# Soft Decoding of LT-Codes for Wireless Broadcast

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***Abstract*— This work investigates reliable wireless broadcast with asynchronous data access. A wireless broadcast system based on *LT-codes*, a realization of the recently introduced *Fountain codes*, is introduced. We review the traditional problem formalization for Fountain codes operating on erasure channels and generalize the framework to arbitrary types of channels coining the terminology and the concept of *information collecting*. The *sum-product algorithm* in order to exploit soft-information available at the wireless receiver for decoding of LT codes is applied. Simulation results on the AWGN, as well as the fading channel, show remarkable performance gains compared to traditional erasure decoding.**

## I. Introduction

Broadcasting in wireless environments faces the problem of receiver heterogeneity resulting in different receiving conditions which is experienced by different loss rates or different signal-to-noise ratios (SNRs) for individual receivers. Advanced adaptation schemes or other reliable transmission modes are usually not feasible in such environments due to the missing feedback channel in the broadcast environment. Furthermore, common broadcast schemes are not designed for the provision of asynchronous data services, *ie*, to tune into an ongoing session at any arbitrary time. Therefore, other means have been considered to provide reliable broadcast and asynchronous data access simutaneously. In [1] and [2] the performance of additional Forward Error Correction (FEC) was investigated to fulfill these requirements and, in [3], it was elaborated that a channel code with potentially limitless redundancy (*rateless*) is able to solve the reliable broadcast problem *and* asynchronous data access. Such a code is referred to as *Fountain code*. Practical approximations of Fountain codes have, for example, been proposed under the acronyms LT-Codes [4] and Raptor codes [5].

Originally, these codes were exclusively proposed for the reliable multicast problem over the wired Internet, and therefore, have almost exclusively been investigated on erasure channels. The loss behavior on the wired Internet is appropriately modeled by an erasure channel as usually packets are dropped in intermediated routers. In contrast, in wireless broadcast it is more common that data is dropped at the receiver due to decoding errors which are in general detected by block check sequences. In wireless broadcast the receivers usually do have access to additional information as the channel is more appropriately modeled by AWGN or fading channels. In this case the receiver can provide soft-information about each received bit which can be exploited by the subsequent channel decoder. For example in [6], investigations of LT-Codes and Raptor codes on noisy channels other than erasure channels have been performed. However, the work only studied bit error rates and block error rates, rather then reliable transmission in combination with asynchronous data access. In this work we provide a framework which generalizes the idea of digital fountains for a general type of channels and coin the idea and notion of *information collecting*. We support our theoretical concept by proposing a wireless broadcast system, which allows both fully reliable and asynchronous data access by applying LT-Codes. The soft-information available at the wireless receiver is incorporated in the decoding algorithm for LT-codes. Specifically we apply the sum-product algorithm for belief-propagation decoding, instead of erasure decoding to obtain Fountain codes for general types of channels.

## II. System Model

We consider a wireless broadcast system with a single transmitter and multiple receivers. The goal of this system is to broadcast a message reliably to all receivers, *ie*, without any residual errors or missing data. Furthermore, we assume that no feedback channel is available to request retransmissions for lost data. However, every single receiver should be able to receive the message error-free, independent of its receiving conditions. Further, every receiver should be able to start receiving the message at arbitrary time. The considered system consisting of a digital fountain transmitter, an inner channel code, and an exemplary receiver is shown in Fig. 1.

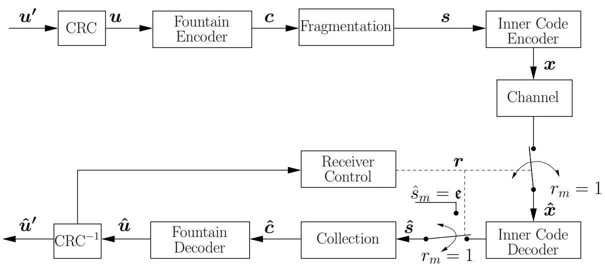


Fig. 1. System model: Wireless broadcast system with asynchronous data access.

