Joint Power Control and Multiuser Receiver Design – Fairness Issues and Cross-Layer Optimization

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***Abstract*— In this paper, we consider the problem of cross-layer optimization for joint power control and adaptive receiver design. An abstract interference model is used for the analysis of the QoS achievable region. We discuss the trade-off between different optimization goals, namely max-min fairness and overall system efficiency. The results lead to iterative algorithms, which are monotonically convergent and computationally efficient.**

I. Introduction

Wireless networks are characterized by elastic link capacities, which are due to inter-link interference caused by the broadcast propagation medium. Thus, the achievable throughput between two network nodes is not only determined by the current channel state, but also by the amount of interference caused by all other nodes in the network. All communication links are highly intertwined. In this respect, wireless communication differs from wireline, where physical links are usually modeled as independent, with fixed capacities.

As a consequence, traditional layer-based design principles are not always the best choice for wireless systems, since they often neglect the interdependencies between layers. This insight has given rise to new cross-layer strategies, which favor a holistic, system-wide approach. Since this leads to very complex, and difficult-to-handle optimization problems, it is important to use abstract models and to define appropriate quality-of-service (QoS) measures. The core of cross-layer concepts is to characterize performance tradeoffs and to better understand the interactions between layers.

II. Resource Allocation and Interference Management

In this paper we focus on the interaction between power control and the joint receive strategy. To this end, we consider some abstract joint receiver design $z \in \mathcal{Z}$, where $\mathcal{Z}$ is the set of all possible receive strategies. Typically, we have $\mathcal{Z} = \mathcal{Z}_1 \times \mathcal{Z}_2 \times \cdots \times \mathcal{Z}_K$ (Cartesian product), where $\mathcal{Z}_k$ is the closed set of possible receive strategies associated with the kth link. The total number of links is K.

Note, that the receive strategy $z_k \in \mathcal{Z}_k$ can be linear or non-linear. Also, we do no specify any particular system design, thus the following results are general enough to hold for a wide range of multiuser systems, e.g. beamforming, synchronous DS-CDMA with fixed transmit sequences, or digital subscriber line (DSL) services, where bundled telephone lines are coupled by mutual interference.

The K communication links are corrupted by mutual interference, which is collected in a K-dimensional vector $\boldsymbol{\mathcal{I}}(z, \boldsymbol{p})$, where $\boldsymbol{p}$ is the power allocation vector. The signal-to-interference+noise ratio (SINR) of the kth link is

$$\text{SINR}_k(z_k, \boldsymbol{p}) = \frac{p_k}{\boldsymbol{\mathcal{I}}_k(z_k, \boldsymbol{p}) + N(z_k)}, \quad k = \{1, 2, \ldots, K\} \ , \tag{1}$$

where $N(z)$ is the effective receiver noise, which takes into account the possible *noise enhancement* caused by z. The structure of the interference function $\boldsymbol{\mathcal{I}}$ will be characterized later in Sec. III.

It can be observed that SINR_k only depends on z_k, which is because the receivers do not interact for a fixed power allocation. Nevertheless, the quantities (1) are tightly intertwined by the powers. Thus, both parameters z and $\boldsymbol{p}$ should be optimized together.

A common design goal is to schedule multiple links within a slot, while meeting minimum required QoS values $Q = [Q_1, \ldots, Q_K]$ and a total power constraint $\|\boldsymbol{p}\|_1 \leq P$. It can be assumed that there is a one-to-one relationship between the QoS and the signal-to-noise-

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plus-interference ratio (SINR), i.e.,

$$QoS_k = \phi(\text{SINR}_k), \quad k = \{1, 2, \ldots, K\} ,$$

where ϕ is a bijective function, depending on the chosen performance criterion[1].

The achievable sum-power constrained QoS region (the set of all jointly achievable $Q > 0$) is

$$\mathcal{Q}(P) = \{Q : C(P, Q) \geq 1\} . \tag{2}$$

where

$$C(P, Q) = \max_{z \in \mathcal{Z}, \boldsymbol{p} > 0} \Big(\min_{1 \leq k \leq K} \frac{\phi\big(\text{SINR}_k(z_k, \boldsymbol{p})\big)}{Q_k} \Big) \quad \text{subject to } \|\boldsymbol{p}\|_1 \leq P . \tag{3}$$

The region (2) is illustrated in Fig. 1 for the 2-user case, assuming that we are interested in maximizing the QoS functions (otherwise max and min must be interchanged). The boundary of $\mathcal{Q}(P)$ is given by $\{Q : C(P, Q) = 1\}$. Note, that the sum power constraint $\|\boldsymbol{p}\|_1 \leq P$ in (3) can be replaced by individual power constraints $\boldsymbol{p}_k \leq P_k$, $1 \leq k \leq K$.

