A COMPARATIVE PERFORMANCE ANALYSIS OF STBC-OFDM SYSTEMS UNDER RAYLEIGH FADING ENVIRONMENTS

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ABSTRACT

In this paper, we compare the performance of three representative STBC schemes, Alamouti's, Tarokh's, and quasi-orthogonal schemes, considering the effect of channel estimation errors when they are applied to the OFDM system.

We first design STBC-OFDM systems employing various modulation techniques to provide transmission rates from 2bps/Hz to 4bps/Hz. The STBC-OFDM systems employ Walsh orthogonal codes covered along the frequency domain in order to discriminate the pilot symbols of different transmit antennas. We finally compare the performance of the three STBC-OFDM systems for various transmission rates under Rayleigh fading environments.

1. INTRODUCTION

Recently, multiple antenna techniques have been extensively studied for high rate data transmission and increasing transmission efficiency [1]-[6]. The space-time block coding (STBC) technique, one of representative multiple antenna techniques, is most attractive for these purposes since it easily provides the diversity at receiver by transmitting a space-time coded signal through multiple antennas. Since it was first proposed by Alamouti [3], its various derivatives have been developed [4]-[6]. On the other hand, the OFDM technique has been widely accepted for the transmission of high rate data due to its robustness to inter-symbol interference. In this context, the STBC-OFDM system may be one of most promising system configurations that can be adopted for 4th generation mobile systems. Therefore, it may be very important to derive the optimum STBC-OFDM system by analyzing and comparing the performance of STBC-OFDM systems in mobile environments.

So far, however, there has been no systematic comparison for STBC-OFDM systems considering the effect of channel estimation errors. In this paper, we first design STBC-OFDM systems by applying three representative STBC schemes, namely, Alamouti's, Tarokh's, and quasi-orthogonal schemes, to the OFDM system. In order to avoid the serious reduction of transmission efficiency due to the preamble, we employ an efficient preamble coding scheme that uses Walsh codes covered along the frequency domain. We finally compare the performance of designed STBC-OFDM systems for various transmission rates under Rayleigh fading environments.

2. STBC TECHNIQUES

2.1 Alamouti's scheme [3]

Alamouti's scheme can be applied to the system with two transmit antennas and its space-time encoding can be described as follow,

$$\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}, \tag{1}$$

where s_1 and s_2 are complex signals to be transmitted and * denotes a conjugate operation. Rows indicate the time domain and columns represent the space domain. The received signal at time t and $t+T$ can be written as

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{2}$$

where $h_1 = \alpha_1 e^{j\varphi_1}$ and $h_2 = \alpha_2 e^{j\varphi_2}$ are complex channel responses for antennas 1 and 2, respectively, and n_1 and n_2 complex noises at times t and $t+T$. The signal vector can be rewritten as,

$$\mathbf{u}' = \begin{bmatrix} u_1 \\ u_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = \mathbf{Hs} + \mathbf{n}' \tag{3}$$

with $E\left[\mathbf{n}'\mathbf{n}'^H\right] = \sigma_n^2 \mathbf{I}_{2\times 2}$. Therefore, the decoded signal can be obtained as

$$\tilde{\mathbf{s}} = \begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{bmatrix} = \tilde{\mathbf{H}}^H \mathbf{Hs} + \tilde{\mathbf{H}}^H \mathbf{n}'. \tag{4}$$

In the above equation, if the channel estimation is perfect, that is, $\tilde{h}_1 = h_1$ and $\tilde{h}_2 = h_2$, then we have

$$\tilde{\mathbf{s}} = \left(\alpha_1^2 + \alpha_2^2\right)\mathbf{s} + \mathbf{n}_{\tilde{\mathbf{s}}} \tag{5}$$

with

$$E\left[\mathbf{n}_{\tilde{\mathbf{s}}}\mathbf{n}_{\tilde{\mathbf{s}}}^H\right] = \left(\alpha_1^2 + \alpha_2^2\right)\sigma_n^2 \mathbf{I}_{2\times 2}. \tag{6}$$

