error rate (BER) as a function of received signal-to-noise ratio per bit E_b/N_0 is used as the measure of performance. The grey coding is assumed in bit mapping. This means that, when symbol errors occur, the received sample usually falls within one of the adjacent decision regions, causing only one bit error. The relation between the symbol error probability and bit error probability can be obtained as $P_{bit} = P_{symbol}/\log_2 M$, where *M* is order of symbol constellation.

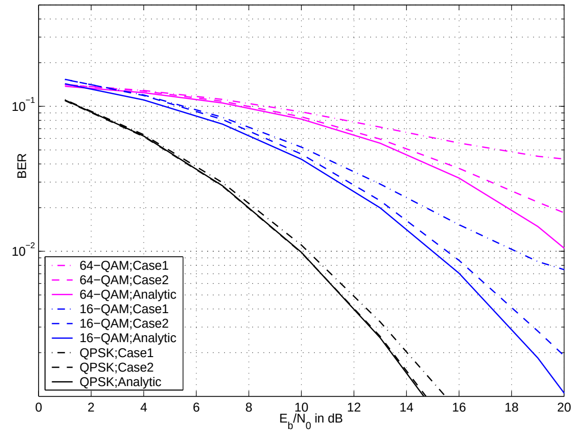

Fig. 4. FBEQ (256 subchannels) Case 1 and Case 2 BER performances

Figure 4 presents the BER performance with QPSK, 16-QAM and 64-QAM. The analytic curve shows the performance of ideal linear equalizer with MSE criterion. It is clear that the FBEQ system of Case 2 improves the performance significantly compared to Case 1. It is also shown that 256 subchannels are sufficient to achieve good performance. The resulting performance is quite close to the analytic BER bound. The performance improvement with Case 2 is a result of the fact that Case 2 is able to cope with frequency selective subband responses. In fact, in this example case, 128 subchannels are sufficient in the QPSK case, and possibly also in the 16-QAM case, when Case 2 equalizers are used.

V. FILTER BANK BASED EQUALIZER VS FRACTIONALLY SPACED EQUALIZER

Performance Comparison

A block diagram of a general FFT-based FSE system is depicted in Figure 5. A pulse shaping filter with the same roll-factor α=35% is used. A cyclic prefix was inserted to each block of signal to eliminate the inter-block interference (IBI). In case of ZF criterion, the system is IBI free as long as the prefix length is longer than the longest channel delay [3]. The receiver pulse shaping filter is combined with

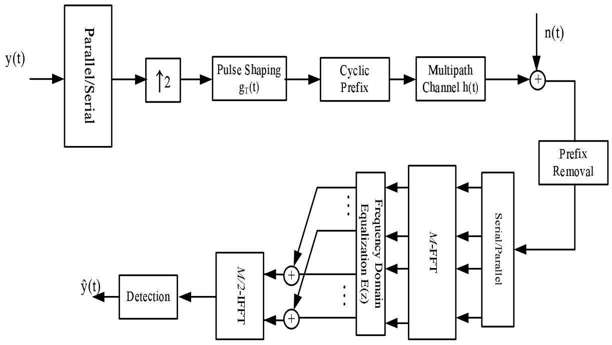

Fig. 5. General Fractionally Spaced Equalizer

the frequency domain equalizer. After the equalization, signal is down-sampled by performing the half length $M/2$ IDFT, i.e. the sampling rate is reduced to the symbol rate in frequency domain. When MSE criterion is used, the output of the equalizer can be expressed as [9]

$$\hat{Y}(k) = \frac{H^*[k]E[k] + H^*[k+M/2]E[k+M/2]}{|H[k]|^2 + |H[k+M/2]|^2 + N_0/\varepsilon_s} \tag{10}$$

for $0 \leq k \leq \frac{M}{2} - 1$, where $E(k)$ and $H(k)$ is length *M* DFTs of the equalizer and channel response, respectively.

The same ITU-R channel model was applied to the FSE system. Figure 6 compares the uncoded BER performance of the FBEQ and FSE systems. The BER performance of FSE does not depend on the DFT size. Case 1 FBEQ has a clear performance degradation compared to FSE with higher modulation. Meanwhile, FBEQ systems of Case 2 can perform equally well as FSE. In addition, in QPSK systems, Case 2 with 128 subchannels can achieve equally good performance. On the other hand, FBEQ has no guard interval overhead which FSE needs to combat IBI. This saves signal bandwidth, especially when the channel has severe delay spread.

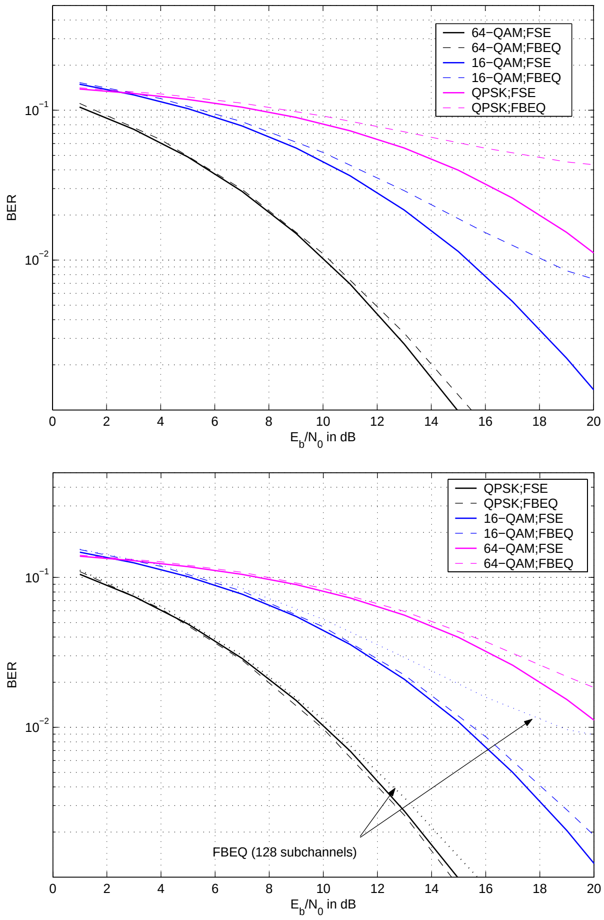

Fig. 6. FBEQ (256-subchannels) vs. FSE (256-DFT). a) Case 1; b) Case 2;

VI. Conclusion

We have applied the AP-ASCET equalizer idea from filter bank based multicarrier systems to frequency-domain equalization of single-carrier transmission systems. It was demonstrated that rather modest filter bank size is sufficient to achieve performance quite close to the analytic performance bound of linear equalizers with MSE criterion. In the studied example case, using Case 2 FBEQ, filter bank size of 128 is sufficient for QPSK, 256 is sufficient for both 16-QAM and 64-QAM compared to FSE performance. In an FSE system, preferably 512 subchannels are needed to get realistic guard interval overhead.

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