An Alamouti-based Hybrid-ARQ Scheme for MIMO Systems

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Abstract— **A new Hybrid Automatic Repeat reQuest (HARQ) transmission scheme for a Multiple Input Multiple Output MIMO system consisting of two transmit antennas and M receive antennas $(2 \times M)$ in a slowly varying channel is proposed. This new scheme is a combination of the pre-combining HARQ scheme proposed in [1] and the Alamouti Space-Time Coding STC [2]. The technique increases the efficiency of HARQ packet transmission by exploiting both the spatial and time diversity of the MIMO channel. It uses the full diversity of Alamouti STC and the added gain of the pre-combining scheme [1] to provide reliable communication. Simulation results show that this new scheme outperforms the basis hopping technique presented in [1] and the soft combining scheme of [5].**

I. INTRODUCTION

A concern in packet data communication system is how to control the transmission errors caused by the channel noise and interferences such that data can be transmitted with a minimum error. HARQ (a combination of forward error correction FEC with the Automatic Repeat request ARQ protocol) schemes are usually considered to exploit both the high coding gain of FEC and the rate flexibility of ARQ protocol. In a pure ARQ protocol a received packet containing error is discarded and a retransmission of the packet is requested. In HARQ, previously received erroneous packets are combined in an intelligent way with the subsequent received packets to improve the decoding reliability. There are mainly two types of HARQ combining scheme: the packet combining [3] and Incremental Redundancy (IR) [4]. In the packet combining, the receiver combines noisy packets to obtain a packet with a code rate which is low enough such that reliable communication is possible even for low quality channels. In IR , systematic bits are sent first and if the receiver detect errors the transmitter will send only the parity bits to allow the receiver to improve the decoding. MIMO systems are known to increase the spectral efficiency and/or the capacity of a communication system. Combined with HARQ, a MIMO system can potentially provide higher throughput packet data services with higher reliability. Furthermore through proper arrangement of the retransmitted packets, one can improve the performance of a MIMO system. In [1], Onggosanusi introduced a packet transmission combining scheme for a Zero-forcing and MMSE receiver. Besides he used a technique termed Basis Hopping that artificially add time diversity in a slowly varying channel by multiplying the transmitted symbol vector by a unitary matrix. It is shown in [1] that this technique together with a pre-combining of the retransmitted packet prior to interference removal by the ZF or MMSE receivers improves the HARQ diversity gain particularly in a slowly varying channel. In [5] , a soft packet combining MIMO HARQ scheme is proposed in which the last two received packets are combined using joint Alamouti space-time decoding. For the case of two transmit antennas and M receive antennas, we propose in this paper to use for preprocessing, instead of multiplying by an unitary matrix when retransmitting subsequent packets (as in [1]), the Alamouti space-time coding. As a result of the MIMO channel diagonalization, the decoding process (particularly for an even total number of transmissions) is dramatically simplified. The rest of the paper is organized as follows. The system model is presented in section II. The MIMO HARQ combining schemes are discussed in Section III. Numerical results are presented in Section IV followed by the conclusion in V

II. SYSTEM MODEL

We consider a MIMO system with 2 transmits antennas and M receives antennas as shown in Figure 1. Information bits are first encoded with a high rate code C_0 for error detection and then with a half rate $(2,1,\mathrm{m})$ convolutional code C_1 for error correction. The coded packet is then demultiplexed into two separate data streams transmitted from the two individual transmits antennas. The two data streams are digitally modulated and simultaneously transmitted from the two antennas. The following assumptions are made in this paper:

- Slow fading channel: the channel matrix $\mathbf{H}$ remains constant upon N transmissions.
- The noise vector $\mathbf{n}$ observed for every transmission are independents
- Channel states are available at the receiver.

The signal packet transmitted from antenna 1 is denoted by s_1 and the one transmitted from antenna 2 is denoted by s_2 . The channel gain between the transmit antenna l and the receive antenna k is denoted by h_{kl} where $l = 1,2$ and $k = 1,2,\ldots,M$. The channel gains are assumed to be uncorrelated complex Gaussian random variables with unit variance. The composite MIMO channel gain can be represented by the following matrix:

This Work is supported in part by Samsung advanced Institute of Technology (SAIT) Korea.