2.2 Tarokh's scheme [4]

Unlike Alamouti's scheme, Tarokh's scheme can be used for three or four antenna systems. In case of employing four antennas, the received signal can be described as,

$$\mathbf{u}=\begin{bmatrix} u_1\\ u_2\\ u_3\\ u_4\\ u_5\\ u_6\\ u_7\\ u_8 \end{bmatrix}=\begin{bmatrix} s_1 & s_2 & s_3 & s_4\\ -s_2 & s_1 & -s_4 & s_3\\ -s_3 & s_4 & s_1 & -s_2\\ -s_4 & -s_3 & s_2 & s_1\\ s_1^* & s_2^* & s_3^* & s_4^*\\ -s_2^* & s_1^* & -s_4^* & s_3^*\\ -s_3^* & s_4^* & s_1^* & -s_2^*\\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}\begin{bmatrix} h_1\\ h_2\\ h_3\\ h_4 \end{bmatrix}+\begin{bmatrix} n_1\\ n_2\\ n_3\\ n_4\\ n_5\\ n_6\\ n_7\\ n_8 \end{bmatrix}, \quad (7)$$

where $h_{1\sim4}$ are complex channel responses for antennas 1, 2, 3, and 4, respectively. We can rewrite the received signal vector as,

$$\mathbf{u}'=\begin{bmatrix} u_1\\ u_2\\ u_3\\ u_4\\ u_5^*\\ u_6^*\\ u_7^*\\ u_8^* \end{bmatrix}=\begin{bmatrix} h_1 & h_2 & h_3 & h_4\\ h_2 & -h_1 & h_4 & -h_3\\ h_3 & -h_4 & -h_1 & h_2\\ h_4 & h_3 & -h_2 & -h_1\\ h_1^* & h_2^* & h_3^* & h_4^*\\ h_2^* & -h_1^* & h_4^* & -h_3^*\\ h_3^* & -h_4^* & -h_1^* & h_2^*\\ h_4^* & h_3^* & -h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} s_1\\ s_2\\ s_3\\ s_4 \end{bmatrix}+\begin{bmatrix} n_1\\ n_2\\ n_3\\ n_4\\ n_5^*\\ n_6^*\\ n_7^*\\ n_8^* \end{bmatrix}, \quad (8)$$

with $E\left[\mathbf{n}'\mathbf{n}'^H\right]=\sigma_n^2\mathbf{I}_{8\times8}$. Therefore, the decoded signal can be obtained as,

$$\tilde{\mathbf{s}}=\tilde{\mathbf{H}}^H\mathbf{H}\mathbf{s}+\tilde{\mathbf{H}}^H\mathbf{n}'. \quad (9)$$

In case of perfect channel estimation, we have

$$\tilde{\mathbf{s}}=2\left(\alpha_1^2+\alpha_2^2+\alpha_3^2+\alpha_4^2\right)+\mathbf{n}_{\tilde{\mathbf{s}}}, \quad (10)$$

with

$$E\left[\mathbf{n}_{\tilde{\mathbf{s}}}\mathbf{n}_{\tilde{\mathbf{s}}}^H\right]=2\left(\alpha_1^2+\alpha_2^2+\alpha_3^2+\alpha_4^2\right)\sigma_n^2\mathbf{I}_{4\times4}. \quad (11)$$

It should be noted that Tarokh's scheme obtains the diversity gain at the cost of reduction of the transmission rate.

2.3 Quasi-orthogonal scheme [5]

The quasi-orthogonal scheme can be used for four antenna systems without loss of transmission rate unlike Tarokh's scheme. The received signal vector can be written as

$$\mathbf{u}=\begin{bmatrix} u_1\\ u_2\\ u_3\\ u_4 \end{bmatrix}=\begin{bmatrix} s_1 & s_2 & s_3 & s_4\\ s_2^* & -s_1^* & s_4^* & -s_3^*\\ s_3 & -s_4 & -s_1 & s_2\\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{bmatrix}\begin{bmatrix} h_1\\ h_2\\ h_3\\ h_4 \end{bmatrix}+\begin{bmatrix} n_1\\ n_2\\ n_3\\ n_4 \end{bmatrix}. \quad (12)$$

