An Interpretation of Channel Overloading and Construction of Some Novel Schemes

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***Abstract*—Channel overloading, which refers to accommodating a larger number of users on a channel than can be achieved with orthogonal signaling, is currently a very hot topic for future wireless systems. A number of approaches have been described in the literature, and the most efficient of them use several sets of orthogonal signal waveforms. Multiuser interference, which is inherent to channel overloading, is handled using maximum-likelihood detection or iterative interference cancellation. In this paper, channel overloading is interpreted as multilevel coding of an expanded signal constellation. This interpretation gives further insight into the performance and limitations of existing channel overloading techniques and allows us to construct some novel schemes.**

***Index Terms*—Channel overloading, combined TDMA/OCDMA, multilevel coding, multiple access, set partitioning.**

I. Introduction

Efficient use of the available radio spectrum is a major requirement for future wireless systems. In a cellular system based on code-division multiple access (CDMA), the number of sequences assigned to users in a cell is typically less than the spreading factor (spreading sequence length), and cells are underloaded. To make a better use of the radio spectrum, it is of considerable interest to assign more sequences than the spreading factor, i.e., to overload the channel. This kind of channel overloading has been actually provisioned in third-generation (3G) wireless standards [1]. In a series of recent papers, one of the present authors and his co-authors analyzed the general problem of channel overloading and proposed to use two sets of orthogonal signal waveforms (see, e.g. [2] and [3]). Maximum likelihood (ML) detection, which has a computational complexity that increases exponentially with the number of users, is intractable for these schemes: For this reason, the authors used a two-stage iterative interference cancellation instead.

An *ad-hoc* channel overloading technique was described by Ross and Taylor in [4]. This technique, which can be interpreted as a special case of channel overloading using multiple sets of orthogonal signal waveforms, makes it possible to increase the number of users on a channel by approximately 33% at best. A low-order polynomial complexity (in $O(K^{3/2})$) optimal detector has been shown to exist for such a construction in [5]. The channel overloading concept described in [2], [3], and other papers of the same authors generally do not have the same limitation in the number of users, but performance degradation becomes significant when the number of users is increased.

In this paper, we give a new interpretation to channel overloading. Various channel overloading techniques are interpreted as a multilevel coding of an expanded signal constellation. Not only this interpretation gives further insight into the performance limitations of existing channel overloading schemes, but it also allows constructing several new schemes, following the rate-splitting multiple access technique presented in [6]. For simplicity, we will limit our study to real signals using binary phase-shift keying (BPSK) modulation. Throughout the paper, we assume perfect timing synchronization and power control and consider an additive white Gaussian noise (AWGN) channel.

The paper is organized as follows: First, in Section II, we describe the concepts proposed in [2]–[4] and give an interpretation of these concepts in terms of multilevel coding of an expanded signal constellation. Section III describes some novel schemes with higher capacity and performance compared to existing schemes. Finally, we give a summary of our results and present our conclusions in Section IV.

II. An Interpretation of Channel Overloading

A. The Principle of Channel Overloading

In multiuser communications, channel overloading refers to accommodating a larger number of users than can be achieved with orthogonal signaling. A simple way to describe it is as follows: Suppose that all users need the same data rate which requires a bandwidth of W Hz in the single-user case. Channel overloading consists of accommodating more than N users on a multiple access channel which has a bandwidth of $N.W$ Hz.

In the sequel, K denotes the number of users. Hence, the case of channel overloading corresponds to $K \geq N$, that is $K = N + M$, the overload factor (OF) being given by $OF = M / N$. Furthermore,

- $\mathbf{a} = (a_1, a_2, ..., a_K)^T$ is the set of BPSK symbols associated to the K users,
- $\mathbf{s} = (s_1, s_2, ..., s_N)^T$ is the transmitted signal block.

The minimum Euclidean distance d_{min} is defined as:

$$d_{\min} = \min_{\mathbf{a}^{(i)} \neq \mathbf{a}^{(j)}} d\left(\mathbf{s}\left(\mathbf{a}^{(\mathbf{i})}\right), \mathbf{s}\left(\mathbf{a}^{(\mathbf{j})}\right)\right) \tag{1}$$

This work was supported by France Telecom R&D under contract no. 42621040.

$$\text{with } d\left(\mathbf{s}\left(\mathbf{a^{(i)}}\right), \mathbf{s}\left(\mathbf{a^{(j)}}\right)\right) = \left\|\mathbf{s}\left(\mathbf{a^{(i)}}\right) - \mathbf{s}\left(\mathbf{a^{(j)}}\right)\right\|.$$

It represents the minimum Euclidean distance between the transmitted sequences $\mathbf{s}\left(\mathbf{a^{(i)}}\right)$ and $\mathbf{s}\left(\mathbf{a^{(j)}}\right)$ corresponding to two different sets of symbols $\mathbf{a^{(i)}}$ and $\mathbf{a^{(j)}}$ associated to the K users.

