An Efficient Adaptive Space Time Coding Scheme for MIMO-OFDM Systems

Ioannis Dagres, Andreas Zalonis and Andreas Polydoros

Abstract— **Within the framework of the Stingray Project, a Hiperman compatible MIMO system, three architectures with different rate-one Space-Time Codes (STC) are presented and their performance assessed. Viewing beam-forming and Alamouti as the two extreme cases of the trade-offs between performance/ complexity/feedback-requirements, an efficient scheme is proposed that reduces complexity and feedback requirements at the mild cost of performance, thus posing as a good compromise candidate between the aforementioned two extremes. All examined schemes are rate-adaptive, based on periodically transmitted channel-state information (CSI) back to the transmitter, and they all employ turbo coding as the outer channel code option.**

Index Terms— **Adaptive, MIMO, OFDM, space time coding**

I. Introduction

THE increasing demand for better spectral utilization and higher QoS requirements motivate the design of increasingly more intelligent and agile communication systems. These are able to adapt and adjust (in real-time) the transmission parameters based on the instantaneous link quality for the ultimate goal of reaching, to the degree possible, the capacity limits of the underlying channel. Referred to as "Adaptive Modulation and Coding" (AMC), many algorithms have been proposed and their performance limits assessed [1-3] for single-input single-output (SISO) systems. In a multiple-input multiple-output (MIMO) system, direct extension of past proposed AMC methods is not straightforward. The corresponding STC choice must be taken into account in order to optimize overall system performance. The present work, developed under the Stingray IST Project, targets the development of a Hiperman [4] compatible, MIMO system for *fixed wireless access* (FWA) applications by use of flexible (that is, adaptive and reconfigurable) schemes, as described herein. These algorithms, that support adaptivity, exploit the *channel state information* (CSI) provided by a feedback channel from the receiver to the transmitter, driven by the needs of the supported service.

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The paper is structured as follows: the system model is presented in Section II, along with the code design approach. In Section III, a comparison of STC schemes is performed based on bit error rate (BER) and capacity, whereas in Section IV the throughput of the proposed system is evaluated.

II. MIMO-OFDM System Model and Design Procedure

A. MIMO-OFDM System Model

The general system model that employs M_T transmit and M_R receive antennas is shown in Fig. 1. Each channel is modeled as a *Finite Impulse Response* (FIR) filter with L taps. For L =1 the channel is flat fading, whereas for L>1 the channel is frequency selective. The impulse response of the channel from the transmit antenna i to the receive antenna j is denoted as $\mathbf{h}_{i,j}$ (with $h_{i,j}(l)$ as the l-th tap of the impulse response), which is assumed constant during one frame interval and changes independently in subsequent frames (quasi-static assumption). The information sequence $\mathbf{s}=\left[s_0,s_1,....,s_{N-1}\right]$ is encoded by a *space time frequency code* (STFC) to produce the N sequences $\mathbf{c}_k=[c_k^{(0)}\ c_k^{(1)}\ ..c_k^{(i)}.\ c_k^{(M_T-1)}]^T$, where $c_k^{(i)}$ is the coded symbol transmitted from the i-th antenna to the k-th OFDM sub-channel. All antennas transmit simultaneously. Viewing the system in the frequency domain, the channel for each sub-carrier consists of L channel-matrix taps of size $M_R\times M_T$, with a matrix-valued transfer function of:

$$\mathbf{H}(e^{j2\pi\theta})=\sum_{l=0}^{L-1}\begin{bmatrix} h_{0,0}(l) & \dots & h_{0,M_R-1}(l) \\ : & . & : \\ h_{M_T-1,0}(l) & \dots & h_{M_T-1,M_R-1}(l)\end{bmatrix}^T e^{-j2\pi l\theta}, \tag{1}$$

$0\le\theta<1$

The received data vector in the frequency domain is:

$$\mathbf{r}_k=\begin{bmatrix} H_k^{0,0} & \dots & H_k^{M_T-1,0} \\ : & . & : \\ H_k^{0,M_R-1} & \dots & H_k^{M_T-1,M_R-1}\end{bmatrix}\mathbf{c}_k+\mathbf{n}_k, \tag{2}$$

$k=0,1,....,N-1$

where $H_k^{i,j}$ is the frequency response of the channel $\mathbf{h}_{i,j}$ at sub-carrier k and $\mathbf{n}_k$ is a complex valued additive white Gaussian noise with $E\{\mathbf{n}_k\mathbf{n}_l^H\}=\sigma_n^2\mathbf{I}_{M_R}\delta[k-l]$.

