Fine Timing and Frequency Synchronization for MIMO System

A. Saemi, V. Meghdadi, JP. Cances, M.J.Syed, G. Ferre and JM. Dumas
University of Limoges, ENSIL-GESTE, Parc ESTER B.P 6804, 87068 Limoges, France

Abstract— **In this paper a new criterion is proposed for precise timing synchronization in multiple-input multiple-output (MIMO) system. Although this criterion is derived for block flat fading channel, according to the simulation results presented in this paper, the proposed method can also be used for frequency selective channel. Using this method, offset frequency estimation is improved in comparison with published results. The simulations are done for IEEE 802.11a wireless LAN specifications about training sequence.**

Index term—**MIMO system, training sequence design, timing synchronization, frequency synchronization**

I. Introduction

High data rate techniques in communication systems for multimedia applications have gained considerable interest in recent years. A technique that has attracted a lot of attention is Multiple Input Multiple Output systems (MIMO) combined with Orthogonal Frequency Division Multiplexing (OFDM).

Each MIMO system requires synchronization in both the time and frequency. Time synchronization deals with finding the packet arrival instant in received noise and interference corrupted signal. Frequency synchronization deals with finding an estimation of the frequency offset between the received carrier and local oscillator. Precision of timing and frequency synchronization subsystems has a major effect on the overall MIMO system performance. In particular in OFDM–MIMO case, the degradation due to imperfection in frequency synchronization is more important and should be improved by all means.

Numerous ideas have been proposed in literature to approach the synchronization problem with different assumptions. Here the focus is on the repetition preamble scheme introduced for OFDM by Moose[1]. He describes a technique to estimate frequency offset by using a repeated training sequence and derives a maximum likelihood estimation procedure. Then, this idea was used for time synchronization by Schmidl [2], Speth [3] and Keller and Hanzo [4]. The general principle of exploiting a double training sequence preceded by a cyclic prefix (CP) for frame synchronization was originally suggested for single-carrier Single Input Single Output (SISO) transmission in [5]. Weinfurtner in [6] introduces several metric functions to detect repeated preambles for course frame synchronization. Weinfurtner compares several metrics by simulation.

Zelst [7] and Schenk in [8] extend the Moose algorithm for frequency synchronization in MIMO-OFDM system present coarse time synchronization using the results of [6]. Fine time synchronization is accomplished after frequency synchronization by minimizing the Inter Carrier Interference (ICI) and Inter Symbol Interference (ISI) [9]. Duc Long in [10] base on [7] proposed an algorithm to improve timing estimation without considering carrier frequency offset.

Training sequence repetition is a suitable technique to identify propagation channels and gives good results about phase rotation due to frequency error. In literature, fine timing synchronization is done after removing the frequency error. We propose in this paper an algorithm to find the fine time of packet arrival in presence of frequency offset and improve the frequency estimation by using the result of fine symbol timing.

In absence of frequency carrier offset, the best way for time synchronization is to calculate correlation between a known reference and received sequence. However the presence of frequency offset reduces the peak of correlation function. In fact, this frequency offset prevents coherent addition of individual term in correlation calculation and results in a drop of the correlation peak. So in this paper, we propose a new technique to estimate fine time of packet arrival before frequency synchronization.

The rest of this paper is as follows. In section II the MIMO system description and signal model are defined. Section III, first introduces the fine time synchronization algorithm and then investigate the frequency estimation improvement. Section IV presents a suitable preamble design algorithm. The performance of our synchronization algorithms is verified by simulation and presented in section V.

II. System Model

The simplified overall MIMO-OFDM digital transmission system is presented in Figure 1. The OFDM symbols are generated by taking Inverse Discrete Fourier Transform (IDFT) of complex input symbol drown from a complex constellation (MPSK or QAM). Since timing recovery frequency estimation is done before applying any special demodulation schemes (e.g. OFDM), our synchronization can be used for any MIMO system. However, we use the specifications of WLAN standard IEEE 802.11a [11] and extend them to be used in a MIMO-OFDM scheme.

