# Design Rules for Efficient Scheduling of Packet Data on Multiple Antenna Downlink

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### ABSTRACT

**We provide a novel analysis of the optimal number of users that should be allocated power in order to achieve the sum-capacity of the MIMO broadcast channel, as well as the optimal power allocation and the optimal transmitter covariance matrices in the asymptotically high power region. We study cases where receivers are equipped with a single or with multiple antennas, and point out the fundamental differences between these systems. This analysis is then applied to multiple users scheduling algorithms for throughput maximization, with the additional goal of providing low-complexity solutions. We also provide a discussion of the similarities and differences with receive antenna selection algorithms and MIMO channels with co-channel interference.**

## I. INTRODUCTION

MIMO systems are envisioned to enable high-speed data transmission on the downlink of fourth generation wireless networks. When the base station is equipped with multiple transmit antennas, a scheduler takes advantage of the latency allowed with packet data communication and provides increased throughput by means of multiuser diversity. Recent information theoretic advances have proved that the nature of the MIMO broadcast channel (BC) requires to revisit scheduling algorithms to be able to transmit to more than one user at a time in each fading state in order to achieve sum-capacity [1][2][3][4]. The very first attempts at combining the advantages of MIMO systems with multiuser diversity used single-user scheduling strategies. However these cannot achieve spatial multiplexing gain if the receivers have a single antenna, and they can even suffer from channel hardening if no care is taken in the exploitation of spatial diversity jointly with multiuser diversity [5].

We revisit scheduling algorithm strategies when the base station is equipped with multiple transmit antennas. Our analysis provides guidelines for the design of near-optimal low-complexity scheduling algorithms for maximum throughput with optimal signaling, i.e. dirty-paper coding [6]. We thus obtain design criteria for scheduling algorithms applicable with sub-optimal transmission schemes, such as transmitter channel inversion.

We tackle the problem by exploring the following two fundamental questions. What is the optimal number of active users in any given fading state or channel realization? What are the optimal power allocation and transmit covariance matrices? Numerical methods exist that allow to obtain the optimal covariance matrices, but a more precise understanding of the underlying solution is still required. The underlying objective of throughput maximization with multiple antennas is to achieve the maximum spatial multiplexing gain available in MIMO systems. Hence, the nature of the problem largely depends on the number of antennas at each mobile user's receiver.

The scope of this paper is limited to throughput maximization. However in cellular systems fairness must be ensured to prevent some users, typically those close to the base station, to hold all the resources. The study presented here is applicable to groups of users with the same statistical channel conditions over a short period of time, in which case fairness in terms of throughput and delay will be provided statistically. One scenario one could think of is to group users is such a way and apply throughput maximization within the groups, while fairness is provided by some other means between groups of users. Proportionally-fair scheduling has been considered in a broader context [9], but due to the assumed short-term statistical equivalence of all user channels, it is not useful in this study.

The remainder of the paper is organized as follows. In Section II we present the channel model. We briefly review recent advances on the MIMO BC in Section III. We provide a novel analysis of the optimal number of active users and their power allocation in the high power region in Section IV. We apply our analysis to the design of low-complexity maximum-throughput scheduling algorithms in Section V. Conclusions are given in Section VI.

## II. SYSTEM MODEL

We consider the downlink of a cellular system, in which the base station has $N$ transmit antennas. There are $K$ users in the sector served by the base station. User $k$ is equipped with $M_k$ receive antennas. We call this channel the $(N,M_k,K)$ MIMO BC. The complex channel gains are assumed to be independent between users and between antenna elements. The channel remains constant in each time slot, and changes randomly from slot to slot (quasi-static fading). The channel between the base station and user $k$ is described by a matrix $\boldsymbol{H}_k$ of size $M_k$ by $N$. The elements of $\boldsymbol{H}_k$ represent small-scale fading and they are modeled as i.i.d. complex Gaussian random variables with zero mean and unit variance. The AWGN variance at each receive antenna is normalized to one. The transmitter has a total power constraint $P$ at each channel use. We assume that the transmitter and the mobile users perfectly know all channel complex fading gains. We define the channel matrix $\boldsymbol{H} = [\boldsymbol{H}_1^T \quad \ldots \quad \boldsymbol{H}_N^T]^T$ when $M_k = 1$ for all users. The absence of path loss and shadow fading makes this model applicable for groups of users with approximately the same SNR averaged over small-scale fading over several time slots. When shadow fading and path loss are accounted for, more elaborate scheduling strategies must be applied, such as proportionally-fair scheduling. As shown in [9], the insights obtained in the analysis of throughput maximization with the simple channel model considered here are still relevant in the analysis of systems assuming more complete channel models.