Consider a source message $\boldsymbol{u'} = (u'_1, \dots, u'_{k'})$ of length $k'$ symbols to be reliably distributed in the broadcast system. A Cyclic Redundancy Check (CRC) sequence of length $k - k'$ symbols is appended resulting in an overall information sequence $\boldsymbol{u} = (u_1, \dots, u_k)$ of length $k$. The finite information sequence $\boldsymbol{u}$ is encoded with a Fountain code $\mathcal{F}$, which is assumed to produce a code word $\boldsymbol{c}$ of length $\tilde{n}$ encoding symbols. The specific property of the Fountain code results in an infinite sequence of encoding symbols, *ie*, $\tilde{n} \to \infty$. Therefore Fountain codes are also referred to as *rateless* since an arbitrary amount of redundancy can be generated. The fragmentation entity processes the continuous symbol sequence into segments $s_m$, each of $b$ symbols length resulting in a segment sequence $\boldsymbol{s} = (s_1, s_2, \dots, s_{\hat{n}})$ with $\hat{n} = \tilde{n}/b \to \infty$. Each segment $s_m$ is possibly protected with an inner channel code of fixed code rate $R = b/B$. In the following assume that each encoded segment is directly modulated on radio slots $\mathbf{x}_m$ consisting of $B$ transmission symbols. This framework could easily be generalized by introducing an additional interleaver between the inner channel code radio slots. For ease of exposition we skip this detail in this work. Assume that the infinite radio block sequence $\boldsymbol{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m, \dots)$ is sequentially broadcasted over the channel, *ie*, the transmitter serves as a perpetual fountain.

Now assume that receivers tune into the ongoing broadcast session at arbitrary time without any coordination among receivers or any information being sent to the transmitter. Some receivers might even decide to interrupt their listening, for example, to take or place a phone call, or to receive e-mails on a different radio resource. Furthermore, assume that the smallest entity the receiver is able to receive is a radio slot $\mathbf{x}_m$. To formalize the behavior of an arbitrary receiver, we assign to each receiver a *receiver pattern* $\boldsymbol{r} = (r_1, r_2, \ldots)$ with $r_m \in \{0, 1\}$. Thereby, $r_m = 1$ indicates that the receiver subscribes to the channel at radio block index $m$, whereas $r_m = 0$ denotes that the receiver does not listen to the broadcast session. The receiver pattern $\boldsymbol{r}$ is determined by some *receiver control* not further specified in the remainder.

A one-dimensional discrete-time signal model is adopted. Let $\mathbf{x}_m \triangleq (x_{m,1}, \ldots, x_{m,B})$, $\hat{\mathbf{x}}_m \triangleq (\hat{x}_{m,1}, \ldots, \hat{x}_{m,B})$ and $\boldsymbol{\nu}_m \triangleq (\nu_{m,1}, \ldots, \nu_{m,B})$ be the transmitted signal, the received signal and the noise during slot $m$, respectively. The additive noise is assumed to be Gaussian, i.i.d., $\nu_{m,i} \sim \mathcal{N}(0, N_0/(2\mathcal{E}_s))$, and the channel signal-to-noise ratio (SNR) is defined as $\mathcal{E}_s/N_0$. The propagation channel is assumed to be slowly time-varying and frequency-flat for each time slot. In particular, the channel gains $\alpha_m$ over slot $m$ are assumed to be constant over the entire slot. Then, the received signal over slot $m$ is given as $\hat{\mathbf{x}}_m = \alpha_m \mathbf{x}_m + \boldsymbol{\nu}_m$. We assume that each receiver has perfect knowledge of the channel gain $\alpha_m$ and has access to the SNR $\gamma_m \triangleq |\alpha_m|^2 \frac{\mathcal{E}_s}{N_0}$ for all slots $m$.

Let us define the L-value or log-likelihood ratio (LLR) [7] of a binary random variable $X \in \{\pm 1\}$ as

$$L(X) \triangleq \log \left( \frac{\Pr\{X = +1\}}{\Pr\{X = -1\}} \right). \quad (1)$$

Channel decoding is then based on the LLR, $L(\hat{x})$, of a received channel symbol $\hat{x}$. This LLR can be separated into a channel part, $\psi(\hat{x})$, and an a priori part, $L_a(\hat{x})$, whereby the latter is usually assumed to be 0. In case of the fading AWGN channel, the channel LLR for the received symbol, $\hat{x}_{m,i}$, can be expressed as

$$\psi(\hat{x}_{m,i}) = 4 r_m \alpha_m \frac{\mathcal{E}_s}{N_0} \hat{x}_{m,i}. \quad (2)$$

This definition already includes that in case the receiver does not listen to the channel the LLR at the receiver is set to zero.