In order to decode the signal, the received signal vector is rewritten as follow,

$$\mathbf{u}'=\begin{bmatrix} u_1\\ u_2^*\\ u_3\\ u_4^* \end{bmatrix}=\begin{bmatrix} h_1 & h_2 & h_3 & h_4\\ -h_2^* & h_1^* & -h_4^* & h_3^*\\ -h_3 & h_4 & h_1 & -h_2\\ -h_4^* & -h_3^* & h_2^* & h_1^* \end{bmatrix}\begin{bmatrix} s_1\\ s_2\\ s_3\\ s_4 \end{bmatrix}+\begin{bmatrix} n_1\\ n_2^*\\ n_3\\ n_4^* \end{bmatrix} \quad (13)$$
$$=\mathbf{H}\mathbf{s}+\mathbf{n}'$$

where $E\left[\mathbf{n}'\mathbf{n}'^H\right]=\sigma_n^2\mathrm{I}_{4\times4}$. Therefore, the decoded signal can be obtained as,

$$\tilde{\mathbf{s}}=\tilde{\mathbf{H}}^H\mathbf{H}\mathbf{s}+\tilde{\mathbf{H}}^H\mathbf{n}'. \quad (14)$$

In the case of perfect channel estimation, we have

$$\tilde{\mathbf{s}}=\begin{bmatrix} \mu & 0 & \phi & 0\\ 0 & \mu & 0 & -\phi\\ -\phi & 0 & \mu & 0\\ 0 & \phi & 0 & \mu \end{bmatrix}\begin{bmatrix} s_1\\ s_2\\ s_3\\ s_4 \end{bmatrix}+\mathbf{H}^H\mathbf{n}' \quad (15)$$
$$=\Delta_4\mathbf{s}+\mathbf{n}_{\tilde{s}}$$

where $\mu=\alpha_A^2+\alpha_B^2+\alpha_C^2+\alpha_D^2$, $\phi=2j\,\mathrm{Im}\left(h_A^*h_C+h_D^*h_B\right)$ and $E\left[\mathbf{n}_{\tilde{s}}\mathbf{n}_{\tilde{s}}^H\right]=\left(\alpha_A^2+\alpha_B^2+\alpha_C^2+\alpha_D^2\right)\sigma_n^2\mathbf{I}_{4\times4}$. From (15), we equivalently have

$$\begin{bmatrix} \tilde{s}_1\\ \tilde{s}_3 \end{bmatrix}=\Delta_2\begin{bmatrix} s_1\\ s_3 \end{bmatrix}+\begin{bmatrix} n_{\tilde{s}_1}\\ n_{\tilde{s}_3} \end{bmatrix},$$
$$\begin{bmatrix} \tilde{s}_4\\ \tilde{s}_2 \end{bmatrix}=\Delta_2\begin{bmatrix} s_4\\ s_2 \end{bmatrix}+\begin{bmatrix} n_{\tilde{s}_4}\\ n_{\tilde{s}_2} \end{bmatrix}, \quad (16)$$

with

$$\Delta_2=\begin{bmatrix} \mu & \phi\\ -\phi & \mu \end{bmatrix}. \quad (17)$$

When using the zero-forcing scheme, the signal vector can be estimated as follow,

$$\begin{bmatrix} \tilde{s}_{zf,1}\\ \tilde{s}_{zf,3} \end{bmatrix}=\Delta_2^{-1}\begin{bmatrix} \tilde{s}_1\\ \tilde{s}_3 \end{bmatrix}=\begin{bmatrix} s_1\\ s_3 \end{bmatrix}+\Delta_2^{-1}\begin{bmatrix} n_{\tilde{s}_1}\\ n_{\tilde{s}_3} \end{bmatrix},$$
$$\begin{bmatrix} \tilde{s}_{zf,4}\\ \tilde{s}_{zf,2} \end{bmatrix}=\Delta_2^{-1}\begin{bmatrix} \tilde{s}_4\\ \tilde{s}_2 \end{bmatrix}=\begin{bmatrix} s_4\\ s_2 \end{bmatrix}+\Delta_2^{-1}\begin{bmatrix} n_{\tilde{s}_4}\\ n_{\tilde{s}_2} \end{bmatrix}, \quad (18)$$

where

$$\Delta_2^{-1}=\begin{bmatrix} \beta & \delta\\ -\delta & \beta \end{bmatrix},\quad \beta=\frac{\mu}{\mu^2+\phi^2},\quad \delta=\frac{-\phi}{\mu^2+\phi^2}. \quad (19)$$