Also, the asymptotic gain of a scheme refers to the savings attainable in the transmitted energy per information bit to the noise spectral density ratio (E_b/N_0) required to achieve a given bit error probability when this scheme is used in comparison with the BPSK performance.

B. Channel Overloading Using Two Sets of Orthogonal Signal Waveforms

In [2], [3] and subsequent papers on the subject by the same authors, a general channel overloading concept was described which uses two sets of orthogonal signal waveforms to accommodate $K = N + M$ users ($OF = M / N$). The waveforms of the first signal set in this scheme are assigned to the first N users, and M waveforms from the second set are assigned to the next M users.

For instance, using TDMA (time-division multiple access) and OCDMA (orthogonal CDMA) for the first set and the second set of users respectively, the transmitted signal block can be written as follows:

$$s_k = a_k + \frac{1}{\sqrt{N}} \sum_{m=1}^{M} a_{N+m} w_{mk} \qquad k = 1, 2, ..., N \tag{2}$$

where w_{mk} designates the kth chip of the Walsh-Hadamard sequence assigned to the mth OCDMA user (w_{mk} taking its values from {-1, +1}).

If we restrict the M parameter to 1, the transmitted signal can be written as:

$$s_k = a_k + \frac{1}{\sqrt{N}} a_{N+1} \qquad k = 1, 2, ..., N. \tag{3}$$

Suppose that the data symbols take their values from the alphabet {-1, +1}. Clearly, the alphabet of the transmitted signal samples will be $A_0 = \left\{\pm 1 \pm \frac{1}{\sqrt{N}}\right\}$. This is a quaternary alphabet with non-uniformly spaced constellation points. Following Ungerboeck's set partitioning concept, this constellation can be partitioned into two subsets $B_0 = \left\{\pm 1 + \frac{1}{\sqrt{N}}\right\}$ and $B_1 = \left\{\pm 1 - \frac{1}{\sqrt{N}}\right\}$. From (3), it is clear that all signal samples of a block take their values either from the subset B_0 (if a_{N+1} = +1) or from B_1 (if a_{N+1} = -1). That is, this channel overloading technique can be interpreted as a block coded modulation (BCM) based on one-step partitioning of a quaternary non-uniform signal constellation, in which the bit that selects the B-subset is a simple repetition code. The transmitted N-dimensional signal thus takes its values from $B_0B_0...B_0$ or $B_1B_1...B_1$. Provided that $N \geq 4$, the minimum Euclidean distance d_{min} of this multilevel code is 2, and channel overloading is achieved without any asymptotic performance loss. However, it is worth noting that when $M > 1$, and depending on the value of N, the minimum distance d_{min} can decrease and reach 0.

C. Construction Proposed by Ross and Taylor

A particular channel overloading scheme was proposed by Ross and Taylor in [4]. This scheme aims at increasing the number of users while keeping d_{min} at the minimum distance of the single user system, if the users are constrained to be of equal energy (implying no asymptotic performance loss). It uses a block length of $N = 4^n$, with n integer. The maximum number of additional users which can be accommodated is equal to $M = \sum_{j=1}^{n} 4^{n-j} = \frac{1}{3}(N-1)$. This provides a maximum overload equal to $OF = \sum_{j=1}^{n} 4^{-j} = \frac{1}{3}\left(1 - \frac{1}{N}\right)$. As a consequence, 1/3 is an upper bound to the overload factor that can be achieved with this channel overloading technique.