Consider a MIMO system with N_t transmit (TX) and N_r receive (RX) antennas. As it is shown in Figure1 fine time

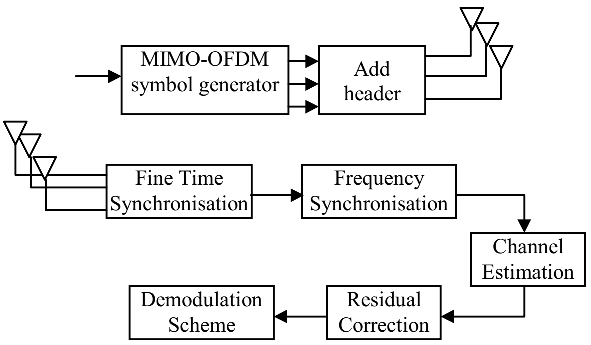

Figure 1- Simplified MIMO-OFDM system

synchronization is the first algorithm to be applied on the received signal. Next, frequency estimation algorithm is applied with the idea mentioned in [8]. Then, Duc Long algorithm, introduced in [10], can be executed to correct the residual offset between estimate and exact timing instant. Since in this case, a channel estimation system is required, we do not develop this idea here.

Using the same notations as used by Zelst *et al* in [7], signal model for the above system description is as follow. Consider an $N_t \times N_r$ system where an N_t dimension complex vector $\mathbf{u}(\tau)$ is sent in a discrete time instant τ. The received complex vector is of dimension N_r and denoted by $\mathbf{r}(\tau)$.

The fading multi-path channel is considered to be quasi static. Suppose the channel length to be L, so we have $N_t \times N_r$ path, each of which can be modeled by an equivalent FIR complex filter of order $L-1$ with $g_{pq}(l)$ as the taps with $l=[0, 1, ..., L-1]$. These taps are assumed to be independent zero mean complex Gaussian random variables with variance $1/2P(l)$ per dimension. The ensemble $P(l)$ with $l=[0, 1, ..., L-1]$ is called the power delay profile (PDP) of the channel and its total power is assumed to be normalized to $\sigma_c^2=1$, which is the average channel attenuation. Suppose $g_{pq}(l)$ is the $(q,p)^{th}$ element of the matrix $\mathbf{G}(l)$ and represents the path between the p^{th} transmit and q^{th} receive antenna. With these notations, the received signal by the q^{th} antenna can be written as:

$$r_q(\tau) = \sum_{p=1}^{N_t} \sum_{l=0}^{L-1} u_p(\tau - l) g_{pq}(l) + v(\tau) \tag{1}$$

where $v(\tau)$ represents complex additive white Gaussian noise (AWGN) at instant τ with variance $(1/2)\sigma_v^2$, stationary and independent from the other antennas. This equation can be written for all the receive antennas as bellow:

$$\mathbf{r}(\tau) = \sum_{l=0}^{L-1} \mathbf{G}(l)\mathbf{u}(\tau - l) + \mathbf{v}(\tau) \tag{2}$$

where $\mathbf{r}$ and $\mathbf{v}$ are column vectors of dimension N_r.

In the presence of frequency error due to local oscillator deviation or Doppler effect, each received symbol undergoes a phase rotation proportional to the delay between the transmit and receive instants and the frequency error. The received complex vector can be written as follow:

$$\mathbf{r}(\tau) = e^{j2\pi\Delta f\tau} \sum_{l=0}^{L-1} \mathbf{G}(l)\mathbf{u}(\tau - l) + \mathbf{v}(\tau) \tag{3}$$

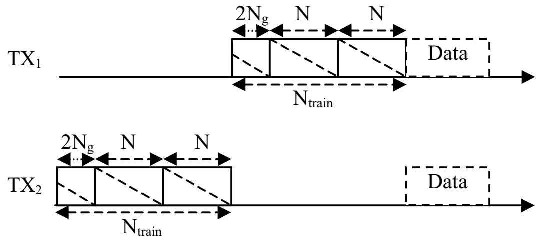

Figure 2- Training sequence format

where Δf is frequency offset.

III. Synchronization

A. Fine Time Synchronization

The idea of calculation of complex correlation of two preambles, proposed in literature [4],[6],[7], is a good way to detect packet arrival in the presence of frequency error. The training sequence used is presented in Figure 2.

This method calculates the correlation of received signal with itself with a delay of N samples. The drawback of this method is that the correlation function does not produce a sharp peak at the arrival of the data packet. In fact, during the CP period, the correlation function remains constant and there is no abrupt falling edge outside this period. This edge can not be localized precisely in a cheap signal quality. This phenomenon, inherent to the structure of training sequence, makes it difficult to find the fine timing. In the other hand, conventional correlation calculation between the received sequence and a noiseless training sequence known by receiver would give a sharp peak only when there is no frequency error. In fact, the phase rotation due to frequency offset degrades the peak generation in correlation calculation.

We propose another robust solution to find the exact time of packet arrival by considering these two aspects at the same time. We use a new function, which has a very sharp response, even in the presence of relatively large frequency offset. We define the function y_q for the q^{th} receive antenna while the p^{th} transmit antenna is considered:

$$y_q(\tau) = \sum_{i=1}^{N-1} r_q^*(\tau - i + 1) c_p(N - i) r_q(\tau - i) c_p^*(N - i - 1) \tag{4}$$

where c_p represents the cyclic prefix sequence. Considering the training sequence as presented in Figure 2, it is clear that the received signal in each instant is only the function of one training sequence of corresponding emitting transmitter. In order to simplify notation assuming the time reference at the instant where the preamble arrives at the receivers ($\tau=0$), So $r_q(n)$ when the p^{th} transmitter is on can be written as follows:

$$r_q(\tau) = c_p(\tau) g_{pq}^1 e^{j2\pi\tau\Delta f T_s} + v_q(\tau) \tag{5}$$

Replacing (5) in (4) and assuming a flat fading channel we obtain:

$$y_q(\tau)\big|_{\tau=N-1} = y_q^{\max} = \sum_{i=1}^{N-1} \left|c_p(N-i)\right|^2 \left|c_p(N-i-1)\right|^2 \left|g_{pq}^1\right|^2 e^{-j2\pi\Delta f T_s} + v_q'(N-1) \tag{6}$$

Hence, $y_q(N\text{-}1)$ presents a sharp peak at packet arrival independent of the channel phase coefficient and frequency error. Using the principle of maximum ratio combining and since the phase effect due to channel and frequency error is already cancelled out, we can add up $y_q(\tau)$ directly to form our decision function for the p^{th} transmit antenna.

$$y^p(\tau) = \sum_{q=1}^{N_r} y_q(\tau) \tag{7}$$

In order to take advantage of the other codes sent by the other transmit antennas, we can sum up a shifted version of all the $y_p(\tau)$ to obtain our final decision function:

$$y(\tau) = \sum_{k=0}^{N_t-1} y^p(\tau - kN_{train}) \tag{8}$$

B. Frequency Synchronization

The goal of this section is to find the carrier frequency offset that appears as Δf in the equation (3). Moose showed in [1] how frequency offset can be estimated for a SISO OFDM system and it is further worked out in [2]. In [8] by extending the Moose algorithm suitable technique is given for estimating of frequency offset in MIMO systems. To this purpose it is proposed to calculate the correlation function over received signal and delayed version of it. This operation should be done in the maximum of the correlation function where we are sure to be in the training sequence. So the correlation function Λ is defined as:

$$\Lambda(m) = \sum_{q=1}^{N_r} \sum_{\tau=m}^{m+N-1} r_q(\tau) r_q^*(\tau - N_p) \tag{9}$$

where m is the instant where the correlation function is maximal. The estimated frequency offset Δf_{est} is given by:

$$\Delta f_{est} = \frac{\theta_{est}}{2\pi N T_s} = \frac{f_s \angle\Lambda(m)}{2\pi N} \tag{10}$$

where $\theta_{est} = \angle\Lambda(m)$ indicates the phase of $\Lambda(m)$ and f_s is sampling frequency. Since the exact timing is not known using the algorithm proposed in the literature, the above estimation is done at the instant where $\Lambda(m)$ is maximal. Here, because of our exact timing algorithm, the estimation can operate over the whole cyclic prefix and training sequence duration. The estimation function now can be written as

$$\Lambda(m) = \sum_{q=1}^{N_r} \sum_{\tau=m}^{m+N+2N_g-1} r_q(\tau) r_q^*(\tau - N_p) \tag{11}$$

where m is the exact time at the end of training sequence.

IV. TRAINING SEQUENCE DESIGN

Training sequence to be used should satisfy the condition corresponding to equation (4). In fact, the sequence is not required to be orthogonal to its shifted version. Defining $c'(n) = c(n)c^*(n-1)$, the required condition is:

$$\sum_{\substack{n=0 \\ m\neq 0}}^{N-1} c'(n) c'^*((n+m) \bmod N) = 0 \tag{12}$$

It guaranties a sharp peak of the function defined by (4) and consequently by (8). One possible code that presents a good periodic correlation properties is proposed by [12]. Applying this code to $c'(n)$, the training sequence $c(n)$ can be calculated by means of a simple recursion algorithm as follow:

- $c(0) = 1$
- $c(i) = \dfrac{c'^*(i-1)}{c^*(i-1)}$

Figure 3- Peak of the y(n) (equation 8) helping to detect the exact timing even in low SNR

V. SIMULATED RESULT

In simulation, we use the same specifications as the IEEE 802.11a. The channel is supposed to be flat fading Rayleigh channel. In order to compare with the literature curves, we use a 4×4 MIMO system. The training sequence is of the form of Figure 2 with $N=64$ and $N_g=32$. The frequency error $\Delta f T_s$ is fixed to $0.2/N_c$ where $N_c=64$ is assumed to be the number of sub-carriers for an OFDM system. Figure 3 presents $y(n)$ as defined by equation (8). The correlation function proposed in literature [4][6][7] is presented also to make clear the decision algorithm. As soon as maximum correlation function [6] exceeds a fixed threshold, we search for a peak in (8) that gives the exact packet arrival.

Next, we use equations (10) and (11) to estimate the frequency offset and compare it with that of Cramer-Rao lower bound. Rife in [13] has shown that Cramer-Rao lower bound in a SISO system is given by:

$$\mathrm{var}(\theta) \geq \frac{1}{SNR \times N} \tag{13}$$

with N the number of observation samples. Here, we have $N_r(2N+2N_g)$ samples. Since differential detection is used to estimate the frequency error, the number of available variables is $N_r(2N+2N_g)-1$ that we substitute in equation (13). Furthermore, for the case of MIMO system, we replace the SNR with SNR per receive antenna. The final formula to calculate the Cramer-Rao bound is now the following:

$$\mathrm{var}(\theta) \geq \frac{1}{SNR \times N_r(2N + 2N_g - 1)} \tag{14}$$

Using equation (10), the lower bound for frequency error variance can be calculated as:

Figure 4- Variance of frequency error estimate for a 4×4 MIMO system

$$\mathrm{var}[(\Delta f_{est} - \Delta f)T_s] \geq \frac{1}{4\pi^2 N^2 SNR \times N_r (2N + 2N_g - 1)} \quad (15)$$

Figure 4 presents the simulation results using the frequency estimator of equation (10). It is compared with results obtained using Moose algorithm extended to MIMO by Zelst and Schenk and also with corresponding theoretical Cramer-Rao bound deduced from equation (15). To obtain the results of Figure 4, perfect symbol synchronization has been supposed. The simulation is repeated to estimate the timing error between exact packet arrival and estimated one. Using our method, no timing error observed for SNR greater than 5 dB.

Although our algorithm is designed for flat fading channels, we wanted to quantify the performance degradation when this algorithm is applied directly to frequency selective channels. The performance is measured by the probability of time synchronization failure, which can be compared with conventional time synchronization algorithm [10] (it should be mentioned that since the residual correction which introduced in [10] is out of scope of this article, it is not considered in these simulations).

TABLE 1

Synchronization failure probability in conventional and proposed algorithms

τ_{rms}	$P_{\text{fail in conventional time synchronization algorithm}}$	$P_{\text{fail in proposed time synchronization algorithm}}$
50 ns	9.4×10^{-3}	1.7×10^{-3}
100 ns	6.4×10^{-2}	1.7×10^{-2}
150 ns	1.2×10^{-1}	6.1×10^{-2}
200 ns	1.8×10^{-1}	1.0×10^{-1}
250 ns	2.4×10^{-1}	2.0×10^{-1}

Table I shows the results. The simulations have been performed for a 2x2 MIMO system with SNR=6 dB. Exponential decay channel was considered with Rayleigh fading coefficients. The rms delay spread range varies between 50 and 250 ns and sampling rate is fixed to 20 MHz. As it can be seen from the table, this algorithm outperforms the conventional method especially in low dispersive channel, as expected. It is to be mentioned that the results for conventional approach are obtained without considering frequency carrier offset.

VI. Conclusion

The problem of timing and frequency synchronization in MIMO system is addressed in this paper. A new correlation function is proposed that results in a fine timing synchronization in presence of carrier frequency offset. The most important property of this function is the fact that it presents a net peak at packet arrival. So, the FFT interval for a MIMO-OFDM system can be identified precisely. Furthermore, it permits us to average our frequency estimator over a wider range, which results in better frequency estimation. Monte Carlo simulations show that this algorithm outperforms the conventional ones even in multipath Rayleigh fading channels.

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