3. STBC-OFDM SYSTEM

3.1 Description of the STBC-OFDM System

Figure 1 describes the block diagram of the STBC-OFDM system considered in this paper. Binary input data are first mapped to one of modulation symbols. An $N\times M$ data matrix is formed through serial-to-parallel conversion of N successive modulation symbols, where N and M represent the FFT size and the number of OFDM symbols in each slot, respectively. OFDM symbols are then space-time encoded for each row using two successive symbols for Alamouti's scheme and four successive symbols for quasi-orthogonal scheme (or eight symbols for Tarokh's scheme). A preamble is inserted at the beginning of each slot for channel estimation. The size of the preamble should be as small as possible to avoid the serious reduction of transmission efficiency. To this end, we assign one OFDM symbol for preamble and encode the preamble in the frequency domain. We use 4×4 Walsh orthogonal codes to encode the preamble for four antenna cases and 2×2 Walsh codes for Alamouti's scheme. One of the Walsh codes is assigned to one of antennas, which is repeated along the frequency axis.

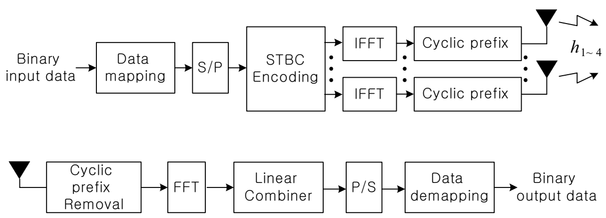

Fig. 1. Block diagram of STBC-OFDM systems.

Figure 2 describes the preamble coding for two and four transmit antenna cases.

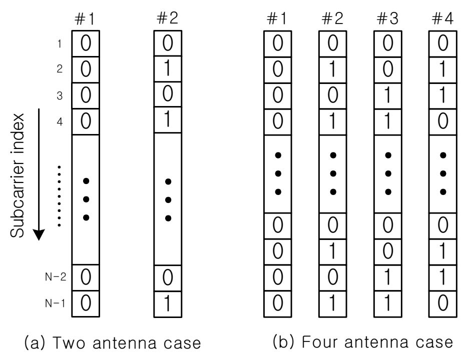

Fig. 2. Preamble coding schemes.

3.2 Channel Estimation

Channel estimation is performed using the preamble. The received signal during the preamble interval can be written as

$$r(t) = \mathrm{Re}\left[\sum_{i=0}^{N_a-1} s^{(i)}(t)\otimes g(t) + n(t)\right] \qquad (20)$$

where $g(t)$ is the channel impulse response and $s^{(i)}(t)$ is the OFDM signal transmitted from the i-th antenna, given by

$$s^{(i)}(t) = \sqrt{E_{s,p}/2}\sum_{k=-N/2}^{N/2-1} d_p^{(k)}(t)W_k^{(i)}(t) \times\exp\left[j\left\{2\pi\left(f_c + k/T_{sym}\right)t\right\}\right]. \qquad (21)$$

In (20) and (21), N_a denotes the number of transmit antennas, $E_{s,p}$ the pilot symbol energy, $d_p^{(k)}(t)$ complex pilot symbol, $W_k^{(i)}(t)$ Walsh function assigned for the i-th antenna, T_{sym} the effective OFDM symbol duration, $n(t)$ the complex noise process with $E\left[n^2(t)\right] = \sigma_n^2 = N_o$. We may express the channel impulse response as an impulse train of the form

$$g(t) = \sum_n a_n \delta\left(t-\tau_n\right) \qquad (22)$$

where the amplitude a_n are complex-valued and τ_n denote time delays.

After performing an FFT operation for a baseband signal, we have the k-th subcarrier component, given by

$$u_k = \sqrt{\frac{E_{s,p}}{2}}\sum_{i=0}^{N_a-1} \alpha_k^{(i)} d_p^{(k)} W_k^{(i)} \exp\left(\varphi_k^{(i)}\right) + n_k \qquad (23)$$

where $\alpha_k^{(i)}$ and $\varphi_k^{(i)}$ are the envelope and phase of the k-th component from the i-th antenna. Without loss of generality, we assume that pilot symbols have equal energy. Assuming that the delay spread is very small such that fading is constant over successive N_a subcarriers, we can estimate the channel response for the l-th group of the j-th antenna as follow,

$$\begin{aligned} Y_{p,l}^{(j)} &= \sum_{k=lN_a}^{(l+1)N_a-1} u_k \left(d_p^{(k)}\right)^* W_k^{(j)} \\ &= \sqrt{E_{s,p}}\, N_a \alpha_l^{(j)} \exp\left(j\varphi_l^{(j)}\right) + n_l^{'}, \\ & l = 0,\ 1,\ 2,\cdots, N/N_a - 1. \end{aligned} \qquad (24)$$

with $E\left[n_l^{'2}\right] = N_a N_o$. We shall employ the low rank least square scheme for enhancing the channel estimation [7]. In order to further improve the channel estimation, a linear interpolation between the channel estimates at two successive slots is applied to each subcarrier.

4. PERFORMANCE EVALUATION

In this section, we design STBC-OFDM systems considering slowly-moving mobile environments and compare their performance for various transmission rates under Rayleigh fading environments. The OFDM system parameters and modulation schemes used for simulation are summarized in Table 1 and 2, respectively. Each slot consists of nine OFDM symbols, one for preamble and eight for information symbols. It is assumed that there are three multipath signals with relative signal strengths of 0.6, 0.3, and 0.1 and relative delays of 0μs, 0.5μs, 1.0μs, respectively, mobile speed is 5km/h, and the number of receive antennas is one, unless specified otherwise. In all examples, pilot symbol energy is set to be the same as the average symbol energy.

Table 1. Key OFDM system parameters

bandwidth	10MHz
FFT size / OFDM symbol	1024 / 102.4μs
guard interval	10.6μs
subcarrier spacing	9.7656KHz
carrier frequency	2.5GHz
transmission rate	2bps/Hz, 3bps/Hz, 4bps/Hz
modulation scheme	QPSK, 8PSK, 16QAM, 64QAM, 256QAM
slot length	9 OFDM symbols (0.1017ms)

Table 2. Modulation schemes v.s. transmission rates

STBC schemes	Modulation schemes		
	2bps/Hz	3bps/Hz	4bps/Hz
Alamouti	QPSK	8PSK	16QAM
Tarokh	16QAM	64QAM	256QAM
quasi-orthogonal	QPSK	8PSK	16QAM

Figure 3 illustrates the performance of three STBC-OFDM systems including the effect of channel estimation errors when transmission rate is 2bps/Hz. Dotted curves represent the performance when there is no channel estimation error. It is seen from the figure that four transmit antenna systems suffer from larger performance degradation due to channel estimation errors since more serious inter-symbol interference arises in four transmit antenna systems. Tarokh's scheme provides the best performance among the three schemes for this 2bps/Hz case. This is because Tarokh's scheme obtains a high diversity gain through four transmit antennas, while employing the modulation scheme with relatively low modulation index such as 16QAM. The performance of the quasi-orthogonal scheme is not as good as that of Tarokh's scheme because of the inter-symbol interference due to its nonorthogonality nature.

Figures 4 and 5 show the performance comparison in case of 3bps/Hz and 4bps/Hz, respectively. The three schemes provide a similar performance in the 3bps/Hz case, while Alamouti's scheme provides the best performance for a 4bps/Hz case.

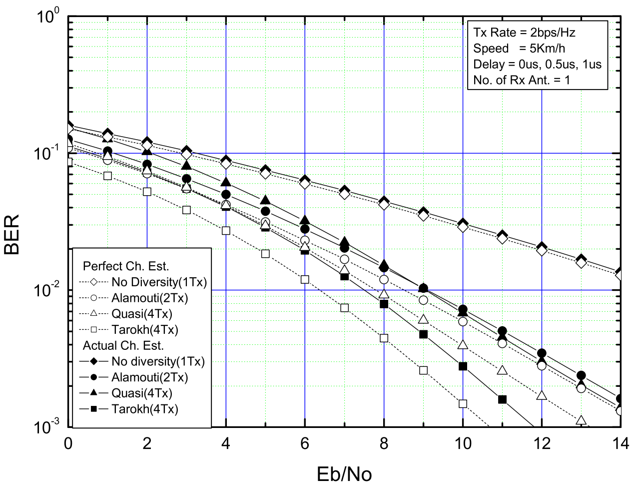

Fig. 3. Performance comparison between STBC-OFDM schemes for 2bps/Hz.