With this definition, it is obvious that each receiver can use one and the same channel decoder independent of its receiving conditions expressed by $\mathcal{E}_s/N_0$ and $\alpha_m$, and its asynchronous access expressed by its receiver pattern $\boldsymbol{r}$. The LLR contains all relevant information about the received symbols and, thus, acts as our general interface from the channel to the channel decoder. Note that only in case of $r_m = 1$, the LLR of the received symbols, $\psi(\hat{x}_{m,i})$, is passed to the inner channel decoder, in order to obtain an estimate $\hat{s}_m$ on each segment. In the case $r_i = 0$, the channel decoder can be bypassed. The segment is declared as erased since no reception is initiated.

We basically consider two modes of operation for the inner channel encoder and decoder pair. In a classical way, the inner channel code, for example a convolutional code, operates with a channel code of rate $R$. Each channel encoded and decoded radio block contains a block check sequence which is not shown in Fig. 1. We assume that the rate loss due to the CRC or any termination bits is negligible. With that block check sequence, and if residual errors are detected within the decoded radio block, the corresponding radio segment is declared as lost. This is indicated by an erasure marker, *ie*, $\hat{s}_m = \mathfrak{e}$. The fountain decoder then operates on a block erasure channel and we refer to this mode as *wireless erasure fountain*.

In a second scenario, the inner channel code of rate $R$ operates applying a soft-in soft-out decoder, in our case a symbol-by-symbol maximum a posteriori decoder. Then, the inner decoder is capable to produce an LLR $\psi(\hat{c}_i)$ for each received fountain symbol, $\hat{c}_i$. Note that in general the LLR of the received symbol, $\psi(\hat{c}_i)$, is a complex function depending on the code, the channel gain $\alpha_m$, as well as on the entire received vector $\hat{\mathbf{x}}_m$. In this case, the fountain decoder operates on a soft output channel and we refer to as soft-input fountain.

Specifically, we consider the case with $R = 1$ for both operation modes, which corresponds to a system where the inner channel code does not exist. In case of the wireless erasure fountain, the block is dropped if any bit errors after the demodulator is detected. In case of the soft-input fountain, the LLR $\psi(\hat{c}_{B \cdot m + i})$ is equal to the channel LLR $\psi(\hat{x}_{m,i})$. The Fountain code then operates directly on a block-fading channel.

Finally, the *collecting unit* concatenates the received segments (including erasure segments) to a continuous sequence $\hat{\boldsymbol{c}} = (\hat{c}_1, \hat{c}_2, \ldots)$, or more specifically the corresponding LLR is collected. The gathered LLR $\boldsymbol{\psi}(\hat{\boldsymbol{c}})$ is passed to the fountain decoder which outputs $\hat{\boldsymbol{u}} = (\hat{u}_1, \ldots, \hat{u}_k)$ as an estimate of $\boldsymbol{u}$. The CRC check enables the fountain decoder to trigger the receiver control: If the information collected from the channel is still not sufficient, the fountain decoder waits for further information.

In the following we will introduce some useful performance measures on Fountain codes. The performance of practical erasure Fountain codes as well as their modification to be used in a soft-input fountain, and some further implementation issues are discussed.

## III. Performance of Fountain Codes

### A. Fountain Codes on Erasure Channels

In [3] the concept of Fountain codes has been introduced. Recently, a survey on digital fountains and their applications has been presented in [8]. In the following, we recall some basic properties of Fountain codes for erasure channels. We will show that the concepts for erasure channels are easily generalized to a general type of channels by the notion of information collecting.