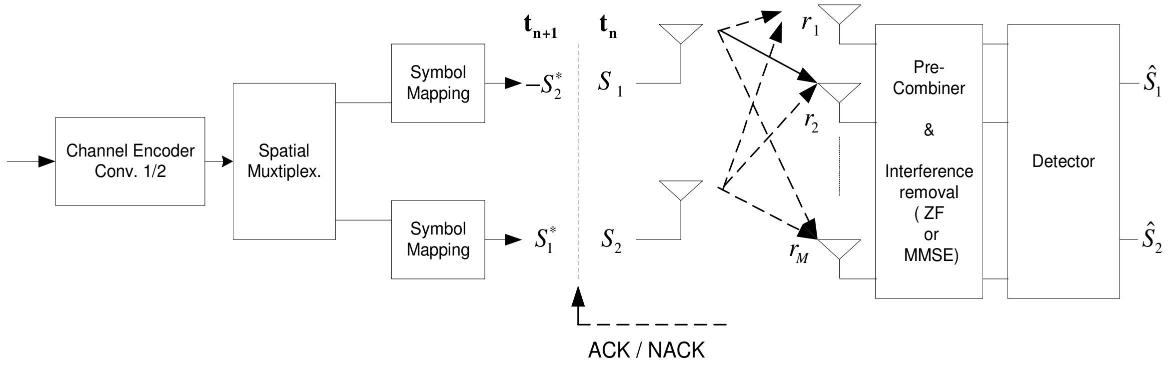

Figure 1. System Model

$$\mathbf{H} = \begin{pmatrix} \mathrm{h}_{11} & \mathrm{h}_{12} \\ \vdots & \vdots \\ \mathrm{h}_{M1} & \mathrm{h}_{M2} \end{pmatrix} \tag{1}$$

The baseband received signal vector is given by:

$$\mathbf{r}^{(i)} = \mathbf{H}^{(i)}\mathbf{s}^{(i)} + \mathbf{n}^{(i)} \ , \ i = 1,2,\cdots,N \tag{2}$$

where $\mathbf{n}^{(i)}$ is the AWGN vector associated with the *i-th* transmission , $\mathbf{s}^{(i)}$ is the transmitted packet at the *i-th* transmission , N is the number of maximum transmissions, $\mathbf{H}^{(i)}$ is the channel matrix at the *i-th* transmission which is assumed constant for the duration of N transmissions ($\mathbf{H}^{(i)} = \mathbf{H}$ for $i = 1,2,\ldots,N$). The received signal vectors are decoded at each transmission. If errors are detected the receiver requests a retransmission of the packet. The retransmitted packet and the previous erroneous packet are then combined together at the *symbol-level.*

III. MIMO HARQ COMBINING SCHEMES

A. *The Basis Hopping with Pre-Combining Scheme*

In [1], a transmission technique termed Basis Hopping is used to artificially add diversity in the slowly varying channel. The transmit signal vector is first multiplied by an unitary matrix $\mathbf{V}$. The received vector signal is then given by:

$$\mathbf{r}^{(i)} = \mathbf{H}\,\mathbf{V}^{(i)}\mathbf{s}^{(i)} + \mathbf{n}^{(i)} \ , \ i = 1,\ldots\ldots,N \tag{3}$$

The matrix $\mathbf{V}^{(i)}$ is different for every retransmission and it changes the MIMO channel from $\mathbf{H}$ to $\mathbf{H}\mathbf{V}^{(i)}$. By doing so, the unitary matrix $\mathbf{V}^{(i)}$ introduces time diversity upon retransmission. The received signal vectors $\mathbf{r}^{(i)}$ are combined before the interference cancellation by MMSE or ZF receiver.