## III. Background on the MIMO Broadcast Channel

The MIMO BC is a degraded broadcast channel. It was recently proved that dirty-paper coding achieves the capacity region of that channel [4]. Dirty-paper coding is a theoretical random coding technique for interference cancellation at the transmitter of a non-causally known interference source [6]. In particular, the sum-capacity of the MIMO BC is achievable by dirty-paper coding with successive encoding at the transmitter and optimal transmit covariance matrices. These optimal matrices can be obtained numerically through efficient algorithms [7]. The problem is in general solved on the dual sum-power MIMO multiple-access channel (MAC) where it is convex. The optimal covariance matrices obtained for the sum-power MIMO MAC can then be transformed to give the optimal covariance matrices on the MIMO BC [3], such that the rate vectors achieved on both channels are the same. The transmit covariance matrices for the MIMO BC depend on the chosen encoding order. The power allocated to a given user is equal to the trace of its transmit covariance matrix, and there is no conservation of the power allocation between the MIMO MAC and the MIMO BC. Thus it is not sufficient to study the optimal power allocation on the sum-power MIMO MAC. However, a user that is allocated no power on either channel is also allocated no power on the dual channel. Thus, the optimal number of active users remains the same.

We briefly review the MAC to BC transformations [3]. Successive decoding is used on the MIMO MAC. The decoding order is the following: user 1 is decoded first, user 2 is decoded second, and so on until user $K$ is decoded last. The same rate vector is achieved on the MIMO BC using the covariance matrices obtained with the MAC to BC transformations when user 1 is encoded last, user 2 is encoded second to last, and so on with user $K$ being encoded first. The optimal covariance matrix of size $M_i \times M_i$ of user $i$ on the MIMO MAC is $\boldsymbol{P}_i$. It does not depend on the decoding order chosen on the MAC. The rate vector achievable on the MAC is determined by the decoding order. The reversed encoding order must be used on the BC in order to achieve the same rate vector, with the optimal covariance matrix of size $N \times N$ of user $i$ on the MIMO BC given by

$$\boldsymbol{\Sigma}_i = \boldsymbol{B}_i^{-1/2} \boldsymbol{F}_i \boldsymbol{G}_i^* \boldsymbol{A}_i^{1/2} \boldsymbol{P}_i \boldsymbol{A}_i^{1/2} \boldsymbol{G}_i \boldsymbol{F}_i^* \boldsymbol{B}_i^{-1/2}, \qquad (1)$$

where the singular value decomposition of the effective channel is

$$\boldsymbol{B}_i^{-1/2} \boldsymbol{H}_i^* \boldsymbol{A}_i^{-1/2} = \boldsymbol{F}_i \boldsymbol{\Lambda}_i \boldsymbol{G}_i^* . \qquad (2)$$

$\boldsymbol{F}_i$ and $\boldsymbol{G}_i$ are unitary matrices, and $\boldsymbol{\Lambda}_i$ is a diagonal matrix with nonnegative main diagonal entries. Let $\boldsymbol{I}_N$ be the identity matrix of size $N \times N$. $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ represent the interference experienced by user $i$ on the BC and on the MAC respectively:

$$\boldsymbol{A}_i = \boldsymbol{I}_{M_i} + \boldsymbol{H}_i \left( \sum_{k=1}^{i-1} \boldsymbol{\Sigma}_k \right) \boldsymbol{H}_i^* \qquad (3)$$

$$\boldsymbol{B}_i = \boldsymbol{I}_N + \sum_{k=i+1}^{K} \boldsymbol{H}_k^* \boldsymbol{P}_k \boldsymbol{H}_k . \qquad (4)$$

## IV. Asymptotically Optimal Number of Active Users and Power Allocation

### A. Single-antenna receivers

This section is devoted to studying the asymptotically optimal power allocation required to achieve the sum-capacity of the ($N$,1,$K$) MIMO BC in the limit when the total transmit power becomes large. On the dual sum-power MIMO MAC the covariance matrix of user $i$ is a scalar $p_i$. Define

$$r_i = \lim_{P \to \infty} \frac{p_i}{P} . \qquad (5)$$

In order to prove our results, we make the following assumptions:

- There are at least as many users as transmit antennas: $K \geq N$.
- At least $N$ users are allocated a non-vanishing fraction of the total transmit power on the dual sum-power MIMO MAC. We number these users such that $r_i > 0$ for $i = K, \ldots, K-N+1$.