The transmitter generates an infinite number of encoding symbols from an information message $\boldsymbol{u}$ consisting of $k$ source symbols, which are broadcasted consecutively. An *ideal Fountain code* has the property that the receiver is able to reconstruct the entire source message reliably from any $k$ received encoding symbols. This allows receivers to subscribe to the ongoing broadcast session at arbitrary time and to collect any $k$ encoding symbols not even in a consecutive manner. Note, this property corresponds to the well-known Maximum Distance Separable (MDS) property of channel codes. If symbols are erased, an ideal Fountain code receiver will just wait until any $k$ encoding symbols are received. One attractive property of Fountain codes in broadcast environments results in the fact that receivers with good channel conditions, *ie*, without any erasures, only have to await $k$ encoding symbols before being able to reconstruct the information message, whereas receivers experiencing symbol erasures have to wait longer, according to their experienced channel conditions.

Although ideal Fountain codes have not been found so far, practical codes are capable to approach the performance of ideal codes very well. Usually only a slightly increased number of $\tilde{k}$ packets has to be received. Practical approximations of a *digital fountain* for erasure channels have been proposed by LT-Codes and Raptor codes [3], both approaching $\epsilon \triangleq (\tilde{k} - k)/k \to 0$. More details on LT-codes will be presented in Section IV.

### *B. Generalized Framework for Asynchronous Reception: Information Collecting*

The properties of Fountain codes, as well as the practical realizations by LT and Raptor codes, have up to now almost exclusively been discussed for erasure channels. However, consider the wireless broadcast framework as introduced in Section II. Whereas, the wireless erasure fountain is perfectly covered by this framework, it is no more obvious what the expected performance, as well as practical realizations, for the soft-input fountains are. Both issues will be covered in the following.

Initially, we will concentrate on the case with $R = 1$. Assume that within each radio block $m$, a certain receiver observes a channel state $\gamma_m$. Further assume that $\gamma_m$ is a realization of a random variable $\Gamma_m$, e.g., fading in wireless transmission. For continuous transmission, *ie*, the receiver listens to consecutive $n$ slots from the beginning, resulting in a receiver pattern $r_m = 1\,\forall_{1\leq m\leq n}$ and $r_m = 0\,\forall_{m>n}$, it is known [9] that for $n \cdot B$ being sufficiently large there exist so-called rate-compatible codes such that the probability $p_{o,s}(k,n)$ that the information of size $k$ symbols cannot be decoded when receiving $n$ consecutive radio slots is given as

$$p_{o,s}(k,n) = \Pr\left\{ B \sum_{m=1}^{n} R_I(\Gamma_m) \leq k \right\} \tag{3}$$

with $R_I(\gamma)$ the mutual information between the input and the output for this specific channel state $\gamma$. This includes for example the binary-input AWGN channel with $\gamma$ the SNR. The performance obviously depends on the channel state distribution $\Gamma_m$. Generalizations of the channel state $\gamma$ and the rate transfer function $R_I(\gamma)$ will be discussed later.

From (3), it is obvious that there also exist codes with a similar behavior for asynchronous reception, *ie*, a receiver subscribes to arbitrary radio blocks. Then, the probability $p_o(k, \boldsymbol{r})$ that the information of size $k$ cannot be decoded when subscribing to the broadcast according to a specific receiver pattern $\boldsymbol{r}$ is given by

$$p_o(k,\boldsymbol{r}) = \Pr\left\{ B \sum_{m=1}^{\infty} r_m \cdot R_I(\Gamma_m) \leq k \right\}. \tag{4}$$

We refer to Fountain codes fulfilling this property as *asymptotically optimal fountain codes*. This framework generalizes the idea of asynchronous reception which has almost exclusively been considered for erasure channels up to now. From a receiver point of view, it basically collects blocks with certain channel states $\gamma_m$ until sufficient information is available to successfully decode the specific fountain. We refer to this receiver operation as *asynchronous information collecting*. The minimum number of blocks $n_{\min}$, necessary at the receiver for successful decoding, depends on the observed channel state sequence $\gamma_m$. This can equivalently be expressed by the maximum achievable rate $R_{\max} = k/n_{\min}$.

The performance bound in (4) provides some interesting insights. In the case that $\Gamma_m = \Gamma_0$ corresponds to the AWGN channel, $n_{\min}$ is deterministic and $R_{\max}$ corresponds to the channel capacity $R_I(\Gamma_0)$ of the binary input AWGN channel.