B. *The Soft Packet Combining scheme*

In [5] an HARQ combining scheme similar to the one suggested in this paper is proposed. The first MIMO packet is sent as $[\mathbf{s}_1 \ \mathbf{s}_2]^T$; if the packet contains error, the retransmission of the same packet is sent as $[-\mathbf{s}_2^* \ \mathbf{s}_1^*]^T$ and the first and second transmissions are jointly decoded as an Alamouti space-time block code. If the second transmission is still in error, a third transmission is sent as $[\mathbf{s}_1 \ \mathbf{s}_2]^T$ and the second and third transmissions are jointly decoded as Alamouti space time block code. The first space-time decoding output and the second space-time decoding output are combined together using Chase combining [3]. This technique always combines the last two received packets using Alamouti space-time coding.

C. *The proposed HARQ combining scheme*

In the proposed scheme, the transmitted coded data stream is split into two sub-packets and sent from the two transmit antennas. The received signal vector at the *i-th* transmission is given by:

$$\mathbf{r}^{(i)} = \mathbf{H}\,\mathbf{s}^{(i)} + \mathbf{n}^{(i)} \ , \ i = 1,2,\cdots,N \tag{4}$$

where $\mathbf{r}^{(i)} = [r_1^{(i)} \ r_2^{(i)} \ \cdots \ r_M^{(i)}]^T \ \in \mathbb{C}^{\mathrm{M}}$

$\mathbf{s}^{(i)} = [\mathrm{s}_1 \ \mathrm{s}_2]^T$

$\mathbf{n}^{(i)} = [\mathrm{n}_1^{(i)} \ \mathrm{n}_2^{(i)} \ \cdots \ \mathrm{n}_M^{(i)}]^T \sim \ \mathcal{N}_C[\mathbf{0}_M, \sigma^2\mathbf{I}_M]$

After the *i-th* transmission, a linear $(\mathrm{ZF\ or\ MMSE})$ is used at the receiver to remove the interference , separate the two transmitted data packets and independently decode them. After decoding if the received packets contain no error, the packets are accepted and a positive Acknowledgment (ACK) is sent to the transmitter otherwise the receiver sends a Negative Acknowledgement (NACK) and the transmitter resends the packet using Alamouti Space Time Coding scheme, i.e. new packets composed of $[-\mathrm{s}_2^* \ \mathrm{s}_1^*]^T$ are sent from the two transmit antennas. The received signal vector at the $(i+1)$-*th* transmission is given by:

$$\mathbf{r}^{(i+1)} = \mathbf{H}\,\mathbf{s}^{(i+1)} + \mathbf{n}^{(i+1)} \ , \tag{5}$$

where $[S_1 \;\; \mathbf{s}^{(i+1)} = [-\mathrm{s}_2^* \;\; \mathrm{s}_1^*\,]^T$

By taking the conjugate of the new received vector packets we obtain :

$$\mathbf{r}^{(i+1)*} = \mathbf{H}^* \mathscr{J} \mathbf{s}^{(i)} + \mathbf{n}^{(i+1)*} \quad , \quad i = 1,\ldots\ldots,N \qquad (6)$$

where $\mathbf{H}^* = \begin{pmatrix} \mathrm{h}_{11}^* & \mathrm{h}_{12}^* \\ \vdots & \vdots \\ \mathrm{h}_{\mathrm{M}1}^* & \mathrm{h}_{\mathrm{M}2}^* \end{pmatrix}$, $\mathscr{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

and $\quad \mathbf{r}^{(i+1)*} = [r_1^{(i+1)*} \; r_2^{(i+1)*} \; \cdots \; r_M^{(i+1)*}]^T$

From (6) , it is clear that taking the conjugate of the received vector $\mathbf{r}^{(i+1)}$ is equivalent to re-sending the previous vector signal through the new channel $\mathbf{H}^*\mathscr{J}$ that add a time diversity.

The received vector $\mathbf{r}^{(i+1)}$ is first processed by the linear receiver front end i.e. multiply symbol wise by $(\mathbf{H}^*\mathscr{J})^H = \mathscr{J}^T\mathbf{H}^T$ and then a symbol level combining is employed to provides the soft symbol decision for $\mathrm{s}^{(i)}$ ($\mathbf{H}^*$ is the conjugate of $\mathbf{H}$, $\mathbf{H}^H$ is the transpose conjugate of $\mathbf{H}$ and $\mathbf{H}^T$ is the transpose of $\mathbf{H}$). This is equivalent to combining the vector $\mathbf{r}^{(i+1)*}$ with the previously received vector $\mathbf{r}^{(i)}$ using the pre-combining scheme of [1] . If after decoding the combined packets, no error occurs, the packets are accepted and a ACK is sent otherwise the transmitter resends the packets as in the *i-th* transmission i.e. $\mathbf{s}^{(i+2)} = [\mathrm{s}_1 \; \mathrm{s}_2]^T$ The newly received signal vector $\mathbf{r}^{(i+2)}$ is combined with the two previous received vectors $\mathbf{r}^{(i+1)*}$ and $\mathbf{r}^{(i)}$. The procedure continues ($[\mathrm{s}_1 \; \mathrm{s}_2]^T$ sent during odd transmission and $[-\mathrm{s}_2^* \;\; \mathrm{s}_1^*\,]^T$ sent during even transmission and all the received vector signal are combined as in [1]) until the packets are correctly decoded or until a preset maximum allowed number of retransmission attempts is reached.

D. Analysis of the Proposed Alamouti-based MIMO HARQ

The channel at every odd transmission is given by $\mathbf{H}$ and at every even transmission by $\mathbf{H}^*\mathscr{J}$. At the *i-th* transmission, applying the matched filter $\mathbf{H}^H$ to the received vector $\mathbf{r}^{(i)}$ results in the reception of the signal vector $\mathbf{x}^{(i)} = \mathbf{H}^H\mathbf{r}^{(i)}$. We assume that the transmitted signal vector $\mathbf{s} = [s_1 \; s_2]^T$ is zero-mean with $\mathrm{E}[\mathbf{ss}^H] = diag(\lambda_1, \lambda_2)$. Without loss of generality we assume that $\lambda_1 = \lambda_2 = 1$. After N transmissions, both receivers (ZF or MMSE) can be described by the following detection principle:

$$\hat{\mathbf{s}} = (\mathbf{C} + \alpha \mathbf{I}_2{}^{-1})^{-1}\,\mathbf{x} \qquad (7)$$

where $\hat{\mathbf{s}}$ is the soft decision statistic,

$$\mathbf{C} = \sum_{i=1}^{N} \mathbf{P}^{(i)H}\mathbf{P}^{(i)} \qquad \text{is a } 2 \times 2 \text{ matrix,} \qquad (8)$$

$$\mathbf{P}^{(i)} = \begin{cases} \mathbf{H} & for\ odd\ i\ , i = 1,3,5\ldots\ldots \\ \mathbf{H}^*\mathscr{J} & for\ even\ i\ , i = 2,4,6\ldots\ldots \end{cases}$$

$$\mathbf{x} = \sum_{i=1}^{N} \mathbf{x}^{(i)} = \mathbf{C}\,\mathbf{s} + \mathcal{N}_C[\mathbf{0}_2, \sigma^2\mathbf{C}] \qquad (9)$$

$$\alpha = \begin{cases} 0 & \text{for} \quad \text{ZF} \\ \sigma^2 & \text{for} \quad \text{MMSE} \end{cases},$$

and $\mathbf{I}_2$ is the 2×2 identity matrix. Using (8) and (9) in (7) yields:

$$\hat{\mathbf{s}} = (\mathbf{C} + \alpha\mathbf{I}_2)^{-1}\,\mathbf{C}\;\mathbf{s}$$
$$+\mathcal{N}_C[\mathbf{0}_2, \sigma^2(\mathbf{C} + \alpha\mathbf{I}_2)^{-1}\,\mathbf{C}\,(\mathbf{C} + \alpha\mathbf{I}_2)^{-1H}\,] \qquad (10)$$