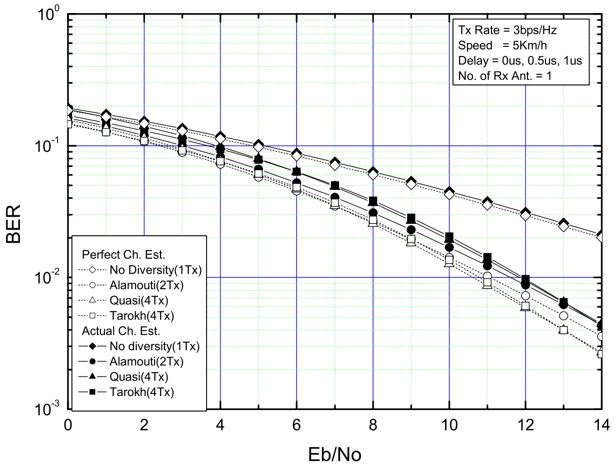

Fig. 4. Performance comparison between STBC-OFDM schemes for 3bps/Hz.

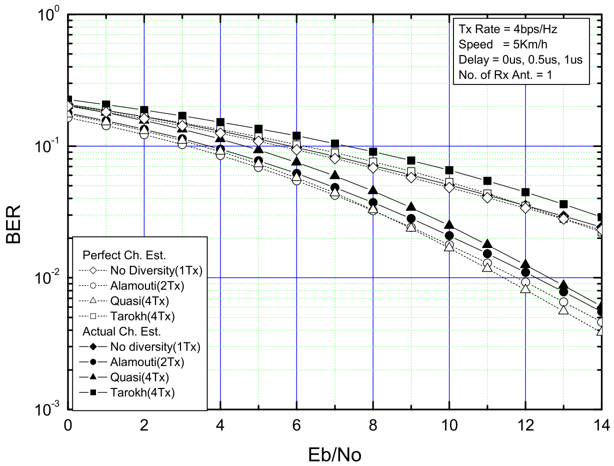

Fig. 5. Performance comparison between STBC-OFDM schemes for 4bps/Hz.

This is because the performance loss due to employing the modulation scheme with high order index becomes more significant than the diversity gain for four transmit antenna systems. The performance of Tarokh's scheme is degraded seriously below that of no-diversity case for a 4bps/Hz.

Figure 6 shows the performance comparison for the transmission rate of 2bps/Hz, in case where relative delays are 0μs, 0.5μs, 7.0μs, respectively. It is seen from the figure that Alamouti's scheme still provides a significant diversity gain, while the performance of four antenna systems is degraded seriously. This is because the correlation between subcarriers decreases as the delay spread increases, so that the channel response estimated by averaging four subcarriers in four antenna systems involves a significant estimation error.

Figure 7 illustrates the performance comparison in case where the mobile speed is 50km/h and relative delays are 0μs, 0.5μs, 1.0μs, respectively. It is shown that the performance of Tarokh's scheme is degraded seriously in this high speed case, while Alamouti's scheme is little affected by the mobile speed. This is due to the fact that as the mobile speed increases, the channel state during eight OFDM symbols for

Tarokh's scheme or four OFDM symbols for quasi-orthogonal scheme changes significantly, which violates the basic assumption of STBC schemes on the channel.

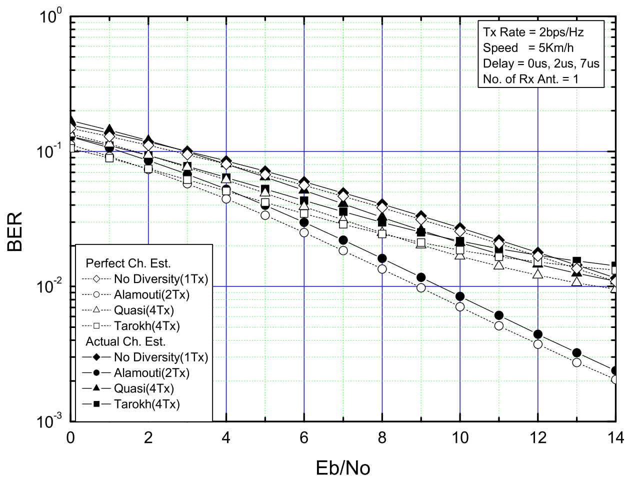

Fig. 6. Performance comparison between STBC-OFDM schemes for 2bps/Hz in case of large delay spread.

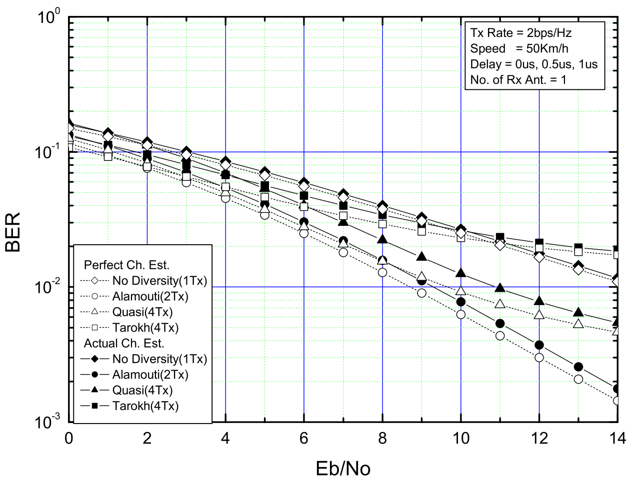

Fig. 7. Performance comparison between STBC-OFDM schemes for 2bps/Hz in case of 50km/h.

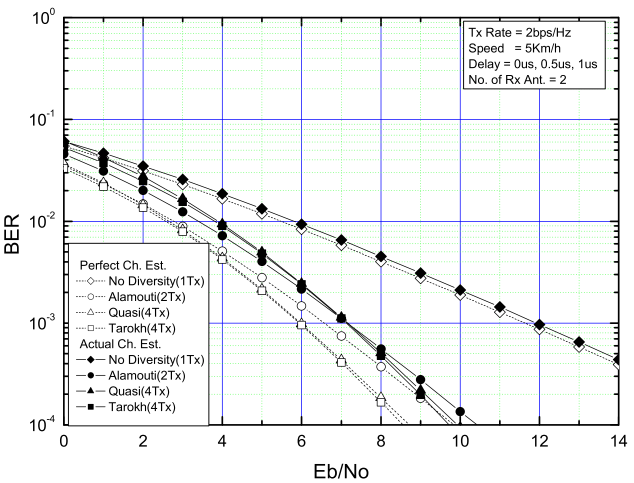

Fig. 8. Performance comparison between STBC-OFDM schemes for 2bps/Hz in case of two rx antennas.

Figure 8 illustrates the performance comparison when two receive antennas are employed. The figure shows that the three schemes provide a similar performance even for 2bps/Hz case unlike the single receive antenna case. This is because the diversity gain obtained through multiple transmit antennas becomes less significant as the diversity gain increases due to multiple receive antennas, while the performance degradation due to high-order modulation is unchanged. Therefore, Alamouti's scheme is better than four antenna schemes when employing the receive antenna diversity.

5. CONCLUSIONS

In this paper, we compared the performance of three representative STBC schemes when they were applied to the OFDM system under Rayleigh fading environments. It was shown that Alamouti's and quasi-orthogonal schemes provide the better performance than Tarokh's scheme in case of high transmission rate, although the performance of the quasi-orthogonal scheme degrades seriously when delay spread is large or mobile speed is relatively high. On the contrary, Tarokh's scheme provides the best performance for a 2bps/Hz case if the delay spread is small as well as the mobile speed is low.

The performance degradation of the four transmit antenna systems is rather obvious for the designed OFDM system in case where the delay spread is large or the mobile speed is relatively high, which may sometimes limit their usage. However, the performance of four transmit antenna systems may be improved by using the combination of space-time and space-frequency coding schemes.

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