If the observed channel state $\gamma_m$ is random, $n_{\min}$ will be a random variable as well. A valid measure for the receiver is the average amount of slots $\overline{n} \triangleq \mathbb{E}\{n_{\min}\}$, necessary to decode an information message of size $k$, or the maximum achievable average rate $\overline{R}_{\max} \triangleq k/(B\overline{n})$. For example, a block-fading AWGN channel with iid Rayleigh fading, resulting in an exponential distribution of the SNR $\Gamma_m$, is known to be a good model for TDMA-based wireless channels with frequency hopping [10]. In case of random channel states, the performance not only depends on the channel state distribution $\Gamma_m$, but also on the information length $k$ and the radio slot size $B$. Only for $B$ sufficiently smaller than $k$, $n_{\min}$ will become deterministic again, as an ergodic sequence of channel realizations $\gamma_m$ has been observed by the receiver. In this case, the maximum achievable $\overline{R}_{\max}$ is equivalent to the capacity of the fully interleaved Rayleigh fading channel, *ie*, $\overline{R}_{\max} \to R_{\max}$. Note that due to the convexity of $R_I(\gamma)$, $R_{\max}$ serves as an upper-bound of the achievable rate of any Fountain code with arbitrary $B$ and arbitrary message length $k$.

Finally, we would like to emphasize that the channel SNR $\gamma$, as well as the mutual information $R_I(\gamma)$, might be replaced by other channel states as well as an appropriate information transfer function $R_I(\gamma) \to \mathcal{T}$. For example, in case that the channel and the inner channel code with code rate $R = b/B$ are operated as an erasure channel with erasure probability $p$ and payload size $b$, the channel state is appropriately modelled as $\Gamma_m \in \{0,1\}$ with $\Pr\{\Gamma_m = 0\} = p$ and $\Pr\{\Gamma_m = 1\} = 1 - p$. The transfer function is appropriately represented as $\mathcal{T}(\gamma) = R \cdot \mathbf{1}\{\gamma = 1\} = R \cdot \gamma$. If the system is operated as a soft-input Fountain coding system with an inner channel code with code rate $R < 1$, the channel state $\gamma_m$ is mapped to some channel state $\tilde{\gamma}_m \geq \gamma_m$. Thus, the transfer function results in $\mathcal{T}(\gamma) = R \cdot R_I(\gamma)$.

This leaves the question on good practical rateless codes for wireless transmission. One option, which we consider in the remainder of this work, is the application of commonly available rateless erasure codes, but instead of using erasure decoding, the decoding process is modified to include soft information $\psi$. We introduce soft decoding of LT codes in Section IV-C and compare the performance to the asymptotically optimal Fountain code in Section V.

## IV. LT-Codes

In the following, we give a short summary on the encoding, the graphical representation and decoding algorithms for LT-codes $\mathcal{F}_{\text{LT}}$. For a detailed description of LT-codes we refer to [4] and [11].

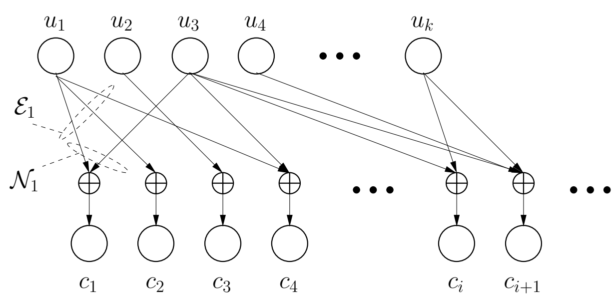


Fig. 2. Tanner graph of an LT-code. Encoding symbols are the $\oplus$ connection of information symbols.

### *A. Encoding*

Let $\boldsymbol{u} = (u_1, \ldots, u_k)$ be the information symbols and $\boldsymbol{c} = (c_1, c_2, \ldots)$ the encoding symbols, respectively, with $\boldsymbol{c} = \mathcal{F}_{\text{LT}}(\boldsymbol{u})$. Then, the generation of encoding symbol $c_i$ is defined as follows [4]:

- Randomly draw a degree $d_i$ for the encoding symbol $c_i$ from a degree distribution $\rho(d)$. A well designed degree distribution $\rho(d)$ was presented in [4].