- If the total number of transmissions N is even , it can be easily shown that :

$$\mathbf{C} = \tfrac{N}{2}\mathrm{h}^2\,\mathbf{I}_2 \qquad (11)$$

where $\mathrm{h}^2 = \sum_{k=1}^{M} |\mathrm{h}_{k1}|^2 + |\mathrm{h}_{k2}|^2$

The soft decision statistic $\hat{\mathbf{s}}$ is then given by :

$$\hat{\mathbf{s}}_{(even)} = \beta\,\mathbf{s} + \mathcal{N}_C[\mathbf{0}_2, \sigma^2 \frac{\frac{N}{2}h^2}{(\frac{N}{2}h^2 + \alpha)^2}] \qquad (12)$$

where $\beta = \dfrac{\frac{N}{2}h^2}{\frac{N}{2}h^2 + \alpha}$ is a scalar. No matrix inversion is needed meaning that the ZF or MMSE processing is unnecessary. The signal to noise ratio is given by :

$$SNR_{(even)} = \frac{\beta^2}{\sigma^2 \dfrac{\frac{N}{2}h^2}{(\frac{N}{2}h^2 + \alpha)^2}} = \frac{\frac{N}{2}h^2}{\sigma^2} \qquad (13)$$

Obviously, for an even number of transmission, the performances of the ZF and MMSE receivers are the same. This is because the channel is diagonalized by the Alamouti space time coding (the interferences are cancelled already). Also the SNR performance is proportional to the number of transmission i.e. the more transmissions the better performance we get. This is also shown in the simulation depicted in the next section.

- If the total number of transmissions N is odd, we have

$$\mathbf{C}_{(odd)} = \begin{pmatrix} a_1^2 & b \\ b^* & a_2^2 \end{pmatrix} \tag{14}$$

where

$$a_1^2 = \tfrac{N+1}{2}\sum_{k=1}^{M}|h_{k1}|^2 + \tfrac{N-1}{2}\sum_{k=1}^{M}|h_{k2}|^2$$

$$a_2^2 = \tfrac{N+1}{2}\sum_{k=1}^{M}|h_{k2}|^2 + \tfrac{N-1}{2}\sum_{k=1}^{M}|h_{k1}|^2$$

$$b = \sum_{k=1}^{M} h_{k1}^* h_{k2} \tag{15}$$

For the ZF receiver (*N* odd case), the decision statistic is given by:

$$\hat{\mathbf{s}}_{(odd)}^{ZF} = \mathbf{s} + \mathcal{N}_C[\mathbf{0}_2, \sigma^2 \mathbf{C}_{(odd)}^{-1H}] \tag{16}$$

The signal to noise ratio is given by:

$$SNR_{(odd)ZF}^{(1)} = \frac{1}{\sigma^2[\mathbf{C}_{(odd)}^{-1H}]_{2,2}} = \frac{(a_1^2\, a_2^2 - |b|^2)}{\sigma^2 a_1^2} \tag{17}$$

$$SNR_{(odd)ZF}^{(2)} = \frac{1}{\sigma^2[\mathbf{C}_{(odd)}^{-1H}]_{1,1}} = \frac{(a_1^2\, a_2^2 - |b|^2)}{\sigma^2 a_2^2} \tag{18}$$

where $SNR_{(odd)ZF}^{(p)}$, $p = 1, 2$ is the signal to noise ratio corresponding to the *p-th* detected signal packet. From (17) and (18) one can notice a signal enhancement by the term $(a_1^2\, a_2^2 - |b|^2)$ and a noise enhancement by a_1^2 for $SNR_{(odd)ZF}^{(1)}$ and a_2^2 for $SNR_{(odd)ZF}^{(2)}$

For the MMSE receiver , the decision statistic is given by:

$$\hat{\mathbf{s}}_{(odd)}^{MMSE} = (\mathbf{C}_{(odd)} + \sigma^2\mathbf{I}_2)^{-1}\,\mathbf{C}_{(odd)}\,\mathbf{s} + \mathcal{N}_C[\mathbf{0}_2, \sigma^2(\mathbf{C}_{(odd)} + \sigma^2\mathbf{I}_2)^{-1}\,\mathbf{C}_{(odd)}\,(\mathbf{C}_{(odd)} + \sigma^2\mathbf{I}_2)^{-1H}] \tag{19}$$