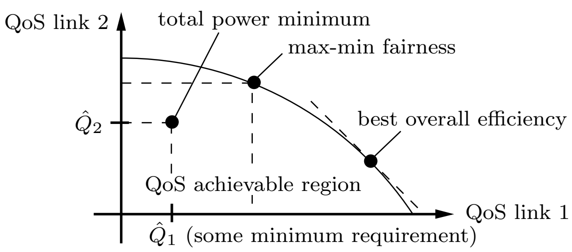

Fig. 1. The resource allocation problem: optimizing over the QoS achievable region

The resource allocation problem consists of finding an optimal trade-off between the K links. Some extreme points, which are illustrated in Fig. 1, will be described in the following.

1) Max-min fairness: means that all QoS are balanced at the same level. This strategy is a special case of (3) with equal targets $Q_1 = \cdots = Q_K = 1$ (or some other constant):

$$\max_{z \in \mathcal{Z}, \boldsymbol{p} > 0} \Big(\min_{1 \leq k \leq K} \phi\big(\text{SINR}_k(z_k, \boldsymbol{p})\big) \Big) \quad \text{s.t. } \|\boldsymbol{p}\|_1 \leq P . \tag{4}$$

The max-min optimum lies on the boundary of $\mathcal{Q}(P)$. Weak links are assigned more power in order to compensate for their bad channel states. Although this strategy is fair, it has a drawback: If there are users with unacceptably high channel attenuations, then these users will require exceedingly large powers, which severely degrades the performance of the other users (and thus the overall system performance).

2) The power minimization problem:

$$\min_{z \in \mathcal{Z}, \boldsymbol{p} > 0} \sum_{k=1}^{K} \boldsymbol{p}_k \quad \text{s.t. } \min_k \frac{\phi\big(\text{SINR}_k(z_k, \boldsymbol{p})\big)}{Q_k} \geq 1 , \tag{5}$$

seeks for the point within the achievable region $\mathcal{Q}(P)$, which fulfills the QoS requirements exactly with minimum total power. Unlike problem (4), the problem formulation (5) is not necessarily feasible, i.e., $Q > 0$ cannot be supported

There is a close link between the weighted max-min problem (3) and (5). If a target $Q > 0$ is chosen such that $C(P, Q) = 1$, then Q lies on the boundary of the region and is equivalently achieved by (3) and (5). If $C(P, Q) > 1$, then additional degrees of freedom are available, which can be used to further reduce the total transmission power, as in (5).

3) The best overall efficiency: is defined as the maximum achievable sum of all QoS. This problem can be written as

$$\max_{z \in \mathcal{Z}, \, \boldsymbol{p} \geq 0} \sum_{k=1}^{K} \phi\big(\text{SINR}_k(z_k, \boldsymbol{p})\big) \quad \text{s.t. } \|\boldsymbol{p}\|_1 \leq P . \tag{6}$$

This strategy is "unfair" in a sense that it uses the available power resource to maximize the overall system efficiency, which may come at the cost of individual link performance.

It is important to notice that problem (6) differs from the "fair" strategies (4) and (5) in that users with bad channel conditions are allowed to be switched off. There are even system designs for which the optimum (6) is always achieved by transmitting only to the link with the best channel at any given time (see e.g. [1]). In general however, it is efficient to schedule multiple links simultaneously, as illustrated in Fig. 1.

In the following we focus on the subset of users which are active ($Q > 0$). Then (6) is also a special case of the max-min problem (3). That is, by properly choosing the targets Q (which in this case should rather be regarded as weighting factors), strategy (3) achieves the boundary point associated with the best overall efficiency.

III. Interference Model

Before studying the resource allocation strategies (4), (5), and (6) in more detail, we briefly review the underlying interference model in its most general form.