These assumptions are reasonable. As long as there are at least as many users as transmit antennas, it is only possible to exploit the $N$ dimensions available in the MIMO channels by allocating a non-vanishing fraction of the total transmit power to at least $N$ users in the high power region. In fact, it is even possible that more than $N$ users are allocated a non-vanishing fraction of the total transmit power in order to achieve the sum-capacity of the dual sum-power MIMO MAC [8]. After MAC to BC transformations of the MAC power allocation we obtain the optimal BC transmit covariance matrices $\boldsymbol{\Sigma}_1, \ldots, \boldsymbol{\Sigma}_K$. We can prove that [9]:

$$\lim_{P \to \infty} \frac{Tr\left(\boldsymbol{\Sigma}_i\right)}{P} = 0 \text{ if } i \leq K-N \qquad (6)$$

$$\lim_{P \to \infty} \frac{Tr\left(\boldsymbol{\Sigma}_i\right)}{P} > r_i \text{ if } i > K-N \qquad (7)$$

Therefore only $N$ users are allocated a non-vanishing fraction of the total transmit power at the sum-capacity in

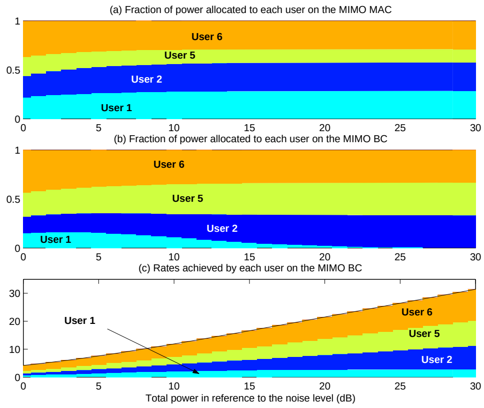


Fig. 1. (3,1,8) MIMO BC optimal power allocation and users rates with encoding order 8 to 1.

the high power region on the MIMO BC. Furthermore we also prove the following property of the optimal BC covariance matrices of these $N$ users. Let the rank-one optimal covariance matrix of user $i$ be:

$$\boldsymbol{\Sigma}_i = tr\left(\boldsymbol{\Sigma}_i\right)\boldsymbol{v}_i^*\boldsymbol{v}_i\,, \tag{8}$$

where $\boldsymbol{v}_i$ is the transmit beamforming vector of user $i$ and $\pi_i = tr\left(\boldsymbol{\Sigma}_i\right)$. We prove that for a given $j > K - N$ [9]:

$$\lim_{P\to\infty} \boldsymbol{H}_j\boldsymbol{v}_i^* = 0 \text{ for all } i \text{ such that } K - N < i < j\,. \tag{9}$$

This result tells us that asymptotically in the high power region on the ($N$,1,$K$) MIMO BC, the optimal transmit beamforming vector of user $i$, who is among the first $N$ users to be encoded, becomes asymptotically orthogonal to the channel matrices of all other users that are allocated an asymptotically non-vanishing fraction of the total transmit power and that are encoded prior to user $i$.

As a consequence of the above property on the ($N$,1,$N$) MIMO BC, we can express the asymptotic rates achieved by the $N$ users in the high power region and we can solve for the first-order asymptotic value of the sum-capacity in a simple way:

$$\lim_{P\to\infty} C_{sum}^{BC} = \sum_{i=1}^{N} \lim_{P\to\infty} R_i^{BC} \approx \sum_{i=1}^{N} log\left|\boldsymbol{I}_{M_i} + \boldsymbol{H}_i\boldsymbol{\Sigma}_i\boldsymbol{H}_i^*\right|. \tag{10}$$

The optimization over the covariance matrices with the orthogonality constraint can be reformulated as:

$$\lim_{P\to\infty} C_{sum}^{BC} \approx \max_{\substack{\pi_1,\ldots,\pi_N \\ \boldsymbol{v}_1,\ldots,\boldsymbol{v}_N}} \sum_{i=1}^{N} log\left(1 + \pi_i \boldsymbol{H}_i\boldsymbol{v}_i^*\boldsymbol{v}_i\boldsymbol{H}_i^*\right)$$

$$\text{subject to } \sum_{i=1}^{N}\pi_i = P \text{ and } \pi_i \geq 0\,,\ i = 1,\ldots,N$$

$$\text{and } \forall i = 1,\ldots,N:\ \boldsymbol{H}_j\boldsymbol{v}_i^* = 0, \forall j > i \text{ and } \left\|\boldsymbol{v}_i\right\| = 1\,. \tag{11}$$