Using (14) and (15) in (19) we get:

$$SNR_{(odd)MMSE}^{(1)} = \frac{[a_1^2(a_2^2 + \sigma^2) - |b|^2]^2}{a_1^2 a_2^4 + 2a_1^2 a_2^2 \sigma^2 + a_1^2 \sigma^4 - |b|^2\, a_2^2 - 2|b|^2\, \sigma^2} \tag{20}$$

$$SNR_{(odd)MMSE}^{(2)} = \frac{[a_2^2(a_1^2 + \sigma^2) - |b|^2]^2}{a_2^2 a_1^4 + 2a_2^2 a_1^2 \sigma^2 + a_2^2 \sigma^4 - |b|^2\, a_1^2 - 2|b|^2\, \sigma^2} \tag{21}$$

The numerator of (20) and (21) shows the signal gain while the denominator shows the noise enhancement

IV. NUMERICAL RESULTS

The performance of the schemes presented in this paper are simulated with a MIMO channel consisting of 2 transmit antennas and 2 receive antennas (2×2 MIMO system); extension to more than 2 receive antennas is trivial. The channel is assumed constant for a maximum of *N* transmissions. The 4 channel gains are i.i.d. complex Gaussian random variables and with uniform power. The size of the information bit packet is taken to be 512 bits. A half rate (2, 1, 4) convolutional code (17, 15) is employed to encode the data packet and a QPSK modulation is used for symbol mapping. The bit error rates are plotted versus E_B/N_0 for different number of transmissions and compared to the Basis Hopping scheme of [1] and the soft packet combining scheme of [5] in Figures 2, 3, 4. As expected with the proposed scheme, when the total number of transmission is even, the performance with ZF and MMSE performance is the same. Moreover no matrix inversion is needed which reduces the complexity of the decoding. When the total number of transmission is odd, despite having non orthogonal combination, the processing by the linear ZF or MMSE take care of the cross interference though with extra decoding complexity due to the matrix inversion. In fact, when *N* is even the linear ZF and MMSE are not needed since the Alamouti space-time coding remove the interference. The bit error rate performance of the proposed scheme is shown in Figure 5 for even and odd number of transmissions (*N*=2, 3, 4, 5) and linear ZF receiver

V. CONCLUSION

HARQ is an important protocol used in packet transmission to provide reliable data communication. MIMO systems are also well known to increase the spectral efficiency and the capacity of a communication system. In this paper a MIMO HARQ technique is proposed for a ($2\times M$) MIMO channel. The new technique exploits both the space-time coding gain of Alamouti STC and the pre-combining gain of [1].It retransmits the HARQ packet using an orthogonal space time code (Alamouti space-time-code is used) and combined all the received packets instead of the last two as in [5]. It simplifies the computation of the decisions statistics especially when the total number of transmissions is even since the Alamouti space-time coding take care of the cross interference. When the total number of transmission is odd, the interference is removed by a linear ZF or MMSE. It does not need a preprocessing of the transmitted data as in [1]. Simulation shows that the performance of our scheme is better than the Basis Hopping scheme proposed in [1] and the soft packet combining of [5]. Note that the technique is valid only in a slow varying channel. Extension to MIMO channel with more than two transmit antennas is under investigation.

REFERENCES

[1] Eko N. Onggosanusi, Anand G. Dabak, Yan Hui, Gibong Jeong, *"Hybrid ARQ transmission and combining for MIMO systems"*, ICC 2003 - IEEE International Conference on Communications, vol. 26, no. 1, May 2003 pp. 3205-3209J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.

[2] S.M. Alamouti. *"A Simple Transmit Diversity Technique for Wireless Communications"* IEEE JSAC, Vol 16(8), pp 1451-1458. October 1998.

[3] D. Chase, "Code combining- a maximum likelihood decoding approach for combining an arbitrary number of noise packets," IEEE Transactions on Communications, vol. COM-33, pp. 385-393, May 1985

[4] Yoon, S. and Bar-Ness Y., " Packet Data Communications over coded CDMA, part II: Throughtput-bound of CDMA in Slotted Aloha with Hybrid Type II ARQ using Rate Compatible Punctured Turbo Coded" IEEE Trans. On Wireless Comm., Vol 3 , pp 1616-1625 Sept 2004

[5] W. Tong, et. al., "Soft packet combing for STC re-transmission to improve H-ARQ performance in MIMO mode",IEEE 802.16 Task Group e Contributing Documents, Doc. IEEE C802.16e-04/113r2, July 2004

