Performance improvement using Turbo Coded BICM-ID with 16-QAM over Gaussian and Flat Fading Rayleigh Channels

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Abstract—**We consider bit interleaved coded modulation with iterative decoding (BICM-ID), using a turbo code instead of the traditional simple convolutional code, for bandwidth efficient transmission over Gaussian and Rayleigh fading channels. BICM-ID has a smaller free Euclidean distance compared to trellis coded modulation (TCM) but a larger diversity order. With iterative decoding, soft bit decision can be employed to significantly improve the conditional intersignal Euclidean distance. Associated with a turbo code, this leads to a large coding gain. We address the association of this type of decoding with a 16-ary quadrature amplitude modulation (QAM). We show significant asymptotical improvement, a second "waterfall" region is seen for mapping techniques reserving better protection for parity bits. No error floor has been observed even for long and exhaustive simulations. We also observe that turbo coded BICM-ID converges to the performance of error-free fed back systems.**

Index Terms—**Bandwidth efficient coded modulation, BICM, fading channels, iterative decoding, QAM modulation, turbo codes.**

I. Introduction

Turbo codes presented in [1] are powerful error correcting codes achieving near capacity performance. Considerable work has been done to integrate these codes into high bandwidth efficient modulations. Several approaches have been considered.

The pragmatic turbo coded modulation approach was first introduced in [2] and consists of a concatenation of a turbo encoder and a mapper. It has the advantage of separating the code from the modulation without significant loss compared to the so-called turbo trellis coded modulation (TTCM) [3].

BICM introduces the use of an interleaver separating the encoder from the modulator as exposed in [4]. It produces significant performance improvement over fading channels due to an increased diversity order and the fact that for this type of channels, the Hamming distance and not the Euclidean distance is the dominant factor. A performance degradation is seen over Gaussian channels due to the "random modulation" caused by bit interleaving [5].

It was shown in [4, 6] that the best mapping for BICM is the Gray mapping. Different types of bit allocation techniques can be used for the Gray mapping depending on the modulation type. QAM and phase shift keying (PSK) schemes present different types of bits protection depending on the position of the allocated bits within the transmitted symbol. In [2] the most protected bit positions are allocated to parity bits. In [7], these positions are allocated to systematic bits.

The latter allocation method, associated with a turbo code, outperforms the former at the beginning of what is known as "*the waterfall region*" due to the fact that systematic bits are used in both component decoders compared to parity bits used only in one. We will show that this is not true asymptotically. Better protection of parity bits has better asymptotical gain due to the decrease in the number of the turbo coding error patterns caused by the code interleaver design[1].

In [8], BICM-ID decoding method has been introduced. This new approach is a serial concatenation of a recursive systematic encoder (RSC), an interleaver and a mapper. As on the receiver side, a scheme with joint iterative decoding and demodulation is used. Extrinsic information at decoder output is calculated for symbols after the first pass, de-interleaved and then fed back to the demodulator as *a priori* information on the channel received symbols.

Inspired by BICM-ID schemes, the authors of [9] have replaced the RSC encoder by a turbo encoder. The new system has two levels of iterations: turbo encoder and demodulator feedback iterations. In [9] a feedback is done after one turbo iteration and the system improved 8 PSK performance by 0.3 dB over Gaussian and Rayleigh flat fading channels.

In our work, we shall use the same approach as [9] with modulation and double binary turbo codes adopted by The European Telecommunication Standard Institute (ETSI) in the digital video broadcasting return channel satellite (DVB-RCS) (EN 301 790) and return channel terrestrial (DVB-RCT) (EN 301 958) standards. We will show significant improvement, compared to non fed back systems, of the bit error rate on Gaussian as well as flat fading channels. A significant asymptotical change in the slope of bit error rate (BER) curve is observed on these channels mainly for bit allocation methods preserving better protection for redundancy bits. Almost vertical asymptotical performance is obtained. No flattening appears even after extensive simulations for moderate signal-to-noise (SNR) ratio. For the waterfall region, no significant gain has been observed for Gaussian channels. The system shows 0.25 to 0.3 dB gain for Rayleigh channels.

In Section II, we review the transmitter of BICM-ID using a double binary turbo decoder as well as the receiver. We also address the maximum attainable performance of the system with respect to the outage capacity. As for Section III, we will show BER curves for double binary turbo coded BICM-ID schemes over Gaussian and flat fading Rayleigh channels for different types of bit allocation strategies. Simulation curves for the maximum attainable performance using perfect demodulator feedbacks are also provided. Section IV concludes the paper.

II. SYSTEM MODEL

A. *The BICM-ID transmitter*

The BICM transmitter studied here is a serial concatenation of the turbo encoder, the bit interleaver π and the memoryless modulator. It uses the pragmatic coded modulation approach. The transmitter is shown in Fig. 1.

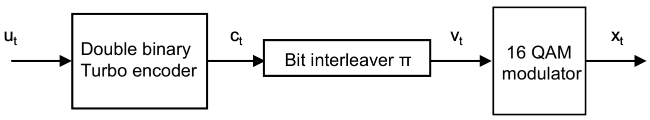

Fig. 1: BICM-ID transmitter.

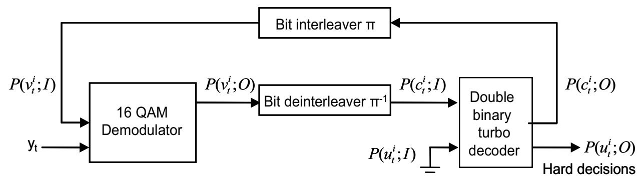

Fig. 2: BICM-ID receiver

The double binary circular recursive systematic encoder [10] encodes two information bits each time. the input $u_t = \{(u_t^1, u_t^2), t = 1,...,N\}$ and its corresponding encoder output or parity bits $r_t = \{r_t^j, j = 1,2 \; and \; t = 1,...,N\}$ are mapped into $c_t = \{c_t^i, i = 1,...4, and \; t = 1,...,N\}$ and interleaved using π . This permutation is based on a parallel bit interleaver:

$$v_t^i = \pi^i(c_w^i)\;;\; with \quad t = f_{\pi^i}(w) \tag{1}$$

f_{π^i} is the permuting function of c^i .

v_t is then mapped to a complex channel symbol x_t chosen from 16-ary constellation χ by a signal label μ .

$$x_t = \mu(v_t), x_t \in \chi \tag{2}$$

where the signal set is $\chi = \{(p,q) \; with \; p,q \in \{-3,-1,1,3\}\}$. Different strategies of bits to symbol mapping can be used. Simulations have been done for different types of mappings, set partitionning, anti-Gray and the modified set partitionning as proposed in [11]. These simulations presented 2 to 3 dB loss compared to the Gray mapping due to the presence of two component encoders separated by an interleaver. An optimization for the first decoder caused a degradation for the second decoder. 16-QAM modulation offers two levels of bit protection in each component axis for the Gray mapping which have been adopted. We shall call scheme U the mapping strategy preserving better bit protection for systematic bits and scheme R the one that offers better protection for parity bits. Other allocation strategies offered performance curves between these two strategies offering the same point of intersection. The received discrete time baseband signal is :

$$\mathbf{y}_t = \rho_t \sqrt{E_s}\,\mathbf{x}_t + \mathbf{z}_t \tag{3}$$

where ρ_t is the fading coefficient, E_s is the symbol energy is an additive white Gaussian noise with spectral density $N_0/2$ in each component axis. For Gaussian channels $\rho_t = 1$. As for flat fading Rayleigh channels, ρ_t is Rayleigh distributed with $E(\rho_t^2) = 1$. We assume perfect channel side information; ρ_t is perfectly known at the receiver.

B. *The BICM-ID receiver*

Maximum likelihood (ML) detection for BICM is done by generating a soft bit metric as shown in [12]. Our receiver then implements soft input soft output (SISO) decoding as shown in Fig. 2. Being independent in each component axis, the can be reduced into two separate amplitude shift keying (ASK) modulations. For each ASK, two bit metrics are calculated using the ML rule.

Without any *a priori* information, the probabilities of the received bits at the demodulator can be calculated using:

$$\begin{aligned} P(\hat{v}_t^i = b \mid y_t) &= \sum_{x_t \in \chi_b^i} P(x_t \mid y_t) \\ &= \sum_{x_t \in \chi_b^i} P(y_t \mid x_t).P(x_t) \end{aligned} \tag{4}$$

where $\chi_b^i = \{x_t \mid (x \; in \; X \; and \; v_t^i = b), with \; b \in \{0,1\}\}$.

The log likelihood ratio (LLR) metrics are calculated using the following:

$$\begin{aligned} \lambda_t^i &= \log \frac{P(\tilde{v}_t^i = 1 \mid y_t)}{P(\tilde{v}_t^i = 0 \mid y_t)} = \log \frac{\sum_{x_t \in \chi_1^i} P(x_t \mid y_t)}{\sum_{x_t \in \chi_0^i} P(x_t \mid y_t)} \\ &= \log \frac{\sum_{x_t \in \chi_1^i} P(y_t \mid x_t).P(x_t)}{\sum_{x_t \in \chi_0^i} P(y_t \mid x_t).P(x_t)} \end{aligned} \tag{5}$$

Knowing the channel statistical characteristics, the LLR metrics can be written as:

$$\lambda_t^i = \log \left(\frac{\sum_{x_t \in \chi_1^i} \frac{1}{\sigma\sqrt{2\pi}}.\exp\left(-\frac{(y_t - \rho_t x_t)^2}{2.\sigma^2}\right).P(x_t)}{\sum_{x_t \in \chi_0^i} \frac{1}{\sigma\sqrt{2\pi}}.\exp\left(-\frac{(y_t - \rho_t x_t)^2}{2.\sigma^2}\right).P(x_t)} \right) \tag{6}$$

where σ^2 is the noise variance. The *a priori* probability $P(x_t)$ is unavailable at the receiver on the first pass of demodulation. Therefore, an equally likely assumption is made. After deinterleaving, (5) is used as the input to the turbo decoder which then generates the *a posteriori* extrinsic probabilities for both information and parity bits.

Following the notation of [13], we denote $P(z;I)$ as the *a priori* probability for a variable z. $P(z;O)$ is the *a posteriori* probability.

On the second pass, the extrinsic *a posteriori* decoder probabilities $P(c_t^i;O)$ are interleaved and fed back as the *a priori* probabilities $P(v_t^i;I)$ to the demodulator. These probabilities are calculated using:

$$P(x_t) = P(\mu(v_t;I)) = \prod_{\substack{l \neq i \\ l=1}}^{m} P(v_t^i = \tilde{v}_t^i(x_t);I) \qquad (7)$$

where $\tilde{v}_t^i(x_t)$ is the value of the *i*th bit of the label corresponding to $x_t = \mu(\tilde{v}_t)$ and m the number of bits per symbol. For this same bit, extrinsic probability is computed using only the *a priori* probabilities of the other bits $(l \neq i)$ of the same channel symbol v_t. After the last iteration, the final decoded outputs are the hard decisions based on the *a posteriori* decoder probabilities.

C. Maximum attainable performance

We can simulate maximum performance curves by using error-free feedbacks instead of decoder output probabilities which can produce demodulator errors for low SNR. For error free feedbacks or what is known as genie assisted decoding, the *a priori* probabilities are calculated by using the following expression:

$$P(x_t) = P(\mu(v_t;I)) = \prod_{\substack{l \neq i \\ l=1}}^{m} v_t^i \qquad Where\ \ v_t^i = \pi(c_t^i) \qquad (8)$$

The outage probability has been calculated using the expression introduced in [14, eqs. (2), (3)]. We have reached relatively accurate values by the means of numerical integration methods.

III. Simulation Results

BER curves have been generated using the DVB-RCS code, the MAP algorithm for 1504 information bits per frame and 8 iterations. One iteration consists of an exchange between the two constituent decoders followed by a feedback to the demodulator.

The outage capacity, for rate 1/2 and for 16-QAM modulation over Gaussian channels, is 2.1 dB. This capacity is around 4.0 dB over Rayleigh flat fading channels. We provide simulation results over Gaussian and Rayleigh channels for BICM-ID with the two different types of bit allocation strategies, scheme U and R, in comparison with the non fed back BICM. In Fig. 3, we can see performance comparison between turbo coded and turbo coded BICM-ID over Gaussian channel.

It should be noted that the turbo coded system used over Gaussian channels implements the pragmatic approach and is characterized by the absence of the interleaver separating the code from the modulation. As we mentioned earlier, the absence of this interleaver generates slightly better performance results compared to non fed back BICM only over Gaussian channels. In Rayleigh flat fading channels, introducing the interleaver produces a higher degree of diversity order and therefore guarantees significant amelioration.

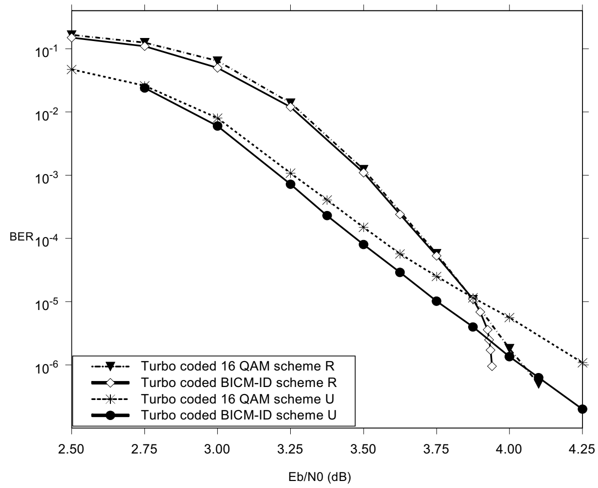

Fig. 3: Performance comparison between turbo coded and BICM-ID for the two different bit allocation strategies over Gaussian channel. DVB-RCS, 1/2 rate code, modulation, and 1504 information bits/frame.

A performance gain is seen for both schemes with respect to non fed back systems. For fading channels, the comparison between non fed back turbo BICM and turbo coded BICM-ID is shown in Fig. 4.

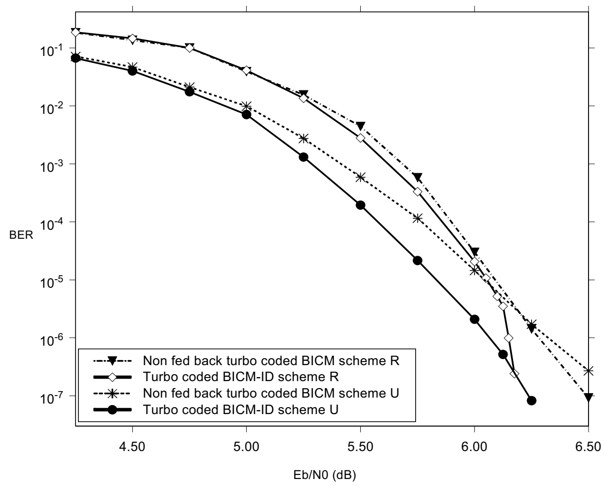

Fig. 4: Performance comparison between turbo coded non fed back BICM and BICM-ID for the two different bit allocation strategies over Rayleigh flat fading channel. DVB-RCS, 1/2 rate code, modulation, and 1504 information bits/frame.

Scheme R outperforms, for turbo coded BICM-ID as well as error free feedbacks over both channel types, the scheme U for BER below 5×10^{-7}.

Simulation curves for turbo coded BICM-ID have been compared to error free fed back BICM-ID which represents an ideal non-realistic system and to the channel capacity. The results of this comparison are shown in Fig. 5 for Gaussian channel and in Fig. 6 for Rayleigh fading channel.