This scheme uses n+1 orthogonal sets of users such that the jth set (j = 0, …, n) comprises 4^{n-j} users. In a fully loaded system, the transmitted signal block can be expressed as:

$$s_k = a_k + \sum_{j=1}^{n} \frac{1}{2^j} a_{\sum_{l=0}^{j-1} 4^{n-l} + \left\lceil \frac{k}{4^j} \right\rceil} \qquad k = 1, 2, ..., N. \tag{4}$$

Thus, for N = 16, 5 symbols are transmitted per block in addition to the original 16 symbols as follows:

$$s_k = a_k + \frac{1}{2} a_{16+\lceil k/4 \rceil} + \frac{1}{4} a_{21} \qquad k = 1, ..., 16. \tag{5}$$

This scheme can be interpreted similarly as the combined TDMA/OCDMA scheme. From (5), the transmitted signal samples take their values from the alphabet $A_0 = \left\{\pm 1 \pm \frac{1}{2} \pm \frac{1}{4}\right\}$ for N = 16. This signal set can be partitioned into two subsets $B_0 = \left\{\pm 1 \pm \frac{1}{2} + \frac{1}{4}\right\}$ and $B_1 = \left\{\pm 1 \pm \frac{1}{2} - \frac{1}{4}\right\}$. A further partitioning step gives $C_0 = \left\{\pm 1 + \frac{1}{2} + \frac{1}{4}\right\}$, $C_2 = \left\{\pm 1 - \frac{1}{2} + \frac{1}{4}\right\}$, $C_1 = \left\{\pm 1 + \frac{1}{2} - \frac{1}{4}\right\}$, and $C_3 = \left\{\pm 1 - \frac{1}{2} - \frac{1}{4}\right\}$. Clearly, selection of a B-subset for the transmitted signal samples is made by symbol a_{21}, which forms a repetition code over the length-16 data block. In addition, the C-subsets from which they take their values are also determined by a coded sequence. Specifically, the transmitted block takes its values from $C_{\alpha_1} C_{\alpha_2} C_{\alpha_3} ... C_{\alpha_{16}}$, where the indices of the C-subsets satisfy the following repetition codes: $\alpha_4 = \alpha_3 = \alpha_2 = \alpha_1$, $\alpha_8 = \alpha_7 = \alpha_6 = \alpha_5$, $\alpha_{12} = \alpha_{11} = \alpha_{10} = \alpha_9$, and $\alpha_{16} = \alpha_{15} = \alpha_{14} = \alpha_{13}$.

III. Construction of Some New Schemes

A. One-Level Partitioning

Following the previous interpretations of existing channel overloading schemes in terms of multilevel coding of an expanded signal constellation, we now investigate a transmission scheme which uses one level of signal superposition and a one-level partitioning of the resulting signal constellation. If we scale by λ the pulse which carries the excess bits, the transmitted signal levels can be expressed as:

$$s_k = a_k + \lambda\, b_k \qquad k = 1, 2, ..., N \tag{6}$$

where the a_k's are the primary (uncoded) bits after BPSK mapping, and the b_k's are the excess (coded) bits after BPSK mapping.

The alphabet of the transmitted signal samples is $A_0 = \{\pm 1 \pm \lambda\}$, which can be partitioned into two subsets $B_0 = \{\pm 1 + \lambda\}$ and $B_1 = \{\pm 1 - \lambda\}$. The minimum Euclidean distance within each of these subsets is 2, and the minimum distance between the B-subsets is $d_{\min}(B_0, B_1)$ = $\min(2\lambda, 2(1-\lambda))$, which is maximized for λ = 1/2. This gives $d_{\min}(B_0, B_1) = 1$. The group of additional bits selects a B-subset for each symbol, and the primary bits determine a point in each subset. The additional bits are encoded using a Reed-Muller (RM) code with a Hamming distance d_H of 4, in order to verify $d_H d_{\min}^2(A_0) = d_{\min}^2(B_i) = 4$. This code is in fact an extended Hamming code of length $N = 2^q$. The number of information bits per code word is thus $M_1 = 2^q - q - 1$. As a consequence, the total number of information bits per block is $N + M_1$, and $OF = M_1/N$.

To illustrate, let us consider the case corresponding to N = 8. We call (i_1, i_2, …, i_8) the N = 8 primary information bits and (i_9, i_{10}, i_{11}, i_{12}) the M_1 = 4 additional information bits ($i_k \in \{0,1\}$ for k = 1, 2, ..., 12). Bits (i_1, i_2, …, i_8) are transmitted serially using BPSK mapping, resulting in (a_1, a_2, …, a_8) (with $a_j \in \{-1,1\}$ for j = 1, 2, ..., 8). The additional bits (i_9, i_{10}, i_{11}, i_{12}) are coded with an RM code of Hamming distance 4. The resulting sequence (x_1, x_2, …, x_8) is given by:

$$\begin{array}{ll} x_1 = i_9 & x_2 = i_9 \oplus i_{12} \\ x_3 = i_{10} & x_4 = i_{10} \oplus i_{12} \\ x_5 = i_{11} & x_6 = i_{11} \oplus i_{12} \\ x_7 = p & x_8 = p \oplus i_{12} \end{array} \tag{7}$$

with $p = i_9 \oplus i_{10} \oplus i_{11}$. After BPSK mapping of (x_1, x_2, …, x_8) ($x_j \in \{0,1\}$ for j = 1, 2, ..., 8), we obtain the symbols (b_1, b_2, …, b_8) (with $b_j \in \{-1,1\}$ for j = 1, 2, ..., 8). Hence, the transmitted sequence is given by:

$$s_k = a_k + \frac{1}{2} b_k \qquad k = 1, 2, ..., 8. \tag{8}$$

Using this design, we obtain OF = 50%. Furthermore, the transmitted energy per bit is equal to 5/6, which implies an improvement of 0.8 dB of the asymptotic performance with respect to BPSK.

The trellis associated to this scheme is shown in Fig. 1 and Fig. 2 for N = 8 and N = 16, respectively. In both cases, the upper half-trellis corresponds to i_{12} = 0, and the lower half-trellis to i_{12} = 1. Each transition corresponds to the transmission of two consecutive samples in the trellis of Fig. 1 and of four samples in the trellis of Fig. 2. Generally, the trellis becomes more and more complex as N is increased.

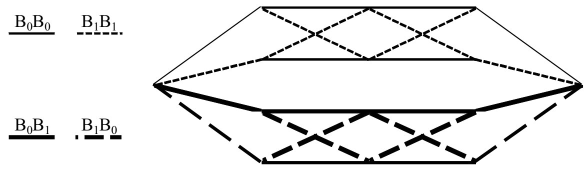

Fig. 1. State trellis for one-level partitioning with N = 8

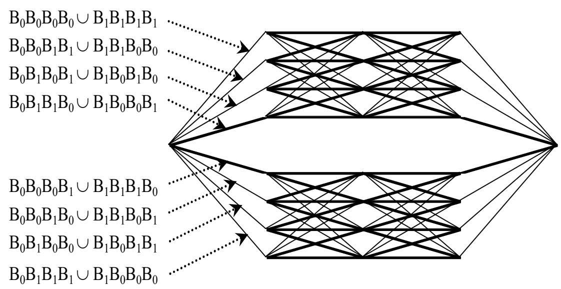

Fig. 2. State trellis for one-level partitioning with N = 16

We now investigate the mean BER performance of TDMA/OCDMA, Ross and Taylor's construction and one-level partitioning scheme after a ML decoding, performed in the latter case by the Viterbi algorithm. We consider an AWGN channel. The BER results, evaluated by means of Monte-Carlo simulations, are given as a function of the ratio E_b/N_0. They can be confronted with the theoretical BPSK performance.

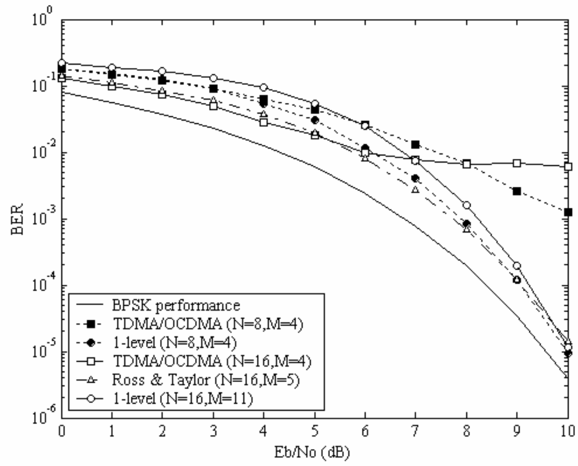

Fig. 3. Mean BER performance

Fig. 3 shows that for (N = 8, M = 4), the one-level partitioning scheme gives much better results than the combined TDMA/OCDMA scheme, whose minimum Euclidean distance is smaller. In particular, one-level partitioning avoids the phenomenon of BER floors, which is due to a minimum Euclidean distance equal to 0 and which can be encountered in the TDMA/OCDMA scheme (e.g. with N = 16, M = 4). Fig. 3 also shows the BER performance obtained with Ross and Taylor's construction and with the proposed one-level partitioning scheme, when the N parameter is fixed to 16 and the number of excess users is taken equal to the maximum allowed by each construction (that is M = 5 for Ross and Taylor's construction and M = 11 for the one-level partitioning scheme). We observe that, although the acceptable overload is much higher in the one-level partitioning case (69% compared to 31% of overload with Ross and Taylor's construction), the performance does not suffer from a significant degradation. Furthermore, the proposed scheme provides an asymptotic gain of 1.3 dB (which could be observed for larger values of E_b/N_0), whereas Ross and Taylor's construction does not give any asymptotic gain.