- Choose $d_i$ distinct information symbols (neighbors), uniformly at random, where $\mathcal{N}_i = \{n_{i,1}, \ldots, n_{i,d_i}\}$ denotes an index set of the selected information symbols.
- Compute the encoding symbol $c_i$ as the bit-wise exclusive-or connection of those $d_i$ information symbols, *ie*,

$$c_i = u_{n_{i,1}} \oplus u_{n_{i,2}} \oplus \ldots \oplus u_{n_{i,d_i}}. \tag{5}$$

After encoding, each information symbol $u_i$ is connected to $l_i$ encoding symbols, where $\mathcal{E}_i = \{e_{i,1}, \ldots, e_{i,l_i}\}$ denotes the set of encoding symbols connected to $u_i$. Figure 2 shows a graphical representation of the encoding procedure.

### B. Decoding on Erasure Channels

Let $\hat{\boldsymbol{c}} = (\hat{c}_1, \hat{c}_2, \ldots)$ be the received encoding symbols, with $\hat{c}_i \in \{c_i, \mathfrak{e}\}$ and $\mathfrak{e}$ indicating an erasure. In order to get the decoding algorithm started, at least one encoding symbol is required with $\hat{c}_i \neq \mathfrak{e} \wedge d_i = 1$. In particular, the decoding process works as described in the following [11]:

1) Look for a $\hat{c}_j$ with $\hat{c}_j \neq \mathfrak{e} \wedge d_j = 1$. If no symbol of degree one is available, the decoding will halt.
2) Find the corresponding neighbor $u_{n_{j,1}}$ of $c_j$.
3) Set the recovered information symbol to $\hat{u}_{n_{j,1}} = \hat{c}_j$.
4) Add $\hat{u}_{n_{j,1}}$ to all other encoding symbols connected to $\hat{u}_{n_{j,1}}$, *ie*, $\forall i \in \mathcal{E}_j : \hat{c}_i = \hat{c}_i \oplus \hat{u}_{n_{j,1}}$.
5) The recovered input symbol $\hat{u}_{n_{j,1}}$ is removed as neighbor from all encoding symbols from the set $\mathcal{E}_{n_{j,1}}$, and the degree of each encoding symbol from the set $\mathcal{E}_{n_{j,1}}$ is reduced by one.
6) Repeat from step 1) until the information message is decoded.

### C. Soft Decoding with Sum-Product Algorithm

As elaborated in Section II, the interface to the channel decoder is usually more general in case of wireless transmission by the availability of soft information $L(\hat{\boldsymbol{c}})$ on the received values $\hat{\boldsymbol{c}}$. This information can be incorporated in the decoding of Fountain codes. Assume a $\oplus$ operation is performed over two random variables $x_1$ and $x_2$, *ie*, $z = x_1 \oplus x_2$, with $x_1, x_2, z \in \{+1, -1\}$. Then, the corresponding L-value $L(z) = L(x_1 \oplus x_2)$ can be calculated by [7]

$$L(x_1 \oplus x_2) = 2\,\mathrm{artanh}(\tanh(L(x_1)/2) \cdot \tanh(L(x_2)/2)). \tag{6}$$

Following this, it turned out [7] to be useful to introduce the symbol $\boxplus$ as the notation for the addition defined by $L(x_1) \boxplus L(x_2) \triangleq L(x_1 \oplus x_2)$. Consider a received encoding symbol $\hat{c}_i$ with its neighbors

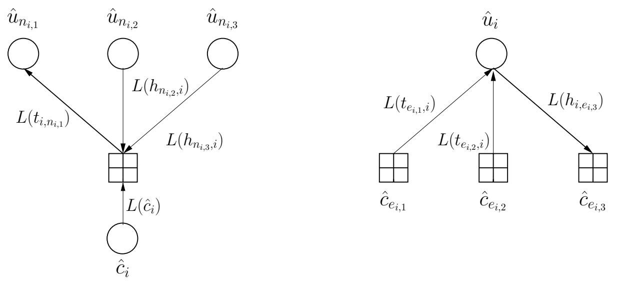


Fig. 3. Message passing on the Tanner graph: $\oplus$ operation replaced by $\boxplus$.