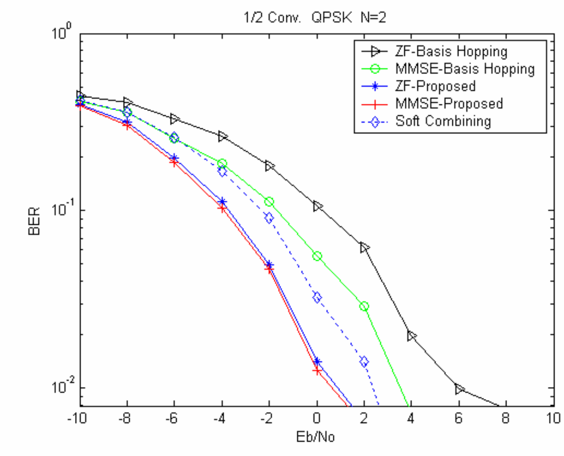

Figure 2. **Simulated performance of the proposed scheme with the Basis Hopping and the Soft Combining schemes for N=2 transmissions**

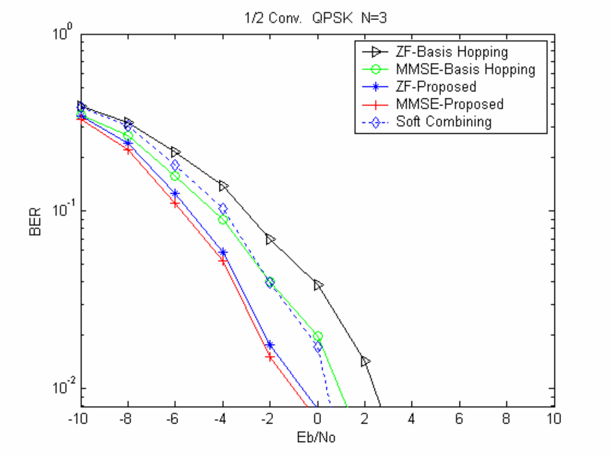

Figure 3. **Simulated performance of the proposed scheme with the Basis Hopping and Soft Combining schemes for N=3 transmissions**

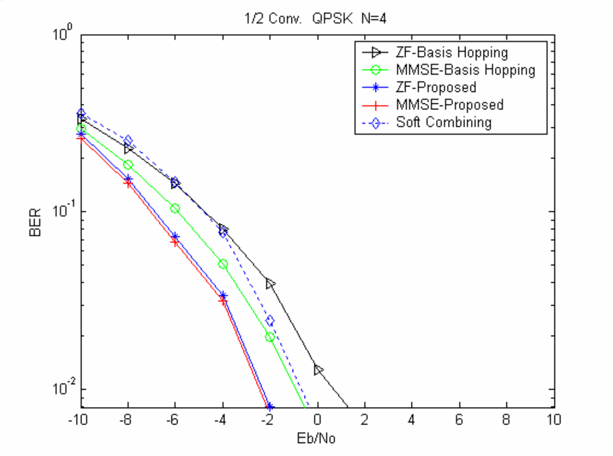

Figure 4. **Simulated performance of the proposed scheme with the Basis Hopping and the Soft Combining schemes for N=4 transmissions**

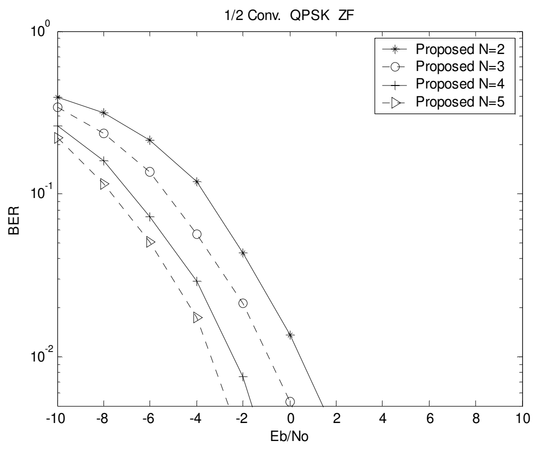

Figure 5. **Simulated Performance of the proprosed scheme with ZF receiver for different total number of transmissions**