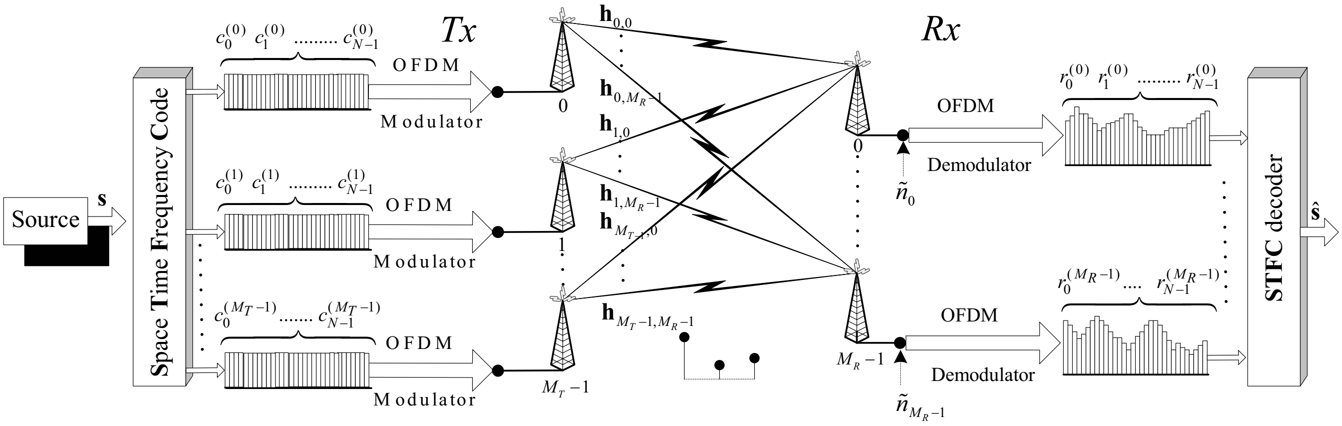

Fig. 1. Generic System Model

B. Code Design Approach

In [5], design criteria were first established for STFC. Direct design based on the derived criteria leads to very complex systems. In order to reduce complexity, common procedure is to use a low-complexity inner code (STC) that transforms the MIMO channel into a SISO, and then employ a strong outer code designed for SISO channels. The outer code will eventually gain back the frequency diversity loss from the use of a conventional STC per sub-carrier. Design separation into these two entities also agrees with the intuition provided by information theory. In fact, [6] shows that for a delay-limited, block-fading channel and for a fixed transmission rate, an optimal scheme that minimizes the outage probability under a constraint on the average transmitted power consists of a standard "Gaussian" code for an AWGN channel, followed by a suitable beam-former that may vary from block to block. Thus, the outer code provides the coding gain and can be a convolutional, turbo, Low Density Parity Check (LDPC) code, block code, etc. For the inner code, the options under consideration are: (a) *beam-forming,* the optimal solution [7] for energy allocation in a N_T x1 system with perfect channel knowledge at the transmitter side, whereby the same symbol is transmitted from both antennas multiplied by an appropriate weight factor in order to get the maximum achievable gain for each sub-carrier; (b) *Alamouti's STBC* [8], where for each OFDM symbol period, two OFDM signals are simultaneously transmitted from the two antennas; and (c) *Transmit Selection Diversity (TSD)*, where the assignment of energy, for each sub-carrier to the antenna, is associated with the largest channel gain at the receiver.

C. Adaptivity Features

The transmission (*Tx*) parameters are suitably selected over a specific time window in order to optimize system performance as the environment (physical channel) changes over time. The adjustment rate, i.e., the rate at which the system is allowed to change the *Tx* parameters, is once per frame (one frame=178 OFDM symbols) and the adjustable *Tx* parameters are {code rate, modulation size}. The optimization procedure is to choose the set which maximizes the system throughput for a given QoS level (here, BER). In a coded frequency-selective OFDM system, exact performance is not easily computed. Even for a frequency-flat channel, the selection is based on simulation curves (BER vs. average SNR) rather than coded-performance evaluation or closed-form expressions. Here, because of the MIMO diversity gain, frequency selectivity (or SNR variation) across the OFDM tones is limited. This is why bit-loading, studied extensively within Stingray project [9], provides marginal performance gain. This is also why we may choose the *effective average* SNR (ESNR) at the receiver (meaning the average SNR at the demodulator after ST decoding) as the criterion for choosing the appropriate parameter values. Performance simulation curves, based on the ESNR, form the basis for the formation of a Table (*Tx* Mode Table--TMT), which consists of ESNR regions and code-rate/modulation size sets for all the specified BER points supported by the system.

III. Performance Comparison of STC Candidates

First, the TSD scheme for a M_T x M_R MIMO-OFDM system is described. Assume transmission from one antenna per subcarrier, and let *T(k)* be a vector of length *N* denoting the transmit antenna for the *k*th sub-carrier. The received data vector in the frequency domain is:

$$\mathbf{r}_k=\sqrt{E_s}\begin{bmatrix} H_k^{T(k),0} \\ \vdots \\ H_k^{T(k),M_R-1}\end{bmatrix} c_k^{T(k)}+\mathbf{n}_k=\sqrt{E_s}\begin{bmatrix} H_k^{T(k),0} \\ \vdots \\ H_k^{T(k),M_R-1}\end{bmatrix} s_k+\mathbf{n}_k, \tag{3}$$

$k=0,1,.....,N-1$

With Maximal-Ratio Combining (MRC) at the receiver and for perfect CSI, the output is

$$\begin{aligned} MRC_k^{out} &= \left[H_k^{T(k),0} \quad H_k^{T(k),1} \quad \dots \quad H_k^{T(k),M_R-1}\right]^* \mathbf{r}_k = \\ &= \sqrt{E_s}\left(\sum_{m=1}^{M_R-1}\left|H_k^{T(k),m}\right|^2\right) s_k + \sum_{m=1}^{M_R-1}\left(H_k^{T(k),m}\right)^* n_k^m, k=0,1,.....,N-1 \end{aligned} \tag{4}$$

The received SNR of the equivalent SISO channel at subcarrier k is:

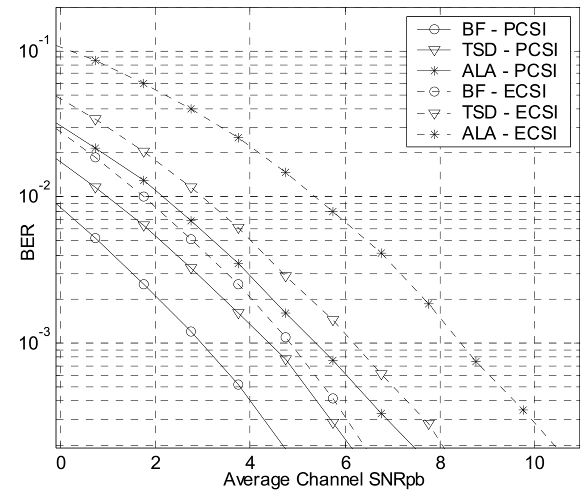

Fig. 2. STC's BER performance for perfect/estimated CSI (PCSI/ECSI) and 4QAM constellation

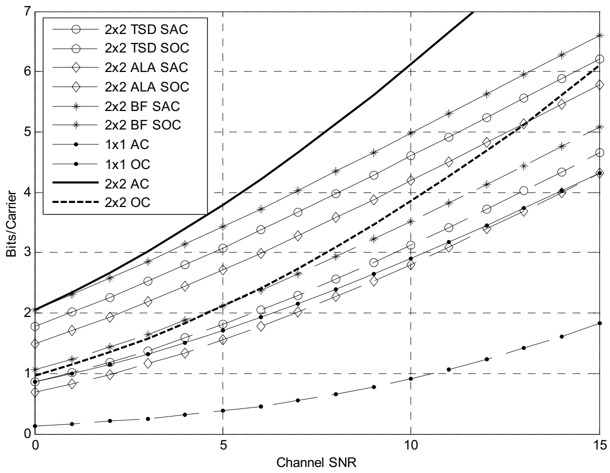

Fig. 3. System Average Capacity and System 1% Outage Capacity of different STC options

$$SNR_k = \frac{\left(\sum_{m=1}^{M_R-1}\left|H_k^{T(k),m}\right|^2\right)^2 E_s}{\left(\sum_{m=1}^{M_R-1}\left|H_k^{T(k),m}\right|^2\right)N_0} = \frac{\left(\sum_{m=1}^{M_R-1}\left|H_k^{T(k),m}\right|^2\right)E_s}{N_0} \tag{5}$$

where $N_0 = 2n_k$ (the same for all subcarriers)

In order to maximize it, *T(k)* must be chosen as

$$T(k) = \arg\max_{t=0,\dots,M_T-1}\left\{\sum_{m=0}^{M_R-1}\left|H_k^{t,m}\right|^2\right\}, \qquad k = 0,\dots,N-1 \tag{6}$$

This is the needed information sent to the transmitter through the feedback channel. At the receiver, zero-forcing is used to demodulate the signal after the combining. The demodulated symbol is

$$s_k^{de\bmod} = \arg\min_{s_i = s_1 \dots s_{\log_2(M_c)}} \left\{\left|s_i - \hat{s}_k\right|\right\}, \quad k = 0,1,\dots,N-1 \tag{7}$$

where M_c is the constellation size and

$$\hat{s}_k = \frac{MRC_k^{out}}{\left(\sum_{m=1}^{M_R-1}\left|H_k^{T(k),m}\right|^2\right)}.$$

Each of the three STC schemes can be treated as an ordinary OFDM SISO system producing (ideally) N independent Gaussian channels. This can be called the Effective SISO-OFDM channel. For Stingray system (2x2) the corresponding ESNR per carrier is:

$$\textit{For TSD:}\; ESNR_k = \frac{\left(\left|H_k^{T(k),0}\right|^2 + \left|H_k^{T(k),1}\right|^2\right)E_s}{N_0} \tag{8}$$

$$\textit{For Alamouti:}\; ESNR_k = \frac{\left(\left|H_k^{0,0}\right|^2 + \left|H_k^{0,1}\right|^2 + \left|H_k^{1,0}\right|^2 + \left|H_k^{1,1}\right|^2\right)E_s}{2N_0} \tag{9}$$

$$\textit{For beam-forming:}\; ESNR_k = \frac{\lambda_k^{\max} E_s}{N_0} \tag{10}$$

where $\lambda_k^{\max}$ is the squared singular value of the 2x2 channel matrix $\begin{bmatrix} H_k^{00} & H_k^{10} \\ H_k^{01} & H_k^{11} \end{bmatrix}$ and $k = 0,1,\dots\dots,N-1$.

In Fig. 2 BER versus Average Channel SNR per bit, simulation curves are presented, for all inner code schemes, and 4-QAM constellation. Both perfect and estimated CSI scenarios are presented. The channel estimation procedure uses preamble structure and is described in [10]. For all simulation results, path delays and the power of channel taps have been selected according to the SUI-4 model for intermediate environment conditions [11]. Note that the channel SNR is independent of the used STC. Having normalized each *Tx-Rx* path to unit average energy, the average channel SNR is simply equal to one over the power of the noise component of any one of the receivers. Alamouti is the most sensitive scheme to estimation errors. This is expected since the errors of all four channel taps are used in the decoding procedure.

Based on ESNR, semi-analytic computation of the average and outage capacity for the effective channel is possible, in order to evaluate a performance upper bound of these inner codes. An outer code will then attempt to approach this bound.

To that effect, we note that the average capacity is given by

$$C^E = E\left\{\frac{1}{N}\sum_{k=1}^{N}\log_2\left(1 + ESNR_k\right)\right\} \text{ bits/carrier} \tag{11}$$

where the expectation operator is over the effective random channel.

The capacity results presented in this Section are based on "quasi-static" assumption mentioned before. For each burst it is also assumed that a sufficiently large number of bits are transmitted, in order for the standard infinite time horizon of information theory to be meaningful. In Fig. 3, the average capacity and the 1% outage capacity of the three competing systems is presented.

For comparison reasons, the average and outage capacity of 2x2 and 1x1 systems with no channel knowledge at the

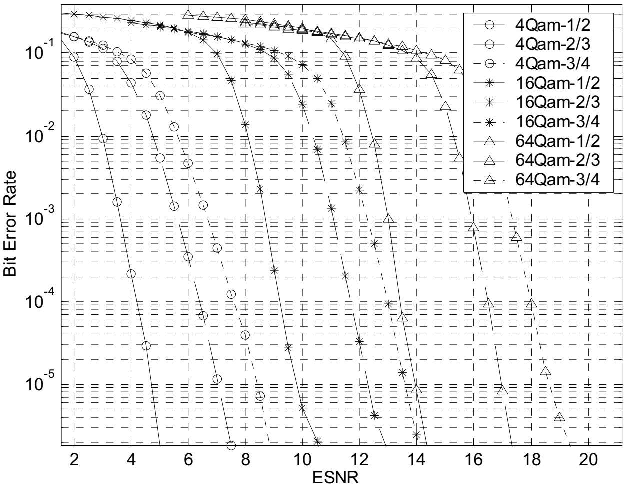

Fig. 4. TSD-Turbo system performance results

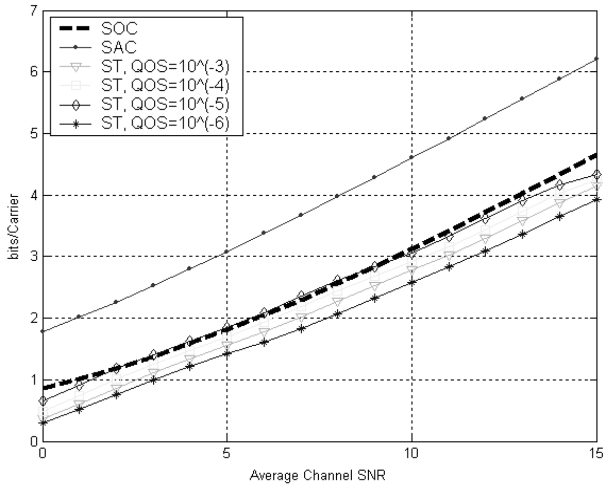

Fig. 5. TSD-Turbo system throughput (perfect CSI-SNR estimation)

transmitter and perfect knowledge at the receiver are presented. The following definitions are used:

SAC (System Average Capacity)**:** Equivalent to the mean or ergodic capacity [12], applied to the effective channel. It serves as an upper bound of systems with boundless complexity or latency that use a specific inner code. This is computed semi-analytically, by averaging over 10000 channel realizations.

SOC (System Outage Capacity)**:** This is the 1% outage capacity of the effective channel.

AC and **OC**: The Average Capacity and Outage Capacity of the real channel.

From Fig. 3 it is clear that all three systems have the same capacity vs. SNR slope. This result is expected, since the rate of all three systems is one. A system that tries to exploit all the multiplexing gain offered by the 2x2 channel may be expected to have a slope similar to the capacity of the real channel (AC, OC).

It is evident that the cost of not targeting full multiplexing is less throughput than what MIMO channels can support. On the other hand, the goal of high throughput has a price of either enhanced feedback requirements or higher complexity. Comparing the three candidate schemes, beam-forming is a high-complexity solution with considerable feedback requirements, while Alamouti has low-complexity with no feedback requirement. TSD has lower complexity than Alamouti, whereas in comparison with beam-forming, has a minimal feedback requirement. The gain over Alamouti is approximately 1.2 dB, while the loss compared to beam-forming is another 1.2 dB.