The error free feedback system uses equation (8) for computing feedbacks to the demodulator. The feedback will be equal to one for the transmitted constellation signal and zero for any other constellation signal.

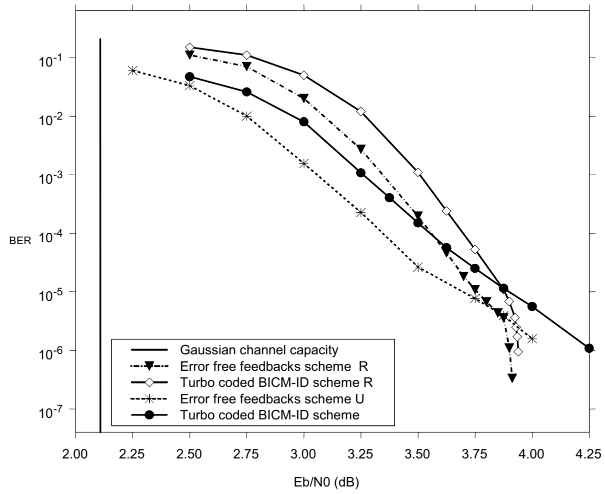

Fig. 5: Performance of turbo coded BICM-ID with respect to the one of error free fed back BICM-ID and to the channel capacity for the two different bit allocation strategies over Gaussian channel. DVB-RCS, 1/2 rate code, modulation, and 1504 information bits/frame.

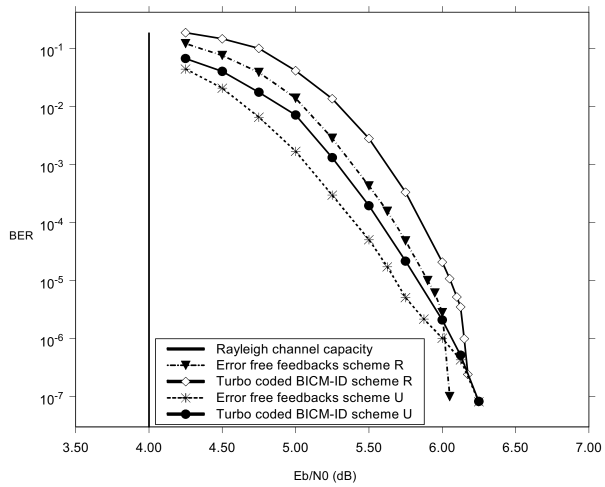

Fig. 6: Performance of turbo coded BICM-ID with respect to the one of error free fed back BICM-ID and to the channel capacity for the two different bit allocation strategies over Rayleigh flat fading channel. DVB-RCS, 1/2 rate code, modulation, and 1504 information bits/frame.

We can clearly notice that the gap between the turbo coded BICM-ID and the error free fed back systems decreases as the SNR increases. This is explained by the fact that, with increased SNR, the decoder generates quasi error free feedback information.

For the turbo coded BICM-ID scheme R over AWGN, no error has been found for an increment of the last E_b/N_0 shown in Fig. 3 by only 0.05 dB and simulations that decoded over 5×10^{10} transmitted information bits. No change in the slope of the BER curve has been seen.

Over fading channels, the performance curve of scheme R shows no error for same increment of the last shown SNR in Fig.4 and simulations that decoded more than 10^{11} information bits. We can assume in this case also the absence of a change in the slope of the BER curve. It should be noted that the gap with respect to the capacity is almost identical for both channel types. It is around 1.9 dB. The use of a 16-state decoder and a larger block size than 1504 information bits would have generated performance curves closer to the channel capacity.

IV. Conclusion

In this paper we have analyzed and simulated the turbo coded BICM-ID approach. We have shown that this approach offers significant performance gains, especially for BER lower than 10^{-7}, over non fed back turbo TCM in Gaussian as well as flat fading channels for a small increase in system complexity. No change in the slope of BER curves appears for exhaustive simulations. We have also observed that, for high SNR, turbo coded BICM-ID converges to the performance of error free fed back systems.

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