From Cell Capacity to Subcarrier Allocation in Multi-User OFDM

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***Abstract*— In this paper we show how a fundamental result from information theory can be used to calculate the aggregate channel capacity in the uplink of an access point or a base station. The fundamental limits for this *cell capacity* provide an upper bound for any practically implementable multiple-access scheme. The capacity loss due to various implementation constraints is evaluated and a method for adaptive subcarrier allocation in OFDMA is presented. This method is based on the famous iterative water-filling algorithm and results in a surprisingly simple algorithm for the subcarrier allocation in OFDMA.**

I. Introduction

The uplink in a cellular system or in a hot spot can be described in information-theoretic terms as a multiple-access channel as depicted conceptually in Fig. 1: Uncoordinated transmitters send independent information to a common receiver. No transmitter cooperation beyond time synchronization is available. We assume that channel state information (CSI) is available both in the receiver and in the transmitters. This information is gleaned in the receiver by channel estimation and can be passed to the transmitters via a downlink signalling channel or is directly obtained there in the case of time-division duplex (TDD).

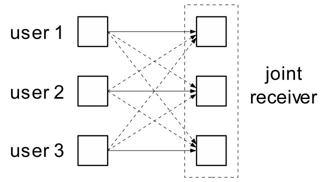

Fig. 1. Multiple access channel: Users send independent information to a common receiver.

The fundamental limits for the amount of transmitted information are given by the capacity region and the sum capacity. For the rather general case of a Gaussian multiple-access channel with intersymbol interference (ISI), the capacity region and the conditions, under which the sum capacity is achieved, have been described by Cheng and Verdú [1]. These results are based on a generalization of the single-user water-filling solution [2][3] and show that the sum capacity is achieved if the available bandwidth is partitioned among the users and the transmit power spectral densities (PSD) follow a water-filling distribution. This indicates that the optimum multiple-access scheme in terms of a maximum sum bitrate is FDMA (frequency division multiple access).

An even more general description of the multiple-access channel has been provided recently by Yu et al. [4], which holds for Gaussian multiple access channels with vector inputs and vector output. Yu et al. describe the capacity region for this case and provide a numerical algorithm which calculates the optimum transmit covariance matrices to attain the sum capacity. The convergence of this algorithm is proven and it is shown that its convergence is fast.

For practical multiple-access schemes, instead of the channel capacity the achievable bitrate is regarded as one of the fundamental figures of merit. The basic results from information theory provide the optimum transmit covariance matrices or the PSD, as well as the amplitude distribution, but they do not give any indication about the most appropriate modulation and coding schemes.

In this paper, we apply the *iterative water-filling* algorithm [4] to an implementable multiple-access scheme based on OFDMA (orthogonal frequency division multiple access) with QAM modulation. The information-theoretic results are adapted to OFDMA with adaptive subcarrier allocation and adaptive modulation and the difference between attainable bitrate and the sum capacity of a cell are shown.

In the following, vectors are denoted by bold-face lower case symbols $\mathbf{x}$, matrices as uppercase bold-face $\mathbf{X}$, the determinant is denoted by $|\mathbf{X}|$, $\mathbf{x}^{\mathrm{T}}$ is the transpose, $\mathbf{x}^{\mathcal{H}}$ the conjugate transpose, x^* the complex conjugate and $\mathrm{diag}(\mathbf{x})$ is a matrix with diagonal $\mathbf{x}$.

II. System Model and Multi-User Water-Filling

The Gaussian multiple-access channel with ISI for U users can be described by

$$y(t) = \sum_{u=1}^{U} h_u(t) \star x_u(t) + w(t) \tag{1}$$

where $h_u(t)$ and $x_u(t)$ are the impulse response and the transmit signal of user u, respectively, and $w(t)$ is additive Gaussian noise. This signal model is also valid for the uplink in a hotspot or in a cellular network where the interference is either negligible or can be modeled as Gaussian noise. The receive signal $y(t)$ can be expressed equivalently in the frequency domain by

$$Y(\omega) = \sum_{u=1}^{U} H_u(\omega) X_u(\omega) + W(\omega) \tag{2}$$

One of the salient features of OFDM is that is decomposes a frequency-selective broadband channel in parallel flat subchannels; thus for OFDMA with N subcarriers, the available frequency band is partitioned into N parallel subchannels:

$$y_n(k) = \sum_{u=1}^{U} H_{n,u} x_{n,u}(k) + w_n(k), \quad n = 1, \ldots, N \tag{3}$$

where n and k are the subcarrier and discrete-time indices, respectively. This discrete-time notation is valid as long as the guard interval is longer than the longest delay spread, which is the case for any carefully designed system. In the following, we denote by *cell capacity* the sum of all user's channel capacities.

A. Multi-user water-filling

The multi-user water-filling solution [1] describes how the PSD $S_u(\omega)$ of each user's transmit signal $x_u(t)$ has to be chosen in order to achieve the maximum cell capacity for a channel described by (1) or (2).

Denoting by $S_w(\omega)$ the PSD of $w(t)$, we can describe the channel with the channel gain to noise ratio (CNR)

$$T_u(\omega) = \frac{|H_u(\omega)|^2}{S_w(\omega)} \tag{4}$$

The transmit power of user u is denoted by $p_u = \mathrm{E}[|x_u(t)|^2] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_u(\omega)\, d\omega$ and is limited to P_u:

$$p_u \leq P_u, \quad \forall\, u = 1, \ldots, U \tag{5}$$

The key idea in the generalization of the water-filling solution to the multi-user case is the definition of the *equivalent channel* by $\hat{H}_u(\omega) \triangleq H_u(\omega)/\sqrt{\lambda_u}$, which leads to the equivalent PSD $\hat{S}_u(\omega) \triangleq \lambda_u S_u(\omega)$ [1]. This scaling of the channel transfer function allows to scale the water-filling diagrams of each user to a "water level" of unity and to combine them to the multi-user water-filling solution:

$$\begin{aligned} \hat{S}_u(\omega) &= \begin{cases} \left[1 - \frac{\lambda_u}{T_u(\omega)}\right]^+ & \frac{\lambda_u}{T_u(\omega)} \leq \frac{\lambda_j}{T_j(\omega)}\ \forall j{\neq}u \\ 0 & \text{otherwise} \end{cases} \\ \lambda_u P_u &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{S}_u(\omega)\, d\omega \end{aligned} \tag{6}$$

Where $[x]^+$ is a shorthand notation for $\max(x, 0)$. The multipliers λ_u are uniquely defined by this equation. Hence, once these multipliers are determined, the bandwidth is partitioned among the users according to the condition in (6): each frequency band is given to the user with the highest equivalent CNR $T_u(\omega)/\lambda_u$.

This is visualized by Fig. 2: The "bottom" of the water-filling diagram is formed by the minimum of $\lambda_u/T_u(\omega)$ and the users are separated at the intersections of these curves, allocating each frequency band to the user with the lowest curve. The area (the "amount of water") corresponds to the maximum equivalent power $\lambda_u P_u$ and the multipliers λ_u have to be scaled such that the "water level" is one. It can also be observed from this diagram that the calculation of the λ_u is not straightforward and highly nonlinear.

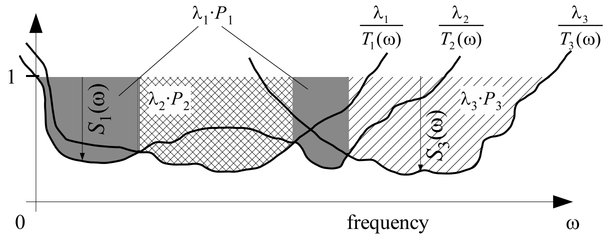

Fig. 2. Multi-user water-filling diagram.

The optimum transmit PSDs can be calculated as $S_u(\omega) = \hat{S}_u(\omega)/\lambda_u$ and the cell capacity is given by

$$\begin{aligned} C &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{ld}\left(1 + \sum_{u=1}^{U} T_u(\omega) S_u(\omega)\right) d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{ld}\left(1 + \sum_{u=1}^{U} \hat{S}_u(\omega) \max_{j=1,\ldots,U}\left(\frac{T_j(\omega)}{\lambda_j}\right)\right) d\omega \end{aligned} \tag{7}$$

The solution (6) can also be applied for the OFDMA case, provided that the number of subcarriers is large. The calculation of the multipliers λ_u in (6), however, is difficult and computationally expensive. An existing heuristic algorithm [5] achieves an approximate solution. However, although this solution was found to perform well in numerous simulations, it has the drawback of lacking of formal proof of convergence.

III. Iterative Water-Filling for the Vector Gaussian Multiple-Access Channel

A more general formulation of the multiple-access channel has been provided recently by Yu et al. [4], which holds for a Gaussian multiple-access channel with vector inputs and vector outputs:

$$\mathbf{y} = \sum_{u=1}^{U} \mathbf{H}_u \mathbf{x}_u + \mathbf{w} \tag{8}$$

Yu et al. provide a numerical algorithm with proven convergence, which maximizes the sum capacity for this signal model and considers a power constraint for each user. The roles of the signal and noise PSDs are now taken by the signal and noise covariance matrices, which are defined as $\mathbf{S}_u \triangleq \mathrm{E}[\mathbf{x}_u \mathbf{x}_u^{\mathcal{H}}]$ and $\mathbf{S}_w \triangleq \mathrm{E}[\mathbf{w}\mathbf{w}^{\mathcal{H}}]$, respectively. The maximum information bitrate of user u, *i.e.* the channel capacity of this user, is given as the mutual information between the received signal $\mathbf{y}$ and the transmitted signal $\mathbf{x}_u$ as $R_u = I(\mathbf{y}; \mathbf{x}_u)$. The maximum sum rate, designated as *cell capacity*, is then given by

$$R = \sum_{u=1}^{U} R_u = \mathrm{ld}\left|\sum_{u=1}^{U} \mathbf{H}_u \mathbf{S}_u \mathbf{H}_u^{\mathcal{H}} + \mathbf{S}_w\right| - \mathrm{ld}\,|\mathbf{S}_w| \tag{9}$$

Yu's *iterative water-filling* (IWF) algorithm finds the signal covariance matrix $\mathbf{S}_u$, which solves the optimization problem:

$$\begin{aligned} &\textit{maximize} && R \\ &\textit{subject to} && \mathrm{trace}(\mathbf{S}_u) \leq P_u \quad \forall u \\ & && \mathbf{S}_u \succeq 0 \quad \text{(pos. semidefinite)} \end{aligned} \tag{10}$$

The result in form of the covariance matrix $\mathbf{S}_u$ indicates the required auto- and cross correlation coefficients of the components of $\mathbf{x}_u$, but it does not give details about the appropriate modulation and coding format. This rather general description simplifies drastically for the OFDMA uplink: By defining the vectorial transmit and receive signals as $\mathbf{x}_u = (x_{1,u}, \dots, x_{N,u})^{\mathrm{T}}$, $\mathbf{y} = (y_1, \dots, y_N)^{\mathrm{T}}$, the channel matrix $\mathbf{H}_u = \mathrm{diag}(H_{1,u}, \dots, H_{N,u})$ and the noise as $\mathbf{w} = (w_1, \dots, w_N)^{\mathrm{T}}$, we can write the receive signal (3) in the more general form (8). This allows to make use of the IWF algorithm for OFDMA. Assuming that the noise as well as the signal samples are uncorrelated, the signal and noise covariance matrices simplify to

$$\begin{aligned} \mathbf{S}_u &= \mathrm{diag}(p_{1,u}, \dots, p_{N,u}), \quad \text{with } p_{n,u} = \mathrm{E}[|x_{n,u}|^2] \\ \mathbf{S}_w &= \mathrm{diag}(\sigma_1^2, \dots, \sigma_N^2), \qquad \text{with } \sigma_n^2 = \mathrm{E}[|w_n|^2] \end{aligned} \tag{11}$$

The power constraint in (10) and the rate sum in (9) become

$$\mathrm{trace}(\mathbf{S}_u) = \sum_{n=1}^{N} p_{n,u} \le P_u \tag{12}$$

$$R = \sum_{n=1}^{N} \mathrm{ld}\left(1 + \sum_{u=1}^{U} T_{n,u} \cdot p_{n,u}\right), \tag{13}$$

which is intuitively satisfying. Hence, the IWF algorithm can be formulated for OFDMA in the following way:

Algorithm Iterative water-filling

1 $p_{n,u} = 0 \quad \forall\, n, u$

2 repeat

3 for $u = 1$ to U

4 $z_n = \sum_{\substack{j=1 \\ j \ne u}}^{U} |H_{n,j}|^2 \, p_{n,j} + N_0$

5 $\mathbf{p}_u = \arg\max_{\mathbf{q}=(q_1,\dots,q_N)} \left\{ \sum_{n=1}^{N} \mathrm{ld}\left(|H_{n,u}|^2 q_n + z_n\right) \right\}$ *subject to* $\sum_{n=1}^{N} q_n \le P_u,\ q_n \ge 0$

6 end

7 until the desired precision is reached

This algorithm calculates the single-user water-filling solution for each user, considering the transmissions of all other users as noise. This is done for all users and then repeated in the next iteration. This power allocation converges asymptotically to the optimum solution.

The objective function which is maximized line 5 to yield the power allocation $\mathbf{p}_u = (p_{1,u}, \dots, p_{N,u})$ can be substituted by the following function without affecting the result:

$$R' = \sum_{n=1}^{N} \mathrm{ld}\left(1 + \frac{|H_{n,u}|^2 q_n}{z_n}\right) \tag{14}$$

This is the channel capacity of a parallel Gaussian channel with transfer coefficients $H_{n,u}$, transmit powers q_n and noise powers z_n, i.e. the CNR is given by $T_n = |H_{n,u}|^2 / z_n$. The optimization problem in line 5 is hence the single-user rate maximization problem:

$$\begin{aligned} &\textit{maximize} && \textstyle\sum_{n=1}^{N} \mathrm{ld}\,(1 + q_n T_n) \\ &\textit{subject to} && \textstyle\sum_{n=1}^{N} q_n \le P, \quad q_n \ge 0 \end{aligned} \tag{15}$$

which has the well-known water-filling solution [2], [3]

$$q_n = \left[q_0 - \frac{1}{T_n}\right]^+, \; q_0 \text{ such that } \sum_n q_n = P \tag{16}$$

This solution is assigned to the power vector $\mathbf{p}_u$ of user u and is closely related to the multi-user water-filling theorem (6), although it has some subtle differences. In (1), (2), frequency is continuous, while in (8) the vector has to be interpreted as a discretization of the frequency axis, analogue to the subchannels in OFDM. The solution (6) is consistent with a pure FDMA multiple-access, while the IWF algorithm does not exclude that power is allocated to more than one user per subchannel (see also [6]). The same result would be obtained by discretizing the frequency axis in the water-filling diagram in Fig. 2: the frequency bins, which contain one or more intersection points of the lowest curves, will be shared by two or more users. The power allocation $\mathbf{p}_u$, which is calculated by the IWF algorithm, can thus be interpreted as a discrete-frequency version of the PSD $S_u(\omega)$ according to the multi-user water-filling solution (6).

IV. Apply Iterative Water-Filling to Adaptive Subcarrier Allocation in OFDMA

In its original form, the IWF algorithm calculates the optimum transmit covariance matrices for maximum cell capacity. If the multiple-access channel can be represented by N parallel flat subchannels according to (3), this simplifies to a simple power allocation for each subchannel and each user. The problem of maximizing the achievable bitrate in OFDMA with adaptive subcarrier allocation and adaptive modulation is closely related to this power allocation problem. However, some additional constraints must be taken into account:

1) The channel capacity is replaced by the achievable bitrate at a given maximum bit error probability $\mathcal{P}_{\mathrm{b}}$.
2) Each subchannel is allocated exclusively to only one user.
3) The number of bits per QAM symbol (code rate) is taken out of a finite set $\mathcal{B}$.

The optimization problem with this basic implementation constraints can be formulated as

$$\begin{aligned} &\textit{maximize} && \sum_{u=1}^{U} \sum_{n=1}^{N} b_{n,u} \\ &\textit{subject to} && \sum_{n=1}^{N} p_{n,u} \le P_u,\ p_{n,u} \ge 0 \\ &&& \sum_{u=1}^{U} \mathrm{sgn}(p_{n,u}) \in \{0, 1\} \\ &&& b_{n,u} \in \mathcal{B},\ \mathcal{P}_{\mathrm{b},u} \le \mathcal{P}_{\mathrm{b,max}} \end{aligned} \tag{17}$$

Rather than trying to solve this rather complex discrete optimization problem, we adapt the IWF algorithm to solve the subcarrier adaptation problem and the Hughes-Hartogs algorithm [7] for bit and power loading of each users' subchannels. This way we separate the optimization problem into two steps with limited complexity:

1) Adaptive subcarrier allocation: each subchannel is allocated exclusively to one user. Hence, each user disposes of a set $\mathcal{N}_u$ of dedicated subchannels.
2) Adaptive coding and modulation for each subcarrier: A single-user bitloading algorithm can be applied to each user's set of subchannels. Instead of the optimum Hughes-Hartogs algorithm, one of the numerous alternatives with less computational complexity can be applied. Channel coding can also be taken into account in this step.

The main link between channel capacity and the achievable bitrate with QAM modulation is given by the gap approximation [8]: the SNR gap Γ is the additional SNR which is required in comparison to the Shannon limit in order to not exceed the symbol error probability $\mathcal{P}_\text{S}$ with QAM modulation and is approximately given by

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\mathcal{P}_\text{S}/4\right)\right]^2 \tag{18}$$

For M-QAM, the symbol error probability $\mathcal{P}_\text{S}$ is approximately related to the bit error probability $\mathcal{P}_\text{b}$ by $\mathcal{P}_\text{S} \approx 1-(1-\mathcal{P}_\text{b})^{\text{ld}M}$. A possible coding gain and a margin for implementation can also be considered with the SNR gap [8].

The first step to adapt the IWF algorithm is to introduce the SNR gap in the calculation of the power allocation vector $\mathbf{p}_u$ in line 5. This basically changes the objective function from maximizing channel capacity to maximizing achievable bitrate. The SNR gap can be simply incorporated in the definition of the CNR:

$$T_n = \frac{|H_{n,u}|^2}{\Gamma_u \cdot z_n} \tag{19}$$

The algorithm provides power allocation vectors for all users, which can be used to derive the subcarrier allocation by assigning each subcarrier to the user with the highest allocated power:

$$a_n = \arg\max_u(p_{n,u}), \quad n = 1, \dots, N \tag{20}$$

Here $\mathbf{a} = (a_1, \dots, a_N)$ denotes the subcarrier allocation vector, which contains in each position the assigned user. This step eliminates the allocation of multiple users to one subchannel. Now, we have for each user a set of dedicated subchannels $\mathcal{N}_u = \{n : a_n = u\}$ and we can apply a single-user bitloading algorithm separately for each user:

$$(\mathbf{b}_u, \mathbf{p}_u) = \text{bitload}(\mathbf{T}, \mathcal{N}_u, \mathcal{B}, P_u),\ u = 1, \dots, U \tag{21}$$

This last step sequentially calculates the bit allocation vector $\mathbf{b}_u = (b_{n,u}, \dots, b_{N,u})$ and a new power allocation vector $\mathbf{p}_u$ for all users. For uncoded QAM the set $\mathcal{B}$ contains integer numbers indicating the number of bits per symbol whereas for coded QAM, $\mathcal{B}$ contains the overall code rates, considering channel coding and QAM modulation. Whether coding is included or not, is of no importance for the preceding steps.

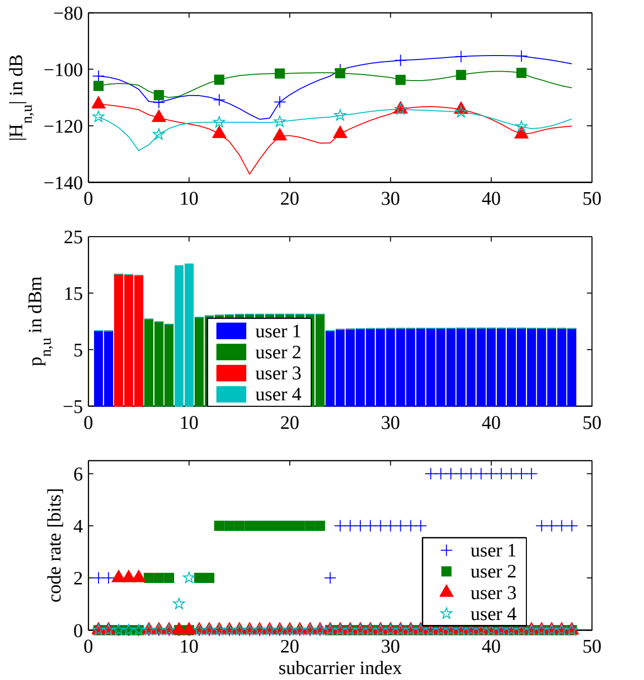

Fig. 3. Multiple access channel: users send independent information to a common receiver.

For the presented algorithm, it is sufficient to consider coding in the SNR gap (18) and in the single-user bitloading (21).

Fig. 3 shows exemplary channel transfer functions, the corresponding power allocation of the IWF algorithm and finally the bit allocations for a WLAN-like scenario with four users. The majority of the subchannels is allocated to the user which is closest to the access point while the users, which are further away, concentrate their transmit power in some few subchannels. In this example, the IWF algorithm allocates only the power of one user to one subchannel, which is not always the case. There is however, a clear tendency of the IWF algorithm to separate the users in frequency, which can be observed from the algorithm itself and from its analogy to the multi-user water-filling solution (6). In the course of the IWF algorithm, a user allocates transmit power to a certain subchannel; in the next step this power will be viewed as noise by the next user, making this subchannel less attractive. We can also see from the bit allocation in Fig. 3 that the vast majority of the bits is allocated to the first user and only few bits are given to the last user, which is farther away from the access point[1].

V. Simulation Results

The presented adapted IWF algorithm has been simulated in a WLAN environment with the following parameters: the transmit power of the transmitters is limited to $P_u = 200\,\text{mW}$, the noise figure of the receiver in the access point is assumed

[1]Note that if the power constraint were not per user, but globally, all power would be allocated to the closest user and the allocation problem would become a trivial one.

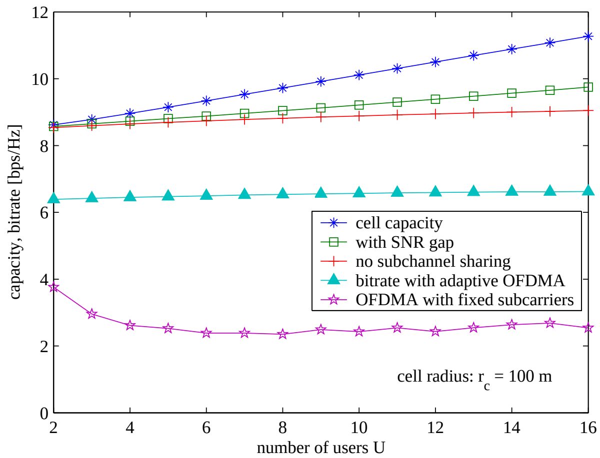

Fig. 4. Maximum achievable bitrate in comparison to cell capacities with OFDMA constraints.

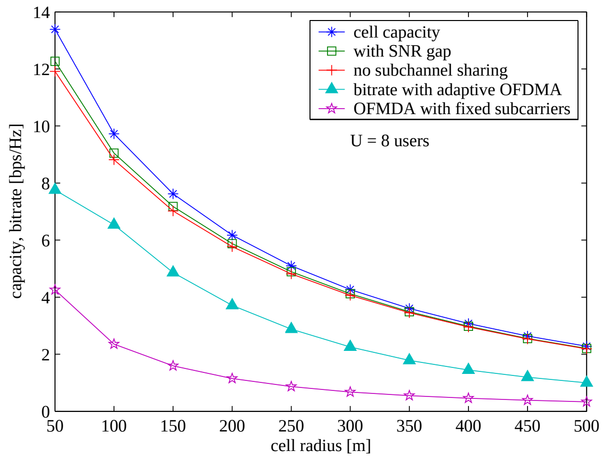

Fig. 5. Cell capacities and sum bitrates as a function of the cell size.

to be $10\,\mathrm{dB}$, the number of subcarriers is $N = 48$ and the channel model 'A' according to [9] has been implemented. Of chief importance for the absolute values of the cell capacities and achievable bitrates are the path loss function [10]

$$L(d) = 31\lg\left(d/d_0\right) + 56\,\mathrm{dB}, \quad d_0 = 1\,\mathrm{m} \tag{22}$$

and the user distances, which follow a deterministic distribution as proposed in [11]:

$$d_u = \left(1 + 9\sqrt{\frac{u-1}{U-1}}\right)\frac{r_\mathrm{c}}{10}, \quad u = 1,\ldots,U \tag{23}$$

This corresponds to a situation where the user density is constant from $10\,\%$ of the the cell radius r_c to the cell border. The results in Fig. 4 and Fig. 5 have been obtained by averaging over 10^5 and 10^6 channel realizations. In both figures, beside the cell capacity as derived by the IWF algorithm, the capacities due to implementation constraints are shown. First of all, the capacity is decreased by the introduction of the SNR gap, which is set to $\Gamma = 4$, corresponding to a symbol error probability of $\mathcal{P}_\mathrm{S} = 10^{-3}$. The capacity is reduced further – although not significantly – by excluding subchannel sharing. The loss due to a limited set of constellation sizes, however, is significant, as can be observed in both figures. In this example, the number of bits per QAM symbol has been chosen out of $\mathcal{B} = \{1, 2, 4, 6, 8\}$. In order to reduce this loss, powerful channel coding can be employed. Finally, the lowest curve represents the bitrate which is achieved with adaptive modulation and *fixed* subcarrier allocation, i.e. the subcarriers are equally distributed among the users. This gives a "fairer" resource allocation at the cost of the sum bitrate.

VI. Conclusion

We have applied the iterative water-filling algorithm to compute the uplink capacity of an isolated cell or a cell within a cellular network, where the interference can be modelled as Gaussian noise. The introduction of additional constraints makes this algorithm applicable to the maximization of the sum bitrate in OFDMA with adaptive subcarrier allocation. We evaluated the gap between the channel capacity and the achievable bitrate with adaptive modulation and compared these results to OFDMA with fixed subcarrier allocation. Apart from maximizing the uplink throughput, the presented method can serve as an evaluation tool to compute an upper bound on the achievable bitrate in the uplink.

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