B. One-Level Partitioning With 50% Overload

We investigate a one-level partitioning scheme allowing a fixed overload of 50%, for all values of N. This scheme, which is inspired from the one-level partitioning scheme with $N = 8$, offers a limited decoding complexity [7].

Let us denote i_k, ($i_k \in \{0,1\}$, $k = 1, 2, \ldots, N+N/2$) the information bits. Bits ($i_1, i_2, \ldots, i_N$) are transmitted serially using BPSK mapping, resulting in ($a_1, a_2, \ldots, a_N$) (with $a_j \in \{-1,1\}$ for $j = 1, 2, ..., N$). The additional bits ($i_{N+1}, \ldots, i_{N+N/2}$) are coded with an RM code of Hamming distance 4. The resulting sequence ($x_1, x_2, \ldots, x_N$) is given by:

$$\left.\begin{aligned} x_{2k-1} &= i_{N+k} \\ x_{2k} &= i_{N+k} \oplus i_{N+N/2} \end{aligned}\right\} \quad k = 1, 2, ..., N/2-1$$
$$x_{N-1} = \bigoplus_{j=1}^{N/2-1} i_{N+j} \tag{9}$$
$$x_N = \bigoplus_{j=1}^{N/2-1} i_{N+j} \oplus i_{N+N/2}$$

After BPSK mapping of ($x_1, x_2, \ldots, x_N$) ($x_j \in \{0,1\}$ for $j = 1, 2, ..., N$), we obtain the symbols ($b_1, b_2, \ldots, b_N$) (with $b_j \in \{-1,1\}$ for $j = 1, 2, ..., N$). The transmitted sequence is given by:

$$s_k = a_k + \frac{1}{2} b_k \qquad k = 1, 2, ..., N. \tag{10}$$

The corresponding trellis is represented on Fig. 4.

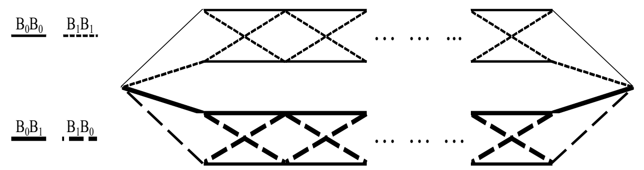

Fig. 4. State trellis for one-level partitioning with 50% channel overload

As previously, the transmitted energy per bit is equal to 5/6, inducing an asymptotic gain of 0.8 dB.

Fig. 5 presents the mean BER performance of this scheme for different values of N. The curve obtained with $N = 16$ is almost identical to the one given by $N = 8$. The results given by $N = 64$ and $N = 128$ tend to degrade, compared to the case of $N = 16$. Actually, this degradation is due to the error coefficient, given the fact that the asymptotic gain is the same in all cases. The asymptotic mean probability of error Pe_{mean} can be derived as:

$$Pe_{mean} = C_{mean} \cdot Q\left(\sqrt{\frac{12}{5N_0}}\right), \tag{11}$$

with $C_{mean} = \begin{cases} \dfrac{3989}{192} & \text{if } N = 8 \\ \dfrac{135N^2 - 368N + 324}{96N} & \text{if } N > 8 \end{cases}$

and $Q(x) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_x^{+\infty} e^{-\frac{u^2}{2}} du$.

Similarly, the asymptotic probabilities of error for set 1 and set 2 users can be written:

$$Pe_1 = C_1 \cdot Q\left(\sqrt{\frac{12}{5N_0}}\right) \text{ and } Pe_2 = C_2 \cdot Q\left(\sqrt{\frac{12}{5N_0}}\right), \tag{12}$$

with $\begin{cases} C_1 = \dfrac{205}{16} & C_2 = \dfrac{9073}{256} & \text{if } N = 8 \\ C_1 = \dfrac{27N - 22}{32} & C_2 = \dfrac{81N^2 - 324N + 1}{32N} & \text{if } N > 8 \end{cases}$.

Because of these coefficients, the asymptotic gain cannot be observed on Fig. 5: The plot of the asymptotic probabilities of error gives clear evidence that this gain is only visible for high SNR values.