[1]Examples are the BER slope for α-fold diversity: $\phi(\text{SINR}_k) = 1/\text{SINR}^{\alpha}$, or the information-theoretical capacity: $\phi(\text{SINR}_k) = \log(1 + \text{SINR}_k)$, assuming Gaussian-distributed signals.

The mutual interference between the communication links is modeled by a coupling matrix $\mathbf{\Psi}(z) \geq 0$ being a function of the receiver design z. The component $\mathbf{\Psi}_{kl}$ is the cross-power coupling coefficient between the lth transmitter and the receiver with index k. Note, that the kth row of $\mathbf{\Psi}(z)$ only depends on the receiver z_k, which is because the receivers are assumed not to interact directly (only indirectly via the power allocation).

The links are not only interconnected by interference, but also by competition for the limited total power. This additional coupling depends on the effective noise powers $\boldsymbol{N}(z) = [N(z_1), \dots, N(z_K)]^T > 0$ As an example, consider a linear beamforming receiver $\boldsymbol{w}$ applied to the output of an antenna array, then the effective noise is proportional to $\|\boldsymbol{w}\|^2$.

Thus, the overall system can be described by an extended coupling matrix

$$\boldsymbol{G}(z) = \left[\ \mathbf{\Psi}(z) \ \middle| \ \boldsymbol{N}(z) \ \right] . \tag{7}$$

Together with the extended power allocation vector

$$\overline{\boldsymbol{p}} = \left[\begin{smallmatrix} \boldsymbol{p} \\ 1 \end{smallmatrix}\right] ,$$

the total interference+noise power experienced by the kth link can be written as

$$\boldsymbol{\mathcal{I}}_k(\overline{\boldsymbol{p}}) = \min_{z_k \in \mathcal{Z}_k} \left[\boldsymbol{G}(z)\, \overline{\boldsymbol{p}}\right]_k , \quad k \in \{1, 2, \dots, K\} , \tag{8}$$

Here, the receiver z_k is chosen so as to minimize the interference (and thus maximize the SINR) of this respective link. This specific receiver design is optimal with respect to problem (3). That means that any other receiver design results in an achievable region, which is a subset of the region defined by (2). Thus, we can focus on interference functions of the form (8) in the following.

Property 1. *The* interference function $\boldsymbol{\mathcal{I}}_k : \mathbb{R}_+^{K+1} \mapsto \mathbb{R}_+$ *has the following properties:*

A1: $\boldsymbol{\mathcal{I}}_k(\overline{\boldsymbol{p}})$ *is strictly positive and continuous on* $\mathbb{R}_+^{K+1}$

A2: $\boldsymbol{\mathcal{I}}_k(\mu\overline{\boldsymbol{p}}) = \mu \boldsymbol{\mathcal{I}}_k(\overline{\boldsymbol{p}})$ *for all* $\overline{\boldsymbol{p}} \in \mathbb{R}_+^{K+1}$ *and* $\mu > 0$.

A3: $\boldsymbol{\mathcal{I}}_k\big(\left[\begin{smallmatrix} \boldsymbol{p}^{(1)} \\ 1 \end{smallmatrix}\right]\big) \geq \boldsymbol{\mathcal{I}}_k\big(\left[\begin{smallmatrix} \boldsymbol{p}^{(2)} \\ 1 \end{smallmatrix}\right]\big)$ *if* $\boldsymbol{p}^{(1)} \geq \boldsymbol{p}^{(2)}$.

A4: $\boldsymbol{\mathcal{I}}_k\big(\left[\begin{smallmatrix} \boldsymbol{p} \\ a \end{smallmatrix}\right]\big) > \boldsymbol{\mathcal{I}}_k\big(\left[\begin{smallmatrix} \boldsymbol{p} \\ b \end{smallmatrix}\right]\big)$ *if* $a > b$.

These properties play an important role for the analysis of the algorithmic solutions presented in Sections IV and V. The framework is partly related to the concept of *standard interference functions* used by Yates in [2]. In addition, we exploit the specific properties of the model (8), thus the QoS region can be described by using techniques from matrix theory.

