Adaptive Bit-Interleaved Coded Irregular Modulation

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***Abstract*—We consider a simple method to improve the adaptiveness and flexibility of bit-interleaved coded modulation (BICM) for various channel models. With state-of-the art adaptive transmission techniques, the raw adaptation of the data rate to the channel characteristics is usually done by selecting an appropriate signal constellation and the fine adaptation is done by changing the code rate. To enable a fine adaptation also with the modulation, irregular modulation schemes have been introduced in [1]. With irregular modulation, different signal constellations may be used within one code block, even if only the average channel state is known at the transmitter. In this paper, irregular modulation schemes are analyzed using EXIT charts, capacity, error bounds and error rate simulations.**

I. Introduction

In most wireless mobile communication systems, the propagation environment and thus the channel characteristics are changing over the time. To maximize the achievable data rate over a time varying channel, the transmission scheme should be adapted using channel estimates available at the transmitter. The adaptation may be done by varying the transmitted power level, the size of the signal constellation or the code rate, [2] and references therein.

We will focus on the adaptation of the signal constellation with the knowledge of only the average channel quality at the transmitter. With state-of-the-art Adaptive Modulation and Coding (AMC), as included in several recent wireless communication standards, the channel code rate can be varied in small steps using appropriate puncturing patterns. However, the choice of the signal constellation allows only a very raw adaptation to the channel quality since the granularity is at least one bit per symbol.

If we use irregular modulation schemes, where different signal constellations may be used within one block of coded bits, a fractional average number of bits per complex symbol may be obtained [1]. Thus, a highly flexible adaptation to the channel quality is now possible with the modulation.

Different signal constellations within one code block have already been widely investigated for fading channels [2][3] and OFDM systems, where according to the channel quality of a sub-carrier, a corresponding signal constellation is chosen [4][5]. For this, the instantaneous channel state should be available at the transmitter. However, we focus on the scenario where only the average channel quality of one data frame is known at the transmitter. We may assign different signal constellations to maximize the achievable rate even if the channel quality is constant.

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As basic system structure, we consider bit-interleaved coded modulation (BICM) [6], i.e. a serial concatenation of an encoder, a bit-interleaver and a mapper, with optional iterative decoding and demapping at the receiver (so called BICM-ID system [7][8]). With irregular modulation, we obtain bit-interleaved coded irregular modulation (BICIM).

First, we will describe the system model. Then, we will construct EXIT charts [9] and investigate the capacity of optimized irregular modulation schemes for different scenarios. Error bounds for irregular modulation based on an Euclidean distance spectrum are derived and simulation results finally demonstrate the performance.

II. System model

We consider a bit-interleaved coded modulation (BICM) system as depicted in Fig. 1 with an optional feedback from the channel decoder to the demapper for iterative decoding and demapping (so called BICM-ID system).

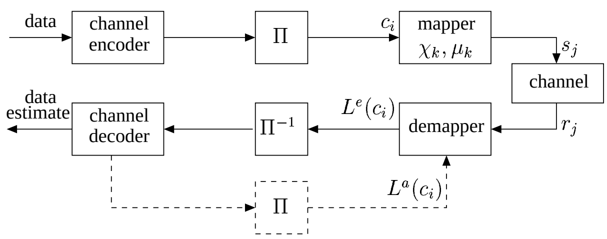


Fig. 1. BICM-ID system model.

A sequence of information bits is encoded by a binary code $\mathcal{C}$ and bit-interleaved by a random interleaver Π. The interleaved sequence of N code bits is divided in P subblocks of length $\alpha_k N$, as illustrated in Fig. 2, where α_k is the ratio of the k-th subblock, $k = 1, \ldots, P$. Each subblock may be mapped to a different 2^{m_k}-ary signal constellation χ_k, with $m_k \in \mathbb{N}$ coded bits per symbol, using different one-to-one binary labeling maps μ_k. Then, an average of $m_{IR} \in \mathbb{Q}$ coded bits per symbol is obtained.

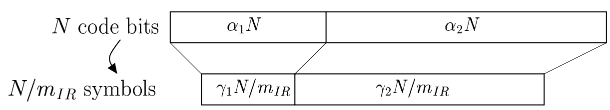


Fig. 2. Construction of irregular modulation schemes.

The ratios α_k must satisfy the conditions

$$\sum_{k=1}^{P} \alpha_k = 1, \qquad \frac{1}{\sum_{k=1}^{P} \alpha_k / m_k} = m_{IR}, \text{ and } \alpha_k \in [0,1], \forall k. \tag{1}$$

The second condition follows from the computation of the symbol block length $N/m_{IR} = \sum_{k=1}^{P} N \cdot \alpha_k / m_k$. The ratios $\gamma_k = \alpha_k \cdot m_{IR}/m_k$ may be used instead of the ratios α_k to determine the segmentation of the code block on a symbol level instead of on a bit level, as illustrated in Fig. 2. Then, the following conditions should be satisfied:

$$\sum_{k=1}^{P} \gamma_k = 1, \qquad \sum_{k=1}^{P} \gamma_k m_k = m_{IR}, \text{ and } \gamma_k \in [0,1], \forall k. \tag{2}$$

The following discussion will include common channel models for which the discrete-time received signal is in general expressed as

$$r_j = h \cdot s_j + n_j. \tag{3}$$

The noise samples n_j are complex-valued, independent and identically distributed (i.i.d.), with zero mean and noise variance σ_n^2 for both real and imaginary part, i.e. $n_j \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_n^2)$. By setting the fading coefficient $h = 1$ in (3), we obtain the standard AWGN channel model. A Rayleigh fading channel model is obtained if the fading coefficients are distributed according to $h \sim \mathcal{N}_{\mathbb{C}}(0, 1/2)$. Depending on whether we assume that the channel quality changes on a slow or fast time scale with respect to the signaling rate, we use a block fading or a symbol (or fully interleaved) fading channel model, where h changes independently after each transmitted block or symbol, respectively. We focus on the scenario where ideal channel state information is assumed at the receiver but only the average channel state of one code block is known at the transmitter.

At the receiver, the APP demapper processes each of the P subblock separately and forwards the soft estimates in terms of log-likelihood ratios (LLRs) $L(c_i) = \log(P(c_i = 0)/P(c_i = 1))$ about the coded bits to the deinterleaver and channel decoder. If a feedback from the channel decoder to the demapper is implemented (BICM-ID system), iterative decoding and demapping can be applied to improve the system performance. Without feedback, Gray mapping is optimal [6]. If the feedback is implemented, other mappings optimized for this system should be used [10].

The following investigations for the basic BICM system can be extended to e.g. MIMO or equalizer systems.

III. EXIT Charts of Irregular Modulation

EXIT charts are widely used to analyze the convergence of iterative systems [9]. The basic idea is to visualize the density evolution of the extrinsic LLRs over the decoding iterations. Instead of tracking the whole density function, the mutual information $I(C; L)$ between the coded bits C at the transmitter and the LLRs L at the receiver is used as the single variable to characterize the distribution of the LLRs.

We will focus on the EXIT function of the demapper with irregular modulation. The EXIT function of the demapper is the average extrinsic mutual information $I_E(I_A, E_b/N_0)$ coming out of the demapper as a function of the average a priori mutual information I_A going into the demapper and the channel quality E_b/N_0 [11], where

$$I_E = \frac{1}{N} \sum_{i=1}^{N} I(C_i; L_i^e) \quad \text{and} \quad I_A = \frac{1}{N} \sum_{i=1}^{N} I(C_i; L_i^a). \tag{4}$$

If the probability density function of the extrinsic LLRs is both symmetric and consistent, I_E (and in a similar way I_A) is easily computed with [12]:

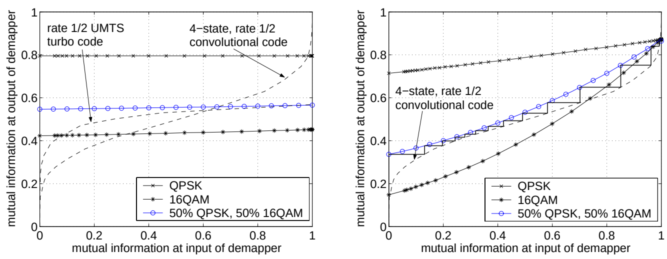
$$I_E = 1 - \frac{1}{N} \sum_{i=1}^{N} \log_2(1 + \mathrm{e}^{c_i \cdot L^e(c_i)}). \tag{5}$$

With (5) or by using the results of [11], the EXIT function I_E of the demapper with irregular modulation is the linear combination of the EXIT functions I_{E_k} of the P subblocks of length $\alpha_k N$ bits:

$$I_E = \sum_{k=1}^{P} \alpha_k \cdot \left(\frac{1}{\alpha_k N} \sum_{i=1}^{\alpha_k N} I(C_{ki}; L_{ki}) \right) = \sum_{k=1}^{P} \alpha_k \cdot I_{E_k}, \tag{6}$$

where C_{ki} and L_{ki} are the ith values of the kth subblock. The desired shape of the EXIT function is obtained by setting the ratios α_k in an appropriate way. The optimization of the ratios α_k can be formulated as a linear programming problem [12].

Fig. 3 depicts as example the EXIT functions of QPSK, 16QAM and a combination of both, with Gray mapping and mappings optimized for iterative demapping and decoding in a BICM-ID system [10][13]. A strong channel code (here a turbo code) is best suited for Gray mapping, a weak channel code (here a 4-state convolutional code) is best suited in combination with a mapping optimized for the BICM-ID system.



(a) (b)

Fig. 3. EXIT functions of regular and irregular modulation, AWGN channel, $E_s/N_0 = 4$dB. (a) Gray mapping (b) Anti-Gray mapping for QPSK and $I16$ [13] mapping for 16QAM, optimized for iterative demapping and decoding in a BICM-ID system.

IV. Capacity of Irregular Modulation

The capacity C of the signal constellation χ is given by the average mutual information [6]

$$C = \frac{m}{N} \sum_{j=1}^{N/m} I(S_j, R_j). \tag{7}$$

With BICM, the system capacity is only slightly lower when Gray mapping is applied [6]. With BICM-ID, the full signal set capacity C can be achieved with all mappings.

The capacity C_{IR} with irregular modulation is the linear combination of the capacities C_k of the P subblocks of length $\gamma_k N/m_{IR}$ symbols:

$$C_{IR} = \sum_{k=1}^{P} \gamma_k \cdot \left(\frac{m_{IR}}{\gamma_k N} \sum_{j=1}^{\gamma_k N/m_{IR}} I(S_{kj}, R_{kj}) \right) = \sum_{k=1}^{P} \gamma_k \cdot C_k, \tag{8}$$

where S_{kj} and R_{kj} are the jth values of the kth subblock. The capacity is closely related to the EXIT chart since the area under the EXIT function of the rate-1 demapper corresponds to C/m if APP demapping is applied [11].

As mentioned in the introduction, the power level, the size of the signal constellation or the code rate should be adapted to the channel conditions to maximize the achievable data rate. Within one frame, power loading and bit loading strategies are usually applied, where the power and the size of the signal constellation are adapted to the instantaneous channel quality. With an equal power distribution and in an AWGN, block fading or symbol fading channel with only average channel state information at the transmitter, equal bit loading within one block would be optimal. Thus, in these cases, irregular modulation schemes may lead to a suboptimum solution since the bit loading within one frame may not be constant. However, we will see that the loss is negligible for some system setups and that the advantages of irregular modulation may predominate.

In the following, we will investigate with simple examples the behavior of irregular modulation schemes in terms of achievable data rates, i.e. in terms of achievable number of information bits per channel use.

A. Continuous code rate, constant number of bits per symbol

First, we vary the channel code rate in function of the channel quality in arbitrary small steps and fix the modulation scheme. If irregular modulation schemes are used, the ratios γ_k, or equivalently, the ratios α_k are fixed and independent from the channel quality.

Fig. 4 shows for this scenario the achievable rates of regular and irregular modulation for an AWGN and fully interleaved fading channel with only average channel state information at the transmitter. As example, irregular modulation schemes with different constant ratios γ_k are investigated.

From an information theoretical point of view, the best would be here to always use 16QAM and just vary the code rate. However, this may not be desired in real world systems, because of e.g. sensitivity to nonlinear distortions or complexity. Irregular modulation schemes offer the possibility to achieve any maximum transmission rate between 2 (QPSK) and 4 (16QAM) bits per channel use in this example with only small performance degradation, especially in combination with low to moderate channel code rates.

In Fig. 4(a), the effect of an optimized power assignment to the different signal constellations is investigated for an AWGN channel with irregular modulation and $\gamma_{QPSK} = \gamma_{16QAM} = 0.5$. In general, more power should be assigned to large signal constellations and less power to small signal constellations. The gain is significant (about 1.3dB) only for data rates of 2 bits per channel use and more, i.e. for a channel code with rate $R > 2/3$ in this example.

In an AWGN channel and block fading channel, the placement of the symbols belonging to different signal constellations is not relevant due to the subsequent interleaver. In a symbol fading channel however, symbols from high order signal constellations should be transmitted when the channel quality is better and symbols from low order signal constellations when the channel quality is worse, according to optimized bit loading strategies [2]. Fig. 4(b) depicts for irregular modulation with $\gamma_{QPSK} = \gamma_{16QAM} = 0.5$ and equal power distribution the achievable rates if the instantaneous instead of only the average channel state information is available at the transmitter and if the symbols are placed accordingly. This scenario may be realistic in an OFDM system, where the estimated quality of the subcareers may be available at the transmitter. The gain is quite substantial (about 3dB) for data rates above 1.5 bits per channel use, i.e. for a channel code with rate $R > 1/2$ in this example.

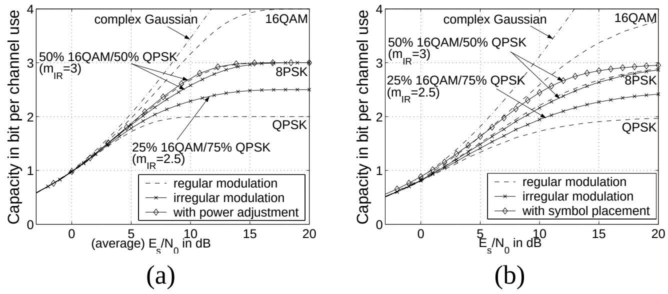


Fig. 4. Capacity of regular and irregular modulation for different values of m and m_{IR} (number of coded bits per symbol). (a) AWGN channel, comparison with optimized power loading; (b) fully interleaved fading channel, comparison with optimized placement of symbols belonging to different signal constellations, equal power distribution.

B. Constant code rate, continuous number of bits per symbol

Now we allow only a small number of code rates and use irregular modulation to adapt the data rate to the channel characteristics. To maximize m_{IR} and thus the achievable data rate, the ratios γ_k should be optimized for every channel state. If more than two signal constellations are combined, it is useful to describe the optimization as a linear programming problem. The problem can be cast into

$$\text{maximize} \quad m_{IR} = \textstyle\sum_{k=1}^{P} \gamma_k m_k, \tag{9}$$

$$\text{subject to} \quad \textstyle\sum_{k=1}^{P} (C_k - R m_k) \cdot \gamma_k \geq 0, \tag{10}$$

$$\textstyle\sum_{k=1}^{P} \gamma_k = 1, \qquad \gamma_k \in [0,1], \forall k. \tag{11}$$

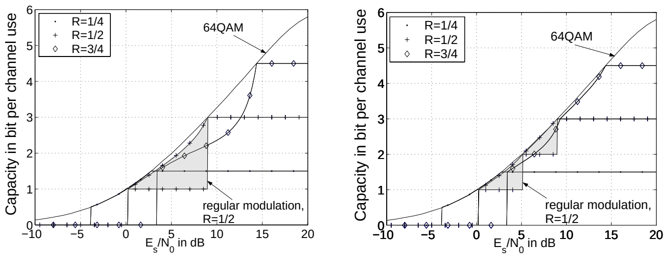
The first condition follows from the constraint of a fixed channel code rate R, stating that

$$C_{IR} = \sum_{k=1}^{P} C_k \gamma_k \geq R \cdot \sum_{k=1}^{P} m_k \gamma_k = R \cdot m_{IR}. \tag{12}$$

The achievable data rates with three fixed code rates are shown in Fig. 5(a), where QPSK and 64QAM modulation are combined, and in Fig. 5(b), where QPSK, 16QAM and 64QAM modulation are used. The capacity of 64QAM, valid for highly adaptive code rates, is depicted as reference.

With irregular modulation schemes, we observe that especially with low rates or a high number of underlying modulation schemes, close to optimum data rates may be achieved. A significant rate loss occurs with large code rates and if only 64QAM and QPSK are combined, e.g. in Fig. 5(a) with $R = 3/4$.

The staircase trajectories in Fig. 5 indicate the maximum achievable rates with regular modulation and a code rate $R = 1/2$. The shaded area is the rate gain of irregular over regular modulation with $R = 1/2$.



(a) (b)

Fig. 5. Achievable data rates of irregular modulation with AWGN channel, three fixed channel code rates and optimized ratios γ_k for the combination of (a) two signal sets (QPSK and 64QAM) and (b) three signal sets (QPSK, 16QAM, 64QAM). Equal power loading, no symbol placement. The shaded area is the rate gain of irregular over regular modulation for a fixed code rate $R = 1/2$.

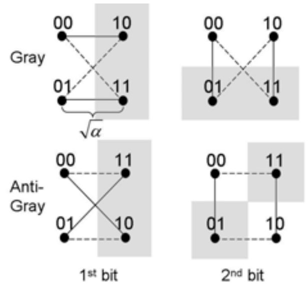
V. Error probability analysis

We use the Euclidean distance spectrum [14] to derive error bounds for irregular modulation. The Euclidean distance spectrum is a precise way to characterize a binary labeling map, similar to the characterization of a channel code through its Hamming distance spectrum. Let us define the three sets D^{ex}, D and Λ:

The set $D^{ex} = \{d_1^{ex}, \ldots, d_n^{ex}\}$ is defined as the set of all possible distinct (*ex*purgated) Euclidean distances between any two distinct signal points of the signal set χ. Let $D = \{d_1, \ldots, d_{n'}\} \supset D^{ex}$ denote the set of all (not necessarily distinct) Euclidean distances between the signal points s_j and $\hat{s}_j$, where $\hat{s}_j$ is defined as a symbol whose bit-label differs in the i-th bit position from the bit-label of $\hat{s}_j$. Finally, let λ_l denote the frequency of the distance d_l^{ex} in the set D and $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$. D^{ex} depends only on the signal constellation, whereas D and Λ characterize the bit mapping. The Euclidean distance spectrum lists the frequency λ_l of the Euclidean distances d_l^{ex}.

We distinguish between the scenarios of no a priori information and ideal a priori information at the demapper. No a priori information is available during the initial demapping step or if the feedback from the channel decoder to the demapper is not implemented, as in the standard BICM system. The case of ideal a priori information (so-called genie or error free feedback case) is a lower bound on the performance at high SNR after several decoder/demapper iterations in the BICM-ID system.

Fig. 6(a) depicts as example the distances from the set D for the two basic QPSK Gray and anti-Gray mappings and for bit position $i = 1$ and $i = 2$. The number of possible distances is reduced with a priori information (solid vs. dashed lines). Table in Fig. 6(b) gives the corresponding distance spectrum. For a more detailed explanation, we refer to [8][14].



(a)

Λ	λ_1	λ_2
No a priori:		
Gray	4	4
Anti-Gray	6	2
Ideal a priori:		
Gray	4	0
Anti-Gray	2	2

(b)

Fig. 6. (a) QPSK mappings with distances from set D. Dashed lines: relevant only without a priori information. Shaded regions: bit i has value 1. (b) Distance spectrum: frequency λ_1 and λ_2 of distances $d_1^{ex} = \sqrt{\alpha}$ and $d_2^{ex} = \sqrt{2\alpha}$, respectively.

The BICM union bound of the probability of bit error for convolutional codes of rate $R = K/N$ is given by [6]

$$P_b \leq \frac{1}{K} \sum_{d=d_{min}}^{\infty} W_I(d) f(d, \mu, \chi), \qquad (13)$$

where $W_I(d)$ denotes the total input weigh of error events at Hamming distance d and d_{min} is the minimum Hamming distance of the code. $f(d, \mu, \chi)$ is the pairwise error probability (PEP) and depends only on the Hamming distance d, the labeling map μ and the signal constellation χ.

Let Φ_Δ be the Laplace transform of the pdf of the metric difference $\Delta(s_j, \hat{s}_j) = |r_j - hs_j|^2 - |r_j - h\hat{s}_j|^2$ and $\psi(s)$ is the average of Φ_Δ taken over all possible metric differences $\Delta(s_j, \hat{s}_j)$. Then, the PEP is given by [15]

$$f(d, \mu, \chi) = P(\Delta \leq 0) = \frac{1}{2\pi j} \int_{\alpha - j\infty}^{\alpha + j\infty} (\psi(s))^d \frac{ds}{s}. \qquad (14)$$

For one specific combination $\Delta(s_j, \hat{s}_j)$, $\Phi_{\Delta(s_j, \hat{s}_j)}(s)$ is given for the general case of a Rician fading channel with Rice factor K by [15]:

$$\Phi_{\Delta(s_j, \hat{s}_j)}(s) = \frac{1+K}{(1+K) - s(N_0 s - 1) d_l^{ex^2}} \cdot \exp\left(\frac{K s(N_0 s - 1) d_l^{ex^2}}{(1+K) - s(N_0 s - 1) d_l^{ex^2}} \right), \qquad (15)$$

with the Euclidean distance $d_l^{ex} = |s_j - \hat{s}_j|$ from the set of possible distances D^{ex}. We set $K \to \infty$ for an AWGN and $K = 0$ for a fully interleaved fading channel. To obtain $\psi(s)$, we average over all possible distances d_l^{ex}, occurring with the frequencies $\lambda_l \in \Lambda$:

$$\psi(s) = \frac{1}{\sum_{l=1}^{n} \lambda_l} \sum_{l=1}^{n} \lambda_l \cdot \Phi_{\Delta(d_l^{ex})}(s). \qquad (16)$$

In an irregular modulation scheme, we average in addition over the P subblocks:

$$\psi_{IR}(s) = \sum_{k=1}^{P} \gamma_k \cdot \frac{1}{\sum_{l=1}^{n} \lambda_{kl}} \sum_{l=1}^{n} \lambda_{kl} \cdot \Phi_{\Delta(d_l^{ex})}(s), \qquad (17)$$

where λ_{kl} are the frequencies of the distances d_l^{ex} in the kth subblock. The PEP in (14) for irregular modulation is now evaluated using the Gauss-Chebyshev quadrature [15].

VI. Simulation Results

Fig. 7 depicts the bit error rate (BER) performance at a data rate of 1.5 bits per channel use and different combinations of modulation schemes and code rates. All code rates are obtained by puncturing from a rate $R = 1/3$ mother code. The channel is AWGN and the interleaver length is 10000 bits. Both the interleaver in the turbo code as well as the interleaver between the code and the mapper are random. A more elaborate interleaver design would improve the performance for all systems and would especially reduce the error floor of the turbo codes.

Two systems are investigated: First, a BICM system similar to the pragmatic approach presented in [16] is investigated, with Gray mapping and the UMTS standard turbo code. The feed-back polynomial is $(13)_o$, the feed-forward polynomial is $(15)_o$ in octal notation. The performance of the different coding and modulation schemes in Fig. 7 differs only in the error floor.

Second, a BICM-ID system with the $(7\ 5)_o$ convolutional code with decoding and demapping iterations is considered. Mappings optimized for ideal a priori information are used: Anti-Gray mapping for QPSK and $I16$ mapping for 16QAM [13]. The different coding and modulation schemes differ in the trade off between convergence at low SNR and low error bound. The irregular modulation scheme converges earlier but has a higher error floor then the 16QAM system. This trade-off can be adjusted by changing the mapping and the memory of the convolutional code. Notice the tight analytic error bounds.

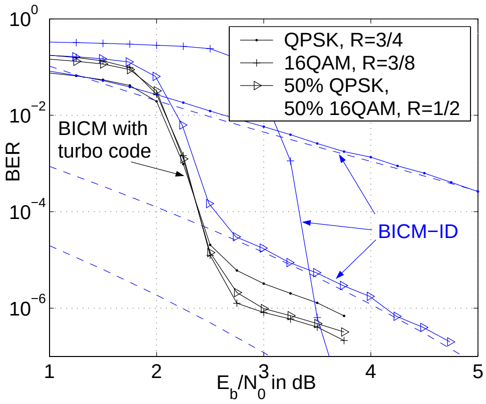


Fig. 7. BER with AWGN channel, different combinations of code rate and modulation scheme, 1.5 bits per channel use. BICM with UMTS turbo code and BICM-ID with 4-state convolutional code, 20^{th} iteration. $E_b/N_0 = E_s/N_0\ /\ (2^{m_{IR}} \cdot R)$.

VII. Conclusion

We investigated a bit-interleaved coded irregular modulation (BICIM) scheme, where different signal constellations may be used within one code block. The analysis included EXIT charts, capacity, error bounds and error rate simulations. We showed that the proposed system offers an additional possibility to adapt the transmission system to the channel quality and has a good performance as long as we use low or medium code rates. In particular, we can state that:

- BICIM is promising if only a limited amount of channel code rates are available or desired. The fine adaptation could be performed with irregular modulation.
- The additional complexity of BICIM is very low. The transmitter should support a certain amount of signal constellations and the rate at which the transmitter has to change its constellation is not excessive if only the average channel characteristics are considered.
- BICIM is well suited for the combination with bit loading and power loading schemes, if additional channel knowledge is available at the transmitter.
- BICIM offers better channel estimation possibilities [17].
- In addition, an irregular mapping scheme [1] may be used, where different mappings are applied within one data frame to optimize the convergence of the iterative decoding and demapping procedure in a BICM-ID system, similar to the effect of irregular codes.

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