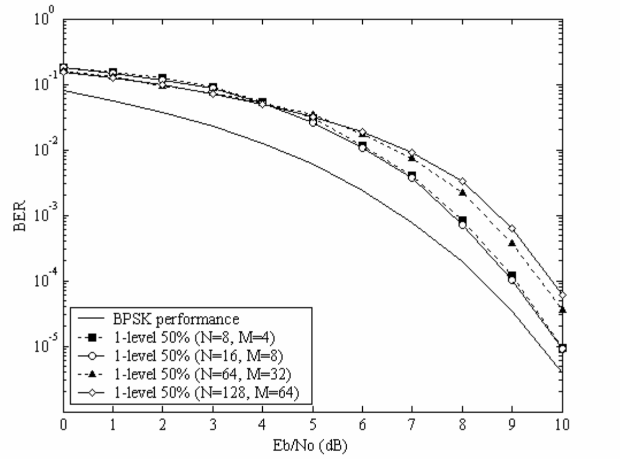

Fig. 5. Mean BER performance of 1-level partitioning scheme with 50% overload and N as a parameter

C. Two-Level Partitioning

A more powerful scheme consists in using two levels of signal superposition and a two-level partitioning of the resulting signal constellation. The signal levels transmitted are:

$$s_k = a_k + \lambda\, b_k + \mu\, c_k \qquad k = 1, 2, ..., N \tag{13}$$

where λ and μ denote the two pulses carrying the excess symbols. The a_k's are the primary bits after BPSK mapping, and the b_k's and c_k's are the excess (coded) bits after BPSK mapping.

In this case, the signal set is $A_0 = \{\pm 1 \pm \lambda \pm \mu\}$, which can be partitioned into two subsets $B_0 = \{\pm 1 \pm \lambda + \mu\}$ and $B_1 = \{\pm 1 \pm \lambda - \mu\}$. A further partitioning step gives

$C_0 = \{\pm 1 + \lambda + \mu\}$, $C_2 = \{\pm 1 - \lambda + \mu\}$, $C_1 = \{\pm 1 + \lambda - \mu\}$, and $C_3 = \{\pm 1 - \lambda - \mu\}$. Clearly, we have:

$$d_{\min}(C_i) = 2 \text{ for } i = 1, 2, 3, 4,$$
$$d_{\min}(B_0) = d_{\min}(B_1) = \min(2\lambda, 2(1-\lambda)),$$
$$\text{and } d_{\min}(A_0) = \min(2\mu, 2(\lambda-\mu), 2(1-\lambda-\mu)).$$

The distance $d_{\min}(B_i)$ is maximized for λ = 1/2, giving $d_{\min}(B_i) = 1$. Next, given this value of λ, $d_{\min}(A_0)$ is maximized for μ = 1/4, providing $d_{\min}(A_0) = 1/2$. The c_k's select a *B*-subset for each symbol, the b_k's select a *C*-subset from the *B*-subset selected by the c_k's, and finally the a_k's determine a point in the *C*-subset determined by the b_k's. The group of bits giving the b_k's is encoded using an RM code with a Hamming distance d_H of 4, in order to verify: $d_H d_{\min}^2(B_i) = d_{\min}^2(C_i) = 4$. As for the group of bits giving the c_k's, it is encoded using an RM code with a Hamming distance of 16, in order to have: $d_H d_{\min}^2(A_0) = d_{\min}^2(C_i) = 4$. This condition implies a block length of at least 16. The number of information bits per code word is thus

$$M_2 = \left(2^q - q - 1\right) + \left(2^q - \frac{5}{6}q - \frac{1}{6}q^3 - 1\right) = 2^{q+1} - \frac{11}{6}q - \frac{1}{6}q^3 - 2$$

For N = 16, the code related to the c_k's is in fact a repetition code of length 16. As for the transmitted energy per bit, it is equal to 21/28 for N = 16, which represents an asymptotic performance gain of 1.25 dB.

D. Comparison of the Different Proposed Schemes

The achievable overloads using one-level (*OF1*) and two-level partitioning (*OF2*) are indicated in Table I. Table I also shows the corresponding asymptotic gains G_1 and G_2, given by: $G_1 = \frac{5 \cdot N}{4 \cdot (N + M_1)}$ and $G_2 = \frac{21 \cdot N}{16 \cdot (N + M_2)}$.

TABLE I
ACHIEVABLE OVERