Optimal Power Allocation in Zero-forcing MIMO-OFDM Downlink with Multiuser Diversity

Peter W. C. Chan, and Roger S. K. Cheng

Abstract—**This paper considers the optimal power allocation for a multiuser MIMO-OFDM downlink system using zero-forcing multiplexing, with the objective to maximize the system capacity. First, we consider the problem of user selection and power allocation for every subcarrier. By converting this combinatorial optimization problem into a convex one, an optimal power allocation algorithm has been derived. Then we introduce and formulate two new problems by extending user selection to antenna selection and dimension selection. We also provide optimal and suboptimal solutions to antenna selection and dimension selection problems respectively, by slightly modifying the proposed algorithm for user selection. Numerical results and analysis are provided to evaluate the system capacity and to study the effect of the number of users and the receive antennas per user on the performance. Comparison among user selection, antenna selection and dimension selection is made.**

Index Terms—**Power Allocation, Resource Allocation, Multiuser MIMO-OFDM system, Multiuser Diversity**

I. INTRODUCTION

Thanks to their promising gain in channel capacity, multiple-input multiple-output (MIMO) systems attract significant recent attention in wireless communications [1][2] and will play an important role in future wireless communications systems. On the other hand, there is a growing interest on employing orthogonal frequency division modulation (OFDM) in MIMO systems since OFDM converts a frequency-selective channel into a set of parallel flat-fading channels, making a lot of MIMO-related algorithms easy to implement. Moreover, it facilitates adaptive modulation which is a potential technique for system throughput enhancement. OFDM has become a mature technology and is adopted in many existing systems, for example, IEEE802.11a/g and considered to be a strong candidate in ultra-wideband standard. Recently, OFDM is also found to be capable of exploiting multiuser diversity in multiuser systems [3][4][5] which has been shown to provide further system capacity gain in multiuser systems.

For the above reasons, MIMO-OFDM is a strong candidate for wideband multiuser wireless communications systems. For downlink, zero-forcing technique or block diagonalization has been proposed for space division multiple access (SDMA) to remove the cochannel interuser interference [6][7][8]. Such approach provides efficient, closed-form solutions that yield a reasonable tradeoff between performance and computational complexity [6].

In this paper, we consider a multiuser MIMO-OFDM downlink system using zero-forcing techniques [6] and [8] to multiplex the signal of multiple users. Our objective is to develop an optimal algorithm to maximize the system capacity by user selection and power allocation on every subcarrier. Multiuser diversity plays an important role in achieving the optimal solution. In [5], an optimal power allocation for multiuser multiple-input single-output (MISO) OFDM system, that is each user has only one receive antenna, has been developed. However, [5] does not address the optimization for systems in which each user has multiple receive antennas. In this paper, we use the techniques similar to [3] and [5] to derive an optimal algorithm for MIMO-OFDM downlink system, using the idea of block diagonalization [6][7][8]. Moreover, we introduce two extensions of the problem: optimization based on antenna selection and dimension selection, instead of user selection. Antenna selection allows the system selects only some of the antennas of a user. Dimension selection allows the system to assign different dimensions of signal space to the different users. Compared to user selection, they have larger degrees of freedom in optimization parameters and provide further capacity gains over user selection. We provide both problem formulations and solutions for them, based on the framework we develop for user selection.

This paper is organized as follows. Section II presents the system model and the formulation of the optimization problem based on user selection. Section III presents and outlines the optimal power allocation algorithm for MIMO-OFDM downlink system. Section IV presents the optimization problems based on antenna selection and dimension selection. Section V presents the numerical results and analysis. Finally, Section VI concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a wireless communication system with a base station with N_T transmit antennas and K mobile users, each having N_R receive antennas. Assume N-subcarrier OFDM is employed so that the MIMO channel on subcarrier n, between the base station and every user k can be characterized by a $N_R \times N_T$ matrix $\mathbf{H}_{k,n}$. For every user k, the base station transmits a m-tuple data vector $\mathbf{d}_{k,n}$ after a $N_T \times m$ dimension linear precoder $\mathbf{M}_{k,n}$ on every subcarrier n. The signal at the k-th user's receiver on subcarrier n is

$$\mathbf{x}_{k,n} = \mathbf{H}_{k,n} \sum_{j=1}^{K} \mathbf{M}_{j,n} \mathbf{d}_{j,n} + \mathbf{w}_{k,n} \tag{1}$$

where $\mathbf{w}_{k,n}$ is $N_R \times 1$ dimension white Gaussian noise at the k-th user receiver. The system block diagram for every subcarrier n is depicted in Figure 1. We assume the base station has perfect knowledge of the channels and the system employs zero-forcing method for spatial multiplexing [6][8]. The zero-forcing method

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constrains $\mathbf{H}_{j,n}\mathbf{M}_{i,n} = \mathbf{0}$ for $i \neq j$ and the multiuser interference is hence eliminated. To achieve this, $\mathbf{M}_{j,n}$ must lie in the null space of $\widetilde{\mathbf{H}}_{k,n}$, which is defined as

$$\widetilde{\mathbf{H}}_{k,n} = \begin{bmatrix}\mathbf{H}_{1,n}^T & \cdots & \mathbf{H}_{k-1,n}^T & \mathbf{H}_{k+1,n}^T & \cdots & \mathbf{H}_{K,n}^T\end{bmatrix}^T . \quad (2)$$

Let $\widetilde{L}_{k,n} = \mathrm{rank}(\widetilde{\mathbf{H}}_{k,n}) \leq (K-1)N_R$. $\widetilde{\mathbf{H}}_{k,n}$ can be expressed as

$$\widetilde{\mathbf{H}}_{k,n} = \widetilde{\mathbf{U}}_{k,n}\widetilde{\mathbf{\Sigma}}_{k,n}\begin{bmatrix}\widetilde{\mathbf{V}}_{k,n}^{(1)} & \widetilde{\mathbf{V}}_{k,n}^{(0)}\end{bmatrix}^H \quad (3)$$

using singular value decomposition (SVD), where $\widetilde{\mathbf{V}}_{k,n}^{(1)}$ holds the first $\widetilde{L}_{k,n}$ right singular vectors, and $\widetilde{\mathbf{V}}_{k,n}^{(0)}$ holds the last $(N_T - \widetilde{L}_{k,n})$ right singular vectors. Hence, $\widetilde{\mathbf{V}}_{k,n}^{(0)}$ forms an orthogonal basis for the null space of $\widetilde{\mathbf{H}}_{k,n}$. Thus, $\mathbf{M}_{k,n}$ can be generally expressed as

$$\mathbf{M}_{k,n} = \widetilde{\mathbf{V}}_{k,n}^{(0)}\mathbf{A}_{k,n} \quad (4)$$

where $\mathbf{A}_{k,n}$ is a $(N_T - \widetilde{L}_{k,n}) \times m$ precoder matrix. By doing so, the downlink system reduces to K parallel non-interfering single user MIMO channels, where the equivalent independent channel for the k-th user on the n-th subcarrier $\mathbf{H}'_{k,n}$ is

$$\mathbf{H}'_{k,n} = \mathbf{H}_{k,n}\widetilde{\mathbf{V}}_{k,n}^{(0)} . \quad (5)$$

Hence, $\mathbf{A}_{k,n}$ can be viewed as a transmit beamformer on $\mathbf{H}'_{k,n}$ for user k on subcarrier n. By SVD, we can write

$$\mathbf{H}'_{k,n} = \mathbf{U}'_{k,n}\mathbf{\Sigma}'_{k,n}\mathbf{V}'^H_{k,n} \quad (6)$$

and the total capacity of the downlink system under zero-forcing constraint is [6]

$$C = \max_{\left\{\substack{\mathbf{\Lambda}_{k,n}:k=1,\cdots,K;n=1,\cdots,N; \\ \sum_{n=1}^{N}\sum_{k=1}^{K} tr(\mathbf{\Lambda}_{k,n})=P_{Total}}\right\}} \sum_{n=1}^{N}\sum_{k=1}^{K}\log_2\left|\mathbf{I} + \mathbf{\Sigma}'^2_{k,n}\mathbf{\Lambda}_{k,n}\right| \quad (7)$$

where P_{Total} is the total power constraint, $\mathbf{\Sigma}'_{k,n}$ is a diagonal matrix whose entries are the channel-to-noise ratios (or, effective channel gains) $\gamma'_{k,n,m}$ of the m-th eigenmode for $m = 1,\cdots,M_{k,n}$, $M_{k,n}$ is the number of eigenmodes of the equivalent MIMO channel for user k on the subcarrier n, $\mathbf{\Lambda}_{k,n}$ is a diagonal power matrix whose entries are the allocated powers whose optimal values are obtained by water filling [9].

Here, we consider the system has the freedom to select different subsets of users on different subcarriers to maximize the capacity. Assume the total number of users on the system is K and the system can select from 1 to K users on every subcarrier. Then there are 2^K possible user assignment sets. Define $\varphi_i \subseteq \{1,2,\cdots,K\}$ for $i = 1,\cdots,2^K$, to be the i-th user assignment set, containing the indices of a set of users. The capacity maximization problem can be formulated as

$$C = \max_{\{\varphi_{i,n}:1,\cdots,N\}}\max_{\left\{\substack{\mathbf{\Lambda}_{k,n}:k=1,\cdots,K; \\ n=1,\cdots,N}\right\}} \sum_{n=1}^{N}\sum_{k\in\varphi_{i,n}}\log_2\left|\mathbf{I} + \mathbf{\Sigma}'^2_{i,k,n}\mathbf{\Lambda}_{k,n}\right| \quad (8)$$

where this maximization is subject to the power constraint $\sum_{n=1}^{N}\sum_{k\in\phi_i} tr(\Lambda_{k,n}) = P_{Total}$. Note that $\mathbf{\Sigma}'_{k,n}$ in (7) is now dependent on the choice of assignment set in (8) and hence the notation is replaced by $\mathbf{\Sigma}'_{i,k,n}$. $\mathbf{\Sigma}'_{i,k,n}$ is determined by deleting the blocks of unselected users in φ_i from $\widetilde{\mathbf{H}}_{k,n}$ and following the steps described above.

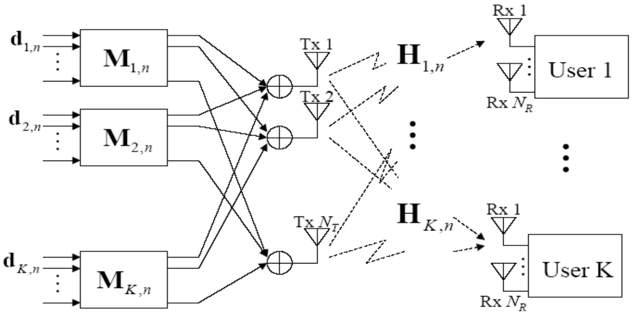

Figure 1. The multiuser MIMO-OFDM downlink system on subcarrier n

III. Optimal Power Allocation Algorithm For Multiuser MIMO System

The problem in (8) is a combinatorial optimization problem. The exhaustive search for the optimal solution requires $O(2^{KN})$ complexity and is prohibitive even for small values of N . To make the problem tractable, we take similar approach as [3][5] to reformulate the problem to be a convex optimization problem. We define

$$C_{i,n}(P_{i,n}) = \max_{\left\{\substack{\mathbf{\Lambda}_{k,n}:k\in\varphi_i; \\ \sum_{k\in\varphi_i} tr(\mathbf{\Lambda}_{k,n})=P_{i,n}}\right\}} \sum_{k\in\varphi_i}\log\left|\mathbf{I} + \mathbf{\Sigma}'^2_{i,k,n}\mathbf{\Lambda}_{k,n}\right| , \quad (9)$$

and rewrite the problem as

$$\max_{\substack{\rho_{i,n}\in[0,1] \\ s_{i,n}\in[0,\infty)}} \sum_{n=1}^{N}\sum_{i=1}^{2^K}\rho_{i,n}C_{i,n}\left(\frac{s_{i,n}}{\rho_{i,n}}\right) \quad (10)$$

subject to $\sum_{n=1}^{N}\sum_{i=1}^{2^K} s_{i,n} = P_{Total}$ and $\sum_{i=1}^{2^K}\rho_{i,n} = 1$, where we define $s_{i,n} = \rho_{i,n}P_{i,n}$ and P_{Total} is the total power constraint. $\rho_{i,n}$ is sharing factor of φ_i on subcarrier n and takes values in [0,1]. By letting

$$\rho_{i,n} = \begin{cases} 0, i^{\text{th}} \text{ user set is not chosen in subcarrier } n \\ 1, \ i^{\text{th}} \text{ user set is chosen in subcarrier } n \end{cases} , \quad (11)$$

we can see the original problem in (8) fits well into the new problem in (10).

Since the total capacity of the parallel channels on a subcarrier is a concave function of allocated power on the subcarrier, $C_{i,n}(P)$ is concave. Hence, the expression in (10) is a concave function because $\rho_{i,n}C_{i,n}\left(\frac{s_{i,n}}{\rho_{i,n}}\right)$ is the perspective function of $C_{i,n}$. The Lagrangian to (10) is

$$L = \sum_{n=1}^{N}\sum_{i=1}^{2^K}\rho_{i,n}C_{i,n}\left(\frac{s_{i,n}}{\rho_{i,n}}\right) - \Omega\left(\sum_{n=1}^{N}\sum_{i=1}^{2^K}s_{i,n} - P_{Total}\right) - \sum_{n=1}^{N}\beta_n\left(\sum_{i=1}^{2^K}\rho_{i,n} - 1\right) \quad (12)$$

where Ω and β_n are Lagrange multipliers. Using Lagrange multiplier method as in [3][5], we can show that the optimal assignment set on subcarrier n is i if it maximizes

$$H_{i,n}(\Omega) = C_{i,n}(C'^{-1}_{i,n}(\Omega)) - \Omega C'^{-1}_{i,n}(\Omega) \quad (13)$$

at the optimal value of Ω . Note that this algorithm has decoupled the search for optimal user assignment sets on different subcarriers and hence requires complexity $O(N2^K)$ without losing the optimality, which is a very dramatic reduction compared to $O(2^{KN})$ required by exhaustive search.

In (9), $C_{i,n}(P)$ is achieved by water filling and obviously the optimal solution has the same water level λ over all subcarriers. One can show that $\lambda = (\Omega \ln 2)^{-1}$ and $P_{i,n}^* = C_{i,n}'^{-1}(\lambda)$ is the allocated power using water filling when assignment set i is selected on the subcarrier n. Hence, (13) can be rewritten as

$$H_{i,n}(\lambda) = C_{i,n}(P_{i,n}^*) - (\lambda \ln 2)^{-1} P_{i,n}^* \tag{14}$$

The optimal user assignment set is given by

$$i^*(n) = \arg\max_i H_{i,n}(\lambda) . \tag{15}$$

when λ is optimal. Given the user assignment set, we have

$$P_{i^*(n),n}^*(\lambda) = \sum_{k \in \varphi_{i^*(n)}} \sum_{m=1}^{M_{i^*(n),k,n}} \left(\lambda - \frac{1}{\gamma'_{i^*(n),k,n,m}} \right)^+ , \tag{16}$$

for all subcarrier n, where $(\cdot)^+$ is the positive function, $M_{i,k,n}$ is the number of eigenmodes (parallel channels) of the equivalent MIMO channel of user k on subcarrier n when the selected user set is $i^*(n)$ on the subcarrier. And the capacity is given by

$$C(\lambda) = \sum_{n=1}^{N} \sum_{k \in \varphi_{i^*(n)}} \sum_{m=1}^{M_{i^*(n),k,n}} \left(\log(\lambda \gamma'_{i^*(n),k,n,m}) \right)^+ . \tag{17}$$

The optimal λ is achieved when total transmit power $P(\lambda) = \sum_{n=1}^{N} P_{i^*(n),n}^*(\lambda)$ is equal to P_{Total}. The algorithm is outlined as follows.

Step 1:

for $n = 1, \cdots, N$ and $i = 1, \cdots, 2^K$,

for $k \in \varphi_i$,

Compute $\mathbf{\Sigma}'_{i,k,n}$ in (8) using (2)-(6).

end

end

Step 2:

Initialize $\lambda > 0$ and $\lambda_l = 0$.

Step 3:

For all subcarrier n, compute $i^*(n)$ using (14) and (15). If the total transmit power $P(\lambda)$ is less than P_{Total}, increase λ by a factor of 2, set $\lambda_l = \lambda$ and repeat Step 3. Otherwise, set $\lambda_u = \lambda$ and proceed to Step 4.

Step 4:

Set $\lambda = (\lambda_u + \lambda_l)/2$ and compute $P(\lambda)$.

If $P(\lambda) < P_{Total}$, set $\lambda_l = \lambda$. If $P(\lambda) > P_{Total}$, set $\lambda_u = \lambda$. Repeat Step 4 until $P(\lambda)$ converges to P_{Total}.

There is chance that two user assignment sets have the same value of $H_{i,n}(\lambda)$, such that one assignment set yields $P(\lambda) < P_{Total}$ and another yields $P(\lambda) > P_{Total}$. In this case, $P(\lambda)$ can never converge to P_{Total} with the above algorithm. The reason for this to happen is the optimal values of some $\rho_{i,n}$ fall within the interval (0,1). This suggests the use of time sharing between these two assignment sets such that the total power constraint is met. This is analogous to the idea to meet the total rate constraint proposed in [3].

IV. Possible Extensions

The algorithm developed in the previous section is based on the selection of users. Nevertheless, it can be extended easily to some systems that allow higher flexibility to further improve the throughput. Rather than user selection, we consider two more special cases: antenna selection and dimension selection.

A. Antenna Selection

If for every selected user and every subcarrier, the system can select some out of N_R receive antennas and ignore the rest of his antennas, the system has larger freedom to select the spatial channels and can accommodate more users in every subcarrier, enhancing the system capacity. In this case, the system selects the rows of $\widetilde{\mathbf{H}}_{k,n}$ in (2) to maximize the capacity. Hence the proposed algorithm should search on the antenna assignment sets, $\{\eta_i : i = 1, \cdots, 2^{KN_R}\}$, instead of the user assignment sets. Denote the set of active users by $\varphi(\eta_i)$ when antenna assignment set i is used, the optimization problem becomes

$$C = \max_{\{\eta_{i,n}:1,\cdots,N\}} \max_{\left\{\substack{\mathbf{\Lambda}_{k,n}: \\ k \in \varphi(\eta_{i,n}); n=1,\cdots,N}\right\}} \sum_{n=1}^{N} \sum_{k \in \varphi(\eta_{i,n})} \log_2 \left| \mathbf{I} + \mathbf{\Sigma}'^2_{i,k,n} \mathbf{\Lambda}_{k,n} \right| \tag{18}$$

subject to $\sum_{n=1}^{N} \sum_{k \in \varphi(\eta_{i,n})} tr(\mathbf{\Lambda}_{k,n}) = P_{Total}$. And the proposed algorithm can be used to find the optimal solution by replacing φ_i by $\varphi(\eta_i)$ in (9), (16) and (17). Note that $\mathbf{\Sigma}'_{i,k,n}$ is now determined from deleting the unselected rows of $\widetilde{\mathbf{H}}_{k,n}$.

B. Dimension Selection

Another situation is that while every selected user uses all of his N_R antennas, the system may allocate different number of substreams or dimensions of received signal space for different users. For each subcarrier, if the dimension assigned to user k is m_k, where $0 \le m_k \le N_R$, the system can assign the dimensions to the users such that $\sum_{k=1}^{K} m_k \le N_T$. Since m_k has $N_R + 1$ possible values, we define the dimension assignment set as $\{\xi_i : i = 1, \cdots, (N_R + 1)^K\}$. The optimization problem becomes

$$C = \max_{\{\xi_{i,n}:1,\cdots,N\}} \max_{\left\{\substack{\mathbf{\Lambda}_{k,n}, \mathbf{W}_{k,n}: \\ k \in \varphi(\xi_{i,n}); n=1,\cdots,N}\right\}} \sum_{n=1}^{N} \sum_{k \in \varphi(\xi_{i,n})} \log_2 \left| \mathbf{I} + \mathbf{\Sigma}'^2_{i,k,n} \mathbf{\Lambda}_{k,n} \right| \tag{19}$$

subject to $\sum_{n=1}^{N} \sum_{k \in \varphi(\xi_{i,n})} tr(\mathbf{\Lambda}_{k,n}) = P_{Total}$, where $\mathbf{W}_{k,n}^*$ is the $m_k \times N_R$ beamforming matrix at the receiver of user k, which determines the orientation of the signal space of user k on subcarrier n. However, the optimal $\mathbf{W}_{k,n}$ is complicated to determine. In this case, a reasonably good suboptimal solution is to select the largest m_k eigenmodes of $\mathbf{H}_{k,n}$ before applying zero-forcing, as proposed in [6] as the coordinated Tx-Rx Block diagonalization algorithm. It means that if $\mathbf{H}_{k,n} = \mathbf{U}_{k,n} \mathbf{\Sigma}_{k,n} \mathbf{V}_{k,n}^*$, $\mathbf{W}_{k,n}$ will be the first m_k columns of $\mathbf{U}_{k,n}$, corresponding to the largest m_k singular values. Then apply the zero-forcing technique in Section II to $\overline{\mathbf{H}}_{k,n} = \mathbf{W}_{k,n}^* \mathbf{H}_{k,n}$ in place of $\mathbf{H}_{k,n}$.

V. NUMERICAL RESULTS

In the simulations, we consider a MIMO downlink system with all user terminals having the same number of receive antennas. The number of subcarriers is 64. The channel used in the simulation is the HiperLan/2 channel model A [10], which is an 8-tap channel with exponential power delay profile, 20MHz sampling frequency and 50ns rms delay spread. In all figures, each curve is denoted by (N_T, N_R, K).

Figure 2 shows the capacity achieved by the proposed optimal algorithm based on user selection. It shows the capacities when N_R =3, and the number of users is $K = 4$ and 8. Under the zero-forcing multiplexing system, the maximum number of spatial channels of the system on every subcarrier, is bounded by the number of transmit antennas. Hence in a user selection system, each selected user will occupy N_R out of the N_T spatial channels and the maximum numbers of active users on each subcarrier is $\lfloor N_T / N_R \rfloor$.

For the purpose of comparison, we also simulate the performance of a round-robin algorithm. This algorithm always selects a set of $\lfloor N_T / N_R \rfloor$ users and allocates all subcarriers to them in every time slot. One user assignment is selected from all those with maximum number of active users, in a round-robin fashion. The performances of the round-robin algorithm when water filling is used to allocate power or not are simulated. When the water filling is not used, equal power allocation over all parallel channels is assumed.

From Figure 2, we can observe that the capacity increases with the number of users for the optimal algorithm but is unchanged for the round-robin algorithm. It is because the round-robin algorithms always average out system capacities achieved by various user assignments. As far as the maximum number of active users does not change, such capacity would be the same. On the other hand, the optimal algorithm always selects the best user assignment to maximize the system capacity. When the number of users increases, the number of possible user assignments increases and the maximum capacity achieved by the best among them increases. Hence the optimal algorithm exploits the multiuser diversity more efficiently.

The capacity gap between the optimal algorithm and the round-robin algorithm seems not significant in terms of percentage at the high SNR regime. However, such gap is significant at the low to moderate SNR regime, which is reasonable operating region in the wideband communications systems. This is illustrated in Figure 3 and 4. Figure 3 plots the ratio of the capacity achieved by the optimal algorithm to that achieved by the round-robin algorithm using water filling for power allocation. At SNR of 10dB, the optimal algorithm has a 40% capacity gain in the (12,3,8) setup. In Figure 4, the round-robin algorithm uses equal power allocation, hence achieving lower capacity and widening the capacity gap significantly. At SNR of 10dB, the optimal algorithm has a capacity improvement over the round-robin algorithm by 60%. When SNR decreases, the significance of water filling becomes more dominant and pushes the capacity ratio more dramatically than the situation in Figure 3. The capacity ratio amounts to 350%, compared to about 180% in Figure 3.

Moreover, the capacity gap increases when the number of receive antennas per user decreases. First, fewer receive antennas per user leads to fewer degrees of freedom in each user's channel and hence the users are more likely to have high correlation with each other. Second, it increases the maximum number of active users and hence increases the number of user assignments containing highly correlated users. Accordingly, the capacity gap between the best user assignment and the worst user assignment widens and only the optimal algorithm can avoid the bad assignments effectively.

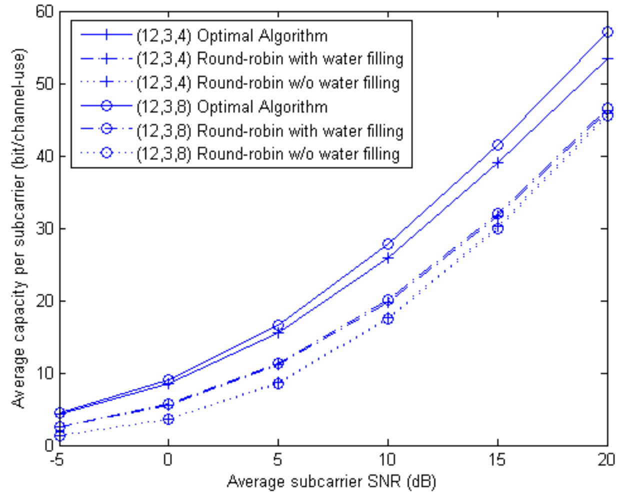

Figure 2. Capacity versus SNR ($N_R = 3$)

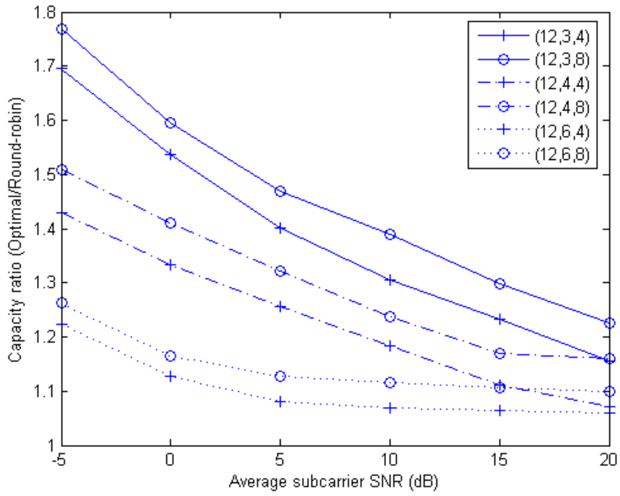

Figure 3. Capacity gain of the optimal algorithm over the round-robin algorithm with water filling

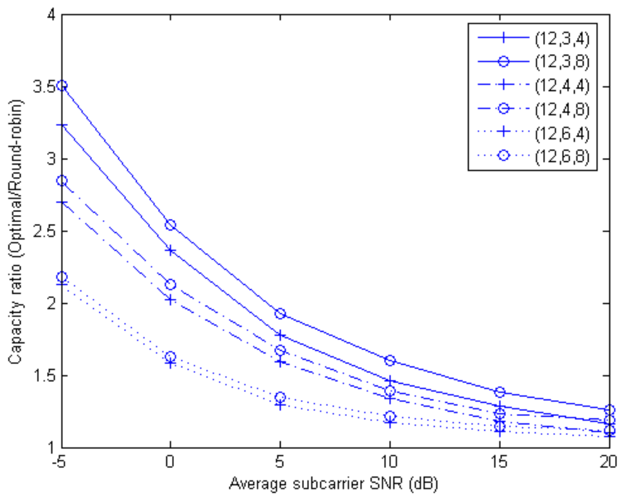

Figure 4. Capacity gain of the optimal algorithm over the round-robin algorithm with equal power allocation

Lastly, when the number of receive antennas per user increases, the capacity loss incurred by the zero-forcing constraint decreases. Hence, the overall system capacity improves for all algorithms.

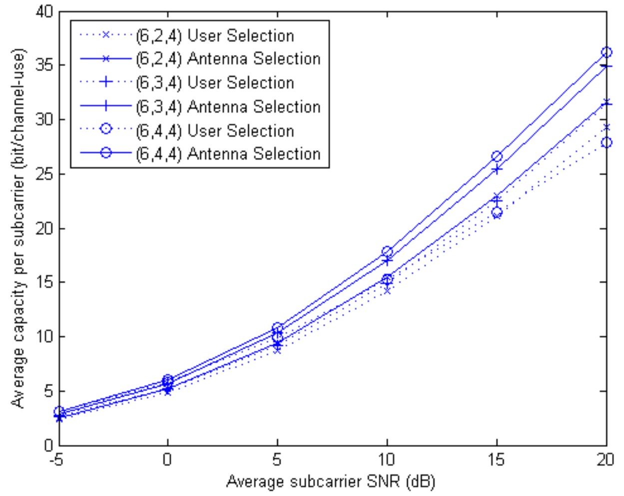

Figure 5. Comparison between antenna selection and user selection

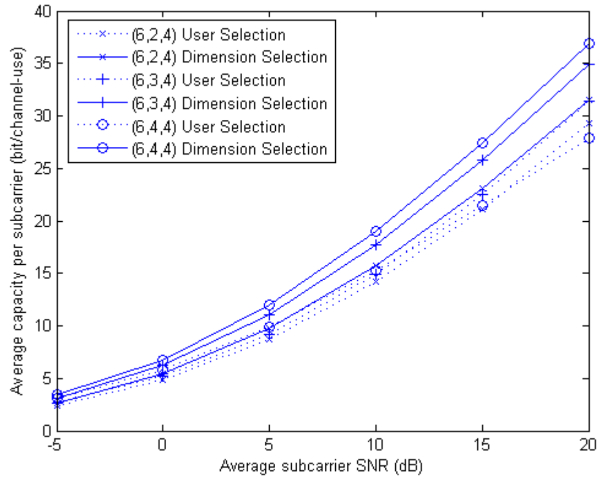

Figure 6. Comparison between dimension selection and user selection

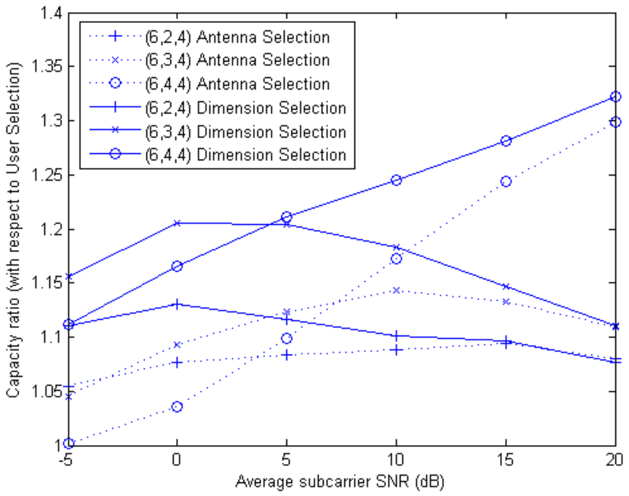

Figure 7. Capacity gain of the two extended approaches over the user selection

Figure 5 and 6 compare the capacity of the user selection system with antenna selection and dimension selection respectively. Since the total numbers of assignment sets for antenna selection and dimension selection are too large for simulation when N_T is 12, we simulate the case when N_T and K are fixed to 6 and 4 respectively. At all SNR, antenna selection and dimension selection outperform user selection. Generally, dimension selection achieves the largest capacity even though it is not optimal. Figure 7 shows the ratio of the capacities of antenna selection and dimension selection to that of user selection. Note that in the (6,4,4) setup, user selection system can select $\lfloor 6/4 \rfloor = 1$ user only for every subcarrier, limiting the capacity. Its capacity is even lower than (6,2,4) when SNR=20dB. Antenna selection and dimension selection do not have such constraint and keep increasing the capacity when N_R increases and hence shows much significant gain over user selection when N_R=4. And at SNR of 10dB, antenna selection and dimension selection have about 14% and 18% capacity gain over the user selection in the (6,3,4) setup.

While dimension selection performs the best, its complexity is lower than antenna selection since the number of assignment sets is much smaller. However, compared to user selection, dimension selection still has a larger complexity of $O(N\,(N_R+1)^K)$ compared to $O(N2^K)$ of user selection. This poses a complexity-performance tradeoff between user selection and dimension selection.

VI. Conclusion

This paper considers the capacity maximization for a MIMO-OFDM downlink system by user selection and power allocation. The combinatorial optimization problem is converted into a convex one and an optimal algorithm has been developed. Numerical results show that the proposed algorithm provides substantial capacity gain over round-robin algorithms. We provide the formulations for two additional optimization problems based on antenna selection and dimension selection, which have the potential to further improve the system capacity, at the expense of complexity. Optimal and suboptimal solutions are developed for the antenna selection and dimension selection respectively.

References

[1] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol.6, pp.311 - 335, March 1998.

[2] A. Goldsmith, S.A. Jafar, N. Jindal and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE Journal on Selected Areas in Communications*, vol.21, no.5, pp. 684 – 702, June 2003.

[3] Cheong Yui Wong, R.S. Cheng, K.B. Letaief and R.D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE Journal on Selected Areas in Communications*, vol.17, no.10, pp. 1747 – 1758, Oct. 1999.

[4] Jiho Jang; Kwang Bok Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on Selected Areas in Communications*, vol.21, no.2, pp.171 – 178, Feb. 2003.

[5] Y. M. Tsang, "Optimal resource allocation in SDMA/ Multi-Input-Single-Output/OFDM systems under QoS and power constraints," *WCNC 2004*, pp.1595 - 1600.

[6] Q.H. Spencer, A.L. Swindlehurst and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Transactions on Signal Processing*, vol.52, no.2, pp. 461 – 471, Feb. 2004.

[7] Q.H. Spencer, C.B. Peel, A.L. Swindlehurst and M. Haardt, "An introduction to the multi-user MIMO downlink," *IEEE Communications Magazine*, vol.42, no.10, pp. 60 – 67, Oct. 2004.

[8] Lai-U Choi, R.D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," *IEEE Transactions on Wireless Communications*, vol.3, no.1, pp. 20 – 24, Jan. 2004.

[9] T.M. Cover and J.A. Thomas, *Elements of Information Theory*. John Wiley and Sons, Inc, 1991.

[10] ETSI Normalization Committee, "Channel models for HIPERLAN/2 in different indoor scenarios," European Telecommunications Standards Institute, Sophia-Antipolis, Valbonne, France, Norme ETSI, doc. 3ERI085B, 1998.