$\hat{u}_{n_{i,1}} \ldots \hat{u}_{n_{i,d_i}}$, represented in a Tanner graph shown in Fig. 3. In the following, we will describe the sum-product algorithm by passing messages (L-values) on the Tanner graph. Let $L(t_{i,j})$ denote an L-value message passed from check node $i$ to variable node $j$ and let $L(h_{i,j})$ denote an L-value message passed from variable node $i$ to check node $j$, respectively. Then, from (5) and (6) follows that $L(t_{i,n})$ is obtained by

$$L(t_{i,j}) = 2\,\mathrm{artanh}\left(\tanh\frac{L(\hat{c}_i)}{2} \cdot \prod_{n \in \mathcal{N}_i, n \neq j} \tanh\frac{L(h_{n,i})}{2}\right), \tag{7}$$

where $L(\hat{c}_i)$ denotes the received L-value about $c_i$ from the channel, with $L(\hat{c}_i) = \psi(\hat{c}_i)$. The L-value $L(h_{i,j})$ only depends on messages passed to the variable node $i$. Hence, $L(h_{i,j})$ is obtained by the sum of L-values passed to variable node $i$, *ie*,

$$L(h_{i,j}) = \sum_{e \in \mathcal{E}_i, e \neq j} L(t_{e,i}). \tag{8}$$

Likewise, the L-value about the decoding decision $L(\hat{u}_i)$ is given by

$$L(\hat{u}_i) = \sum_{e \in \mathcal{E}_i} L(t_{e,i}). \tag{9}$$

The decoding is performed by passing messages from check nodes to variable nodes and vice versa, in an iterative manner. At the beginning of the first iteration, all corresponding L-values are initialized with zeros, except for the received values from the channel, $L(\hat{c}_i)$. Note that decoding is prohibitively complex and it is not practical to attempt decoding after each received radio block. We propose to use (4) to determine whether the received information is already sufficient for a decoding attempt. Only if the collected mutual information is sufficient, decoding will be attempted.

## V. Performance Evaluation

### A. Simulation Parameters

In order to evaluate the performance of the investigated system, as well as the presented decoding schemes, we performed extensive simulations. The symbol size of all introduced symbols within this work was set to 1 bit. For all performed simulations, we selected a radio block size of $B = 160$ bit. We investigate the system performance for two different radio protection modes, uncoded transmission, with $R = 1$, and alternatively coded transmission, with code rate $R = 0.5$. In the latter case, we applied a memory $M = 4$ recursive systematic convolutional code with generator polynomial $[37, 21]_8$ in octal representation. The segment size $b$ was adjusted appropriately with the applied code rate $R$. Decoding of the convolutional code was performed with a Max-Log-Map decoder [12], [13]. The degree distribution of the LT-code was selected according to [11] with parameters $\delta = 0.999$ and $c = 0.03$. We present performance results for information block length $k = 4000$ bit. Theoretically, any arbitrary realization of the receiver pattern could be selected. However, as the channel is assumed to be iid and the code itself randomly generates parity symbols $c$, it is meaningless to distinguish different receiver patterns since the average performance of any receiver pattern $\boldsymbol{r}$ with identical $n = \sum_i r_i$ is the same. In the following, we investigate the performance on the AWGN as well as on the block-fading AWGN channel with Rayleigh distributed fading. In any case, we evaluate the code by determining the maximum achievable rate $\overline{R}_{\max} = k/(B\overline{n})$ for different average receiving conditions.

### B. Simulation Results

Fig. 4 shows simulation results on the AWGN channel. The results are compared with the asymptotically optimal code according to the definition in Section III-B. It is worth to note that for our system parameters, the ultimate capacity bound $R_{\max}$ coincides with the asymptotically optimal bound as the radio block size $B$ is sufficiently

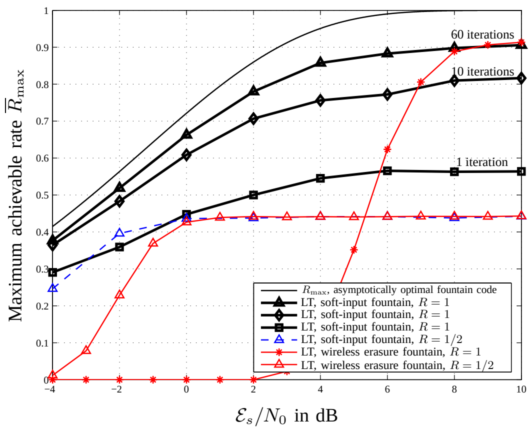


Fig. 4. Maximum achievable rate $\overline{R}_{\max}$ over $\mathcal{E}_s/N_0$ for AWGN channel.

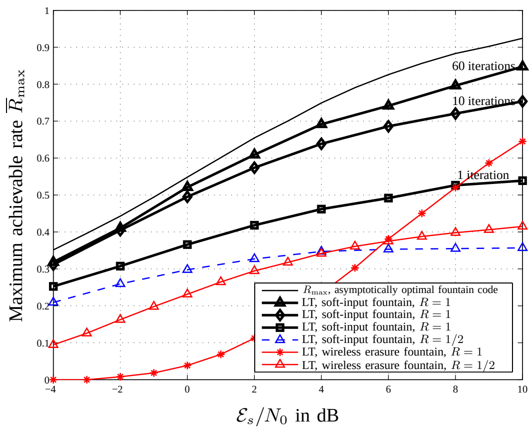


Fig. 5. Maximum achievable rate $\overline{R}_{\max}$ over $\mathcal{E}_s/N_0$ for block-fading AWGN channel.

smaller than the message size $k$ and therefore, the channel sequence is almost ergodic for one message reception. For the simulation results, the maximum achievable rate $\overline{R}_{\max}$ is also shown over different receiving conditions $\mathcal{E}_s/N_0$. Obviously, in any case receivers with better receiving conditions, *ie*, higher $\mathcal{E}_s/N_0$, require less radio blocks $\overline{n}$ for the same system. Therefore, receivers with good conditions are able to decode the message earlier than receivers with bad conditions.

Let us now compare different system designs. For the wireless erasure system without any inner channel code, *ie*, $R = 1$, sufficient throughput can only be achieved for $\mathcal{E}_s/N_0 > 4$ dB. Applying an inner channel code with $R = 0.5$, the system is able to support receivers with lower $\mathcal{E}_s/N_0$ since less radio blocks are declared as erased. However, the maximum rate is limited to $\overline{R}_{\max} = R$ since in an broadcast scenario link adaptation cannot be performed. Unfortunately, the system with erasure decoding remains a huge gap compared to an asymptotically optimal Fountain code.

In the contrast, soft decoding of LT-codes in combination with $R = 1$ approaches the asymptotically optimal Fountain code with an increasing number of performed iterations $I$. The results are shown for $I \in \{1, 10, 60\}$. However, with an increasing number of iterations the complexity is growing, for which an analysis is out of the scope of this work. Finally, radio block protection ($R = 0.5$) in combination with soft decoding of the LT-code does not show any advantages for any receiving conditions. Therefore, it is obvious that it is beneficial to use only a single code, namely a Fountain code, and dispense any inner coding. However, note that this inner code might be necessary for synchronization purposes which is not further considered in this work, but subject of future work.

Fig. 5 shows the simulation results for the block fading AWGN channel with Rayleigh distributed channel gains. Basically, the same tendencies as explained for the AWGN channel can be observed, showing the superiority of the soft-decoding algorithm compared to erasure decoding.

## VI. Conclusions and Future Work

In this work we investigated asynchronous reliable wireless broadcast. We extended the traditional problem formalization of ideal Fountain codes on erasure channels to arbitrary channel types and introduced the principle of information collecting. We reviewed the encoding and decoding algorithms for LT-codes, as traditionally proposed for the communication over erasure channels. The sum-product algorithm has been applied to LT-codes in order to exploit soft information accessible in wireless communication. The performance gains of soft input fountains over wireless erasure fountains have been shown. In addition, it has been shown that one code, namely the Fountain code, will be sufficient in a wireless system if soft input fountains are used. This allows every receiver to operate at its maximum rate. Future work will consider practical issues of this approach, such as soft-decoding techniques with reduced complexity and synchronization issues.

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