IV. Final System Performance Results

The scope of this Section is throughput evaluation of the adaptive system, given specific inner and outer coding choices. The selected inner code is TSD and the outer code is a parallel-concatenated turbo coding scheme with variable rate via three puncture patterns (1/2,2/3,3/4) [13]. The recursive systematic code polynomial used is $(13,15)_{oct}$.

In order to build the TMT, the simulation process proceeds using the ESNR for measuring the system performance and not the average channel SNR. In Fig. 4, BER performance versus ESNR is plotted for all *Tx* mode pairs. Since we assume perfect channel and noise power knowledge, ESNR is in fact the real prevailing SNR. This turns out to be a good performance criterion, since the outer (turbo) code performance is very close to that achieved on an AWGN

TABLE I
TRANSMISSION MODE TABLE IN THE CASE OF PERFECT CHANNEL-SNR ESTIMATION

Thr/put BER	4QAM-1/2	4QAM 2/3	4QAM 3/4	16QAM 1/2
10^-3	>3.6	>5.6	>6.6	>8.6
10^-4	>4.2	>6.4	>7.6	>9.2
10^-5	>4.7	>7	>8.4	>9.8
10^-6	>5	>7.6	>8.9	>10.7

Thr/put BER	16QAM-2/3	16QAM 3/4	64QAM 2/3	64QAM 3/4
10^-3	>11	>12.2	>15.9	>17.3
10^-4	>11.7	>12.9	>16.5	>17.9
10^-5	>12.3	>13.6	>16.9	>18.6
10^-6	>13.1	>14.5	>17.5	>19.8

channel with equivalent SNR. Performance on AWGN environment can be viewed as an upper bound for any given SNR, since these codes are designed for AWGN channels. Based on those curves, and assuming perfect channel-SNR estimation at the receiver, the derived TMT is presented in Table I.

By use of this Table, the average system throughput (ST) for various BER requirements is presented in Fig. 5. The system outage capacity (1%) is a good measure of throughput evaluation of the system and is also plotted in the same figure. The average capacity is also plotted, in order to show the difference from the performance upper bound. The system

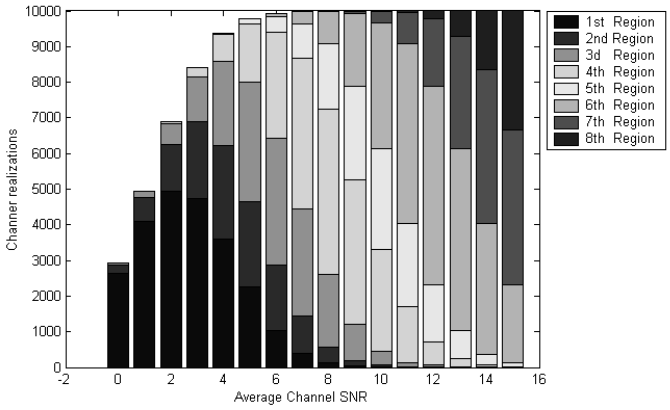

Fig. 6. *Tx* modes histogram

throughput is very close to the 1% outage capacity, but it is 5-7 dB away from the performance limit, depending on the BER level. Since the system is adaptive, probably 1% outage is not a suitable performance target for this system. The SNR gain achieved by going from one BER level to the next is about 0.8 dB. This marginal gain is expected due to the performance behavior of turbo codes (very steep performance curves at BER regions of interest). In Fig. 6, the usage histogram of the regions (or different *Tx* modes) is presented for 10^4 channel realizations versus average channel SNR. The usage of each region is spread over an 8dB SNR. For example, for an average channel SNR of 6 dB, the system is using six *Tx* modes.

V. Conclusion

A (capacity-wise) performance comparison of three rate-1 candidate STC schemes has been presented. The results indicate that all three systems have the same capacity vs. SNR slope, an expected result since the multiplexing gain of all is one. The price to pay for not aiming at full multiplexing is the inefficient utilization of the large throughputs that MIMO channels can support. However, aiming at high throughputs raises the cost of either feedback requirements or system complexity. TSD is thus a good compromise between system complexity and throughput gain. The performance lies in the middle, between the best and more complex rate-1 solution (*beam-forming)*, and the least complex solution (blind *Alamouti)*.

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