Solving the above problem leads to the QR decomposition of the channel matrix $\boldsymbol{H} = \boldsymbol{RQ}$ and waterfilling power allocation. Let the ($i$,$i$) diagonal element of the upper triangular matrix $\boldsymbol{R}$ be $r_{ii}$, then

$$\lim_{P\to\infty} C_{sum}^{BC} \approx N\,log\left(P + \sum_{n=1}^{N}\frac{1}{r_{nn}^2}\right) - N\,log\,N + \sum_{i=1}^{N} log\left(r_{ii}^2\right) \tag{12}$$

As a first order approximation, the asymptotically optimal power allocation on the MIMO BC, given an arbitrary encoding order, is uniform over the $N$ users that are allocated a non-vanishing fraction of the total power in the high power region. Our proof is valid for any $N$.

The asymptotic optimality of uniform power allocation in the high power region had been shown for $N = 2$ in [1] where the transmission strategy used a QR decomposition and dirty-paper coding. Here we prove that not only QR decomposition with dirty-paper coding is asymptotically optimal, but it is the first-order asymptotically optimal procedure to achieve the sum-capacity of the ($N$,1,$K$) MIMO BC in the high power region for any $N$. As a consequence of the orthogonality property we deduced that uniform power allocation is optimal on the ($N$,1,$N$) MIMO BC in the high power region. We can thus directly prove that the asymptotically optimal power allocation on the dual sum-power MIMO MAC is also uniform for any $N$, which was previously known only for large $N$ [10].

We now show some numerical results to illustrate our findings. We consider a fixed channel realization and we let the total transmit power increase to very large values. We observe the fraction of the total power allocated to each user both on the dual MIMO MAC and on the MIMO BC. We consider one realization of the (3,1,8) MIMO BC. Figure 1(a) shows the optimal power allocation on the MIMO MAC as a function of the total transmit power in reference to the noise level. The power allocated to each user is normalized to the total transmit power. The power allocation is independent of the encoding order. Only users {1,2,5,6} are allocated a non-vanishing fraction of the total transmit power in the high power region on the MIMO MAC. Since there are 3 base station antennas, after MAC to BC transformations with encoding order 8 to 1, only users {2,5,6} are still allocated a non-vanishing fraction of the total transmit power in the high power region on the MIMO BC as shown in Figure 1(b). The rate achieved by user $i$ for the set of optimal covariance matrices $\boldsymbol{\Sigma}_1,\ldots,\boldsymbol{\Sigma}_K$ and the encoding order $K$ to 1 is given by:

$$R_i^{BC} = \log\frac{\left|\boldsymbol{I}_{M_i} + \boldsymbol{H}_i\left(\sum_{j\leq i}\boldsymbol{\Sigma}_j\right)\boldsymbol{H}_i^*\right|}{\left|\boldsymbol{I}_{M_i} + \boldsymbol{H}_i\left(\sum_{j< i}\boldsymbol{\Sigma}_j\right)\boldsymbol{H}_i^*\right|} \tag{13}$$

These rates are shown in Figure 1(c). User 1 achieves a constant non-zero rate at high power, which becomes asymptotically negligible with respect to the sum-capacity.

### B. *Multiple-antenna receivers*

With $N$ antennas at each receiver only $N$ one-dimensional channels are allocated a non-vanishing fraction of the total transmit power on the MIMO BC when the total transmit power goes to infinity. These $N$ one-dimensional channels all belong to the same user, as long as that user is allocated a non-vanishing fraction of the total transmit power on the dual MIMO MAC and its MAC covariance matrix is full rank asymptotically, and that user is encoded first by dirty-paper coding. Thus we

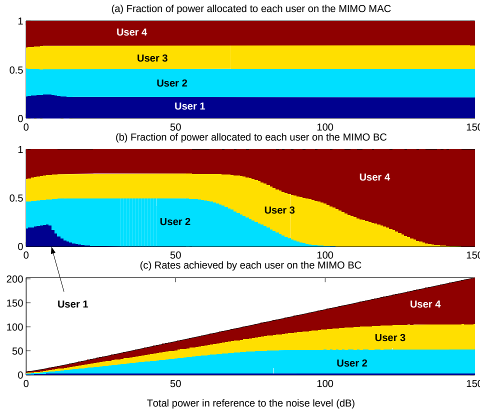


Fig. 2. (4,4,4) MIMO BC optimal power allocation and users rates with encoding order 4 to 1.