IV. The Boundary of the QoS Achievable Region

One important difference between problems (4)-(6) and conventional resource allocation (see e.g. [3]), are the elastic link capacities caused by adaptively choosing the receive strategy $z \in \mathcal{Z}$. This complicates an analytical treatment, since the receivers are linked with the power allocation via the relationship (8). Generally, the optimizer does not even need to be unique. Also, convexity properties, as analyzed in [4], [5], are more difficult to show under the assumption of adaptive receiver design.

A useful way of characterizing the achievable region (2) is by means of the extended coupling matrix

$$\mathbf{\Phi}(z, P, Q) = \begin{bmatrix} \mathbf{\Gamma}_{Q} \boldsymbol{G}(z) \\ \mathbf{1}^T \mathbf{\Gamma}_{Q} \boldsymbol{G}(z)/P \end{bmatrix} , \tag{9}$$

where $\mathbf{1}$ is the K-dimensional all-one vector, and $\mathbf{\Gamma}_{Q} = \mathrm{diag}\{\gamma_1, \dots, \gamma_K\}$, where $\gamma_k = \gamma(Q_k)$ is the minimum SINR level needed by the kth user to satisfy the QoS requirement Q_k, i.e., $\gamma = \phi^{[-1]}$ is the inverse function of ϕ.

The matrix $\mathbf{\Phi}$ has a real and simple maximum eigenvalue, which equals the spectral radius $\rho\big(\mathbf{\Phi}\big)$. The inverse spectral radius can be interpreted as the maximum balanced SINR margin. It is monotonically increasing in the total transmission power P, which is ensured by the last row $\mathbf{1}^T \mathbf{\Gamma}_{Q} \boldsymbol{G}(z)/P$. Thus, the max-min balancing problem (3) is equivalent to an eigenvalue optimization problem

$$C(P, Q) = \frac{1}{\min_{z \in \mathcal{Z}} \rho\big(\mathbf{\Phi}(z, P, Q)\big)} \tag{10}$$

and the QoS achievable region is given by

$$\mathcal{Q}(P) = \{Q : \min_{z \in \mathcal{Z}} \rho\big(\mathbf{\Phi}(z, P, Q)\big) \leq 1\} . \tag{11}$$

Let $\hat{Q}$ a point on the boundary of $\mathcal{Q}(P)$, i.e., $\min_z \rho\big(\mathbf{\Phi}(z, P, \hat{Q}\big) = 1$. Then the set of optimal receive strategies is given by

$$\mathcal{Z}_{\hat{Q}} = \{z : \rho\big(\mathbf{\Phi}(z, P, \hat{Q})\big) = 1\} . \tag{12}$$

A receive strategy $\hat{z}$ achieves the boundary point $\hat{Q}$, i.e., $\hat{z} \in \mathcal{Z}_{\hat{Q}}$, if and only if the following properties hold jointly:

$$\hat{z}_k = \arg\min_{z_k \in \mathcal{Z}_k} \big[\boldsymbol{G}(z)\, \hat{\overline{\boldsymbol{p}}}\big]_k , \quad k \in \{1, 2, \dots, K\} \tag{13}$$

$$\mathbf{\Phi}(\hat{z}, P, \hat{Q})\, \hat{\overline{\boldsymbol{p}}} = \hat{\overline{\boldsymbol{p}}}, \quad [\hat{\overline{\boldsymbol{p}}}]_{K+1} = 1 . \tag{14}$$

The power allocation $\hat{\boldsymbol{p}}$ which fulfills targets $\hat{Q}$ together with a receive strategy $\hat{z} \in \mathcal{Z}_{\hat{Q}}$ is given as the first K components of the right-hand principal eigenvector $\hat{\overline{\boldsymbol{p}}}$, scaled such that its last component equals one.

Note, that due to the possible non-uniqueness of the optimal receiver $\hat{z}$, there may exist different "optimal" matrices $\mathbf{\Phi}$. However, it can be shown, that all have the same principal right eigenvector.

Motivated by the optimality conditions (13) and (14), we propose the following iterative algorithm, which can be used to achieve a boundary point $Q > 0$.

Algorithm 1 Maximizing the jointly achievable QoS margin (problem (3))

initialize: $n := 0$, $\overline{\boldsymbol{p}}^{(0)} := [0, \ldots, 0, 1]^T$
repeat
  $n := n + 1$
  $z_k^{(n)} = \arg\min_{z_k} \Big[\boldsymbol{G}(z) \big[\begin{smallmatrix} \boldsymbol{p}^{(n-1)} \\ 1 \end{smallmatrix} \big] \Big]_k$, for all $k \in \{1, \ldots, K\}$
  $\mathbf{\Phi}(z^{(n)}, P, Q)\, \overline{\boldsymbol{p}}^{(n)} = \lambda_{max}(n)\, \overline{\boldsymbol{p}}^{(n)}$, where $[\hat{\overline{\boldsymbol{p}}}^{(n)}]_{K+1} = 1$
until $\lambda_{max}(n-1) - \lambda_{max}(n) \le \epsilon$

Here, $\lambda_{max}(n) = \rho\big(\mathbf{\Phi}(z^{(n)}, P, Q)\big)$ denotes the spectral radius of the coupling matrix associated with the nth iteration. The algorithm converges monotonically towards the global optimum of (3), i.e.,

$$C(P, Q) = 1/\lambda_{max}(n \to \infty) \ .$$

If Q is a boundary point, then $C(P, Q) = 1$. In general however, the achievable region is not known. In this case, Algorithm 1 can be used to check feasibility. The point Q is feasible if and only if $\lambda_{max}(n \to \infty) \le 1$.

By appropriately choosing the targets $Q > 0$, Algorithm 1 can achieve any point on the boundary of the QoS achievable region (see Fig. 1). Optimal max-min fairness, as discussed in Sec. II, is just a special case, which is achieved by choosing equal targets $Q_k = 1$ (or some other constant). Also the optimal overall system efficiency is a special point on the boundary of $\mathcal{Q}(P)$, and can be achieved by Algorithm 1.

Note, that this performance trade-off can likewise be accomplished by the following optimization strategy:

$$\max_{z \in \mathcal{Z}, \boldsymbol{p} \ge 0} \sum_{k=1}^{K} \alpha_k \cdot \phi\big(\mathrm{SINR}_k(z_k, \boldsymbol{p})\big) \ , \qquad (15)$$

where the weighting factors $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K]$ with $\|\boldsymbol{\alpha}\|_1 = 1$ are chosen so as to trade-off fairness against overall efficiency (see Fig 2).

Strategy (15) was studied in the context of multiuser MIMO uplink scheduling and resource allocation in [6], [7], where it was shown that it maximizes the stability region. An important difference to the trade-off strategy

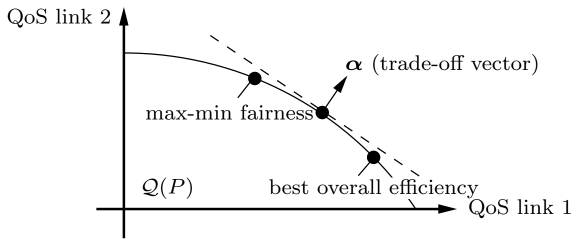

Fig. 2. Tradeoff between max-min fairness and optimal systems efficiency by weighted sum optimization (15).

(3) is that it allows optimization over the extended set $Q \ge 0$. That is, $Q = 0$ is a valid solution and links are allowed to be switched off (recall that the previous analysis was under the assumption of active links).

Thus, strategy (15) is interesting for the problem of joint scheduling and power control. A deeper understanding is desirable. Only a few results are available for joint power control and receiver design. One of the first results was in the context of joint beamforming [8], where it was tried to achieve max-min fairness by solving a sum of inverse SIR. This work was extended in [9], where it was shown how the weights $\boldsymbol{\alpha}$ should be chosen in order to achieve the extreme points "best overall efficiency" and "max-min fairness" (see Fig 2).

V. Power Minimization Strategies

So far we have considered the boundary of the QoS achievable region $\mathcal{Q}(P)$. Next, we focus on how to achieve an arbitrary point $Q \in \mathcal{Q}(P)$ with minimal total transmission power.

We start by reviewing the decentralized algorithm [2] (extended by a feasibility check), which achieves targets Q in a power-efficient way by the following iteration:

Algorithm 2 Decentralized Power Minimization

check feasibility by Alg. 1
initialize: $n := 0$, $\overline{\boldsymbol{p}}^{(0)} := [0, \ldots, 0, 1]^T$
repeat
  $n := n + 1$
  $\overline{\boldsymbol{p}}_k^{(n)} := \gamma_k \boldsymbol{\mathcal{I}}_k(\overline{\boldsymbol{p}}^{(n-1)}), \quad \forall k \in \{1, 2, \ldots, K\}$
until $\max_k(\overline{\boldsymbol{p}}_k^{(n)} - \overline{\boldsymbol{p}}_k^{(n-1)}) \le \epsilon$

Each iteration sequence $\overline{\boldsymbol{p}}_k^{(n)}$, $\forall k$, is monotonically increasing. Note, that for each iteration, only knowledge of the interference level $\boldsymbol{\mathcal{I}}_k$ of the kth link is required. All K sequences jointly converge towards to the unique power allocation $\boldsymbol{p}^{opt}$, which is the allocation that achieves the QoS targets Q with minimum total transmission power (see Fig. 2).

Note, that the interference function is defined in the special way (8), which means that it implicitly contains receiver optimization. Thus, the above algorithm provides a solution to the joint power minimization problem (5). This was already exploited in the context of joint beamforming and power allocation [10], [11], where a similar iteration was used. Algorithm 2 is more general in that it can be applied to arbitrary receiver designs, which fulfill the assumptions stated in Sec. III.

The required number of iterations of Algorithm 2 depends on how "near to infeasible" the QoS targets are. Generally, the convergence behavior deteriorates when the target Q is close to the boundary, i.e., $\lambda_{max}(n \to \infty) \approx 1$.

Multiuser receiver designs are based on joint channel information. If all K channels are known at the receiver (e.g. the base station), then some $Q \in \mathcal{Q}(P)$ can be achieved by the following centralized algorithm (superscript n denotes the nth iteration).

Algorithm 3 Centralized Power Minimization

initialize: $z^{(0)} = \arg\min_{z\in\mathcal{Z}} \rho\big(\mathbf{\Phi}(z,P,Q)\big)$ by solving Alg. 1 and check feasibility $\rho\big(\mathbf{\Phi}(z^{(0)},P,Q)\big) \le 1$
repeat
  $n := n+1$
  $\boldsymbol{p}^{(n)} = \big(\mathbf{\Gamma}_Q^{-1} - \mathbf{\Psi}(z^{(n)})\big)^{-1}\boldsymbol{N}(z^{(n)})$
  $z_k^{(n+1)} = \arg\min_{z_k}\Big[\boldsymbol{G}(z)\big[\begin{smallmatrix}\boldsymbol{p}^{(n)}\\ 1\end{smallmatrix}\big]\Big]_k, \quad \forall k$
until $\|\overline{\boldsymbol{p}}^{(n-1)}\|_1 - \|\overline{\boldsymbol{p}}^{(n)}\|_1 \le \epsilon$

Algorithm 3 approaches the total power minimum (2) up to a desired accuracy. The sequence $\boldsymbol{p}^{(n)}$ can be shown to be component-wise monotonically decreasing [12] and it converges to the unique power minimum (2).

The centralized Algorithm 3 updates the powers at each iteration such that the QoS targets are met exactly. The resulting power sequence is decreasing. The decentralized variant Alg. 2 approaches the optimal allocation by an increasing sequence. In this respect, the convergence behavior of both algorithms differs fundamentally.

VI. Conclusions

We propose cross-layer design strategies for joint resource allocation, power control and multiuser reception. Since these functionalities are tightly intertwined, a fundamental insight into the properties of the QoS achievable region is needed.

Our analysis is based on an abstract interference model. Inter-user interference is characterized by a parameter-dependent coupling matrix of the form (7). That is, the noise can be expressed by a separate column and the interference of the kth user only depends on the receiver z_k. One example is the joint optimization of transmission powers and beamformers, where z simply stands for a bank of linear receivers applied to the outputs of an antenna array.

The results presented here show that the key properties, which lead to the development of efficient algorithms, carry over to a a wide range of system designs. The framework can be used to achieve each point in the achievable QoS region. The computational complexity very much depends on how the parameter z affects the interference coupling $\mathbf{\Psi}(z)$. An example is given in [13], where beamforming is studied in combination with successive interference cancellation/precoding. In this case, the iterative algorithm 3 boils down to a very simple solution with a fixed number of steps.

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