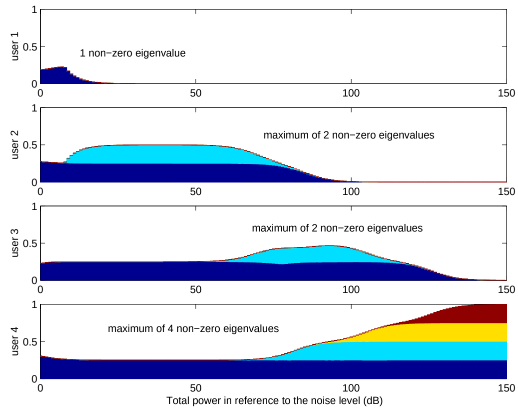


Fig. 3. (4,4,4) MIMO BC optimal power allocation per eigenmode with encoding order 4 to 1.

proved [9] that asymptotically in the high power region, only one user is allocated a non-vanishing fraction of the total transmit power. The latter two assumptions can always be satisfied as long as at least one user has a full-rank channel matrix, which occurs almost certainly in a rich scattering environment.

Let $\lim_{P\to\infty} tr\left(\boldsymbol{P}_K\right) = r_K P$, where $r_K \neq 0$ is a constant. Then we can prove that [9]:

$$\lim_{P\to\infty} \frac{1}{P} \boldsymbol{\Sigma}_K = \frac{1}{N} \boldsymbol{I}_N \tag{14}$$

$$\lim_{P\to\infty} \frac{1}{P} tr\left(\boldsymbol{\Sigma}_j\right) = 0 \text{ for } j = 1,\ldots,K-1. \tag{15}$$

The optimal power allocation on the BC is very different than that on the MAC after MAC to BC transformations. Even though several users could be allocated a non-vanishing fraction of the total transmit power on the MIMO MAC, only one will be allocated a non-vanishing fraction of the total transmit power on the MIMO BC. However, one must be careful in concluding that transmitting to only one user is sufficient to achieve the sum-capacity. We can only say that the ratio of the rate achieved by user $K$ to the sum-capacity tends to one as the total transmit power goes to infinity, but the convergence is slow due to the logarithmic growth of the sum-capacity with power. Moreover, simulations show that the asymptotic result only occurs at very large values of the total transmit power. As the power is large and increases, but as it is still below the threshold where only one user is allocated power on all its $N$ eigenmodes, several one-dimensional channels are allocated power such that this power increases with the total transmit power until the total transmit power reaches the threshold, and these one-dimensional channels belong to more than one user. Beyond that threshold, all the additional power is allocated to only one user.

We consider a realization of the (4,4,4) MIMO BC. Figure 2 shows the optimal power allocation on the MIMO MAC, and on the MIMO BC with encoding order 4 to 1, as well as the users rates, as a function of the total transmit power in reference to the noise level. All four users are allocated a non-vanishing fraction of the total transmit power on the MIMO MAC, but after MAC to BC transformations only user 4 will be allocated a non-vanishing fraction of the total transmit power in the high power region. We notice that at 150 dB, the rates of users 2 and 3, which remain constant, are not negligible compared to the sum-capacity. They will only become negligible at much higher values of the total transmit power. We can take a closer look at the power allocation by observing the ratio of the eigenvalues of the optimal covariance matrices to the transmit power. After MAC to BC transformations with the encoding order 4 to 1, we see in Figure 3 that the power allocation progressively shifts from eigenmodes of users 1, 2 and 3 to all of the fourth user's eigenmodes in the high power region.

In general when users are equipped with different numbers of receive antennas, only $N$ one-dimensional channels are allocated a non-vanishing fraction of the total transmit power in the high power region, and these $N$ one-dimensional channels belong to the $J$ users that are encoded first by dirty-paper coding such that these users are allocated a non-vanishing fraction of the total transmit power on the dual sum-power MIMO MAC and

$$\textstyle\sum_{k=K-J+2}^{K} M_k \leq N \leq \sum_{k=K-J+1}^{K} M_k \,. \tag{16}$$

The proofs of the above results lie in the incremental asymptotic rank of the interference matrices $\boldsymbol{B}_i - \boldsymbol{I}_N$ that are allocated infinite power in the successive encoding process. This is similar in nature to the problem of co-channel interference in MIMO systems [11]. However on the MIMO BC there is additional cooperation at the transmitter that allows to cope with this interference, so that even if users are equipped with one receive antenna it is still possible to allocate infinite power to $N$ users, and that other users achieve a constant rate.

## V. *N*-USER SCHEDULING ALGORITHMS

As seen in [8] it is necessary and sufficient to transmit to $N$ users at a time with any number of receive antennas per user in the medium power region in order to lose only a marginal amount of spectral efficiency relative to the sum-capacity. It might also be required to transmit to no more

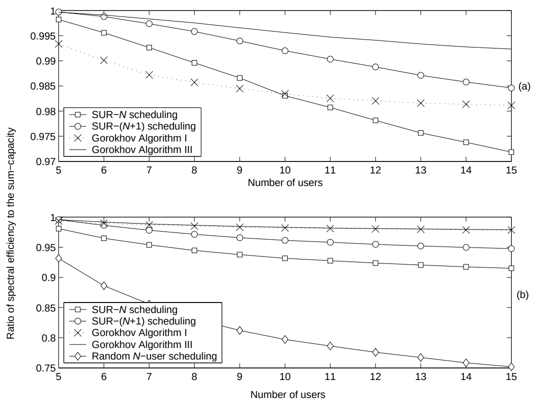


Fig. 4. Ratio of the spectral efficiency with several *N*-user scheduling algorithms and dirty-paper coding to the sum-capacity. Total power in reference to the noise level: (a) 0 dB, (b) 10 dB.

than *N* users at a time due to constraints of linear spatial multiplexing schemes or for complexity reduction. Optimal *N*-user scheduling by exhaustive search of groups of *N* users among *K* incurs a combinatorial complexity, which makes it impractical with a large number of users and even a moderate number of transmit antennas. We consider the following scheduling algorithms to select *N* users in each time slot:

- Random *N*-user scheduling: *N* users are randomly selected.
- Single-User Rates (SUR-*N*) scheduling: the *N* users with the largest individual capacities are selected.
- SUR-(*N*+1) scheduling: first select the *N*+1 users with the largest individual capacities, then the *N* users that offer the largest sum-capacity by exhaustive search.
- Gorokhov's receive antenna selection algorithms [12]: by treating each user as a different antenna in a single receiver with multiple antennas, Gorokhov's low-complexity algorithms allow to select *N* receive antennas out of *K* and to limit the capacity loss.

With 4 transmit antennas and 15 users with a single receive antenna each, random *N*-user scheduling achieves only 75% of the sum-capacity at 10 dB as shown in Figure 4.b. It performs even worse at 0 dB. It is unable to exploit multiuser diversity. Other strategies such as round-robin scheduling that also do not exploit multiuser diversity perform poorly. Scheduling users independently of one another is also not a good solution as seen with SUR-*N* scheduling. The performance is improved with SUR-(*N*+1) scheduling with an increase in complexity. As a consequence of the asymptotic analysis we can conclude that the (*N*,*N*) open-loop cooperative MIMO capacity is a good approximation of the sum-capacity of the (*N*,1,*N*) MIMO BC in the high power region. Hence, receive antenna selection algorithms, such as proposed in [12], are applicable for maximum-throughput joint *N*-user scheduling on the (*N*,1,*N*) MIMO BC in the high power region. Moreover, due to their inherent interference-avoidance properties they also perform well in the low and medium power regions as seen in Figure 4.

## VI. Conclusion

We have shown that in the high power region of the MIMO broadcast channel with *N* transmit antennas, it is asymptotically optimal to allocate a non-vanishing fraction of the total transmit power to *N* users if users are equipped with single receive antennas, or to only one user if users are equipped with *N* receive antennas. In the single receive antenna case an additional orthogonality property was proved as a first order approximation of the sum-capacity, which showed that the channel between the *N* users that are allocated a non-vanishing fraction of the total power is completely orthogonalized by the joint action of dirty-paper coding and optimal beamforming. We observed that in the medium power region it is best to transmit to *N* users simultaneously in each fading state to approach the sum-capacity. As a consequence of this fact and of a high power approximation of the sum-capacity, receive antenna selection algorithms provide an efficient way of scheduling *N* users with low-complexity and near-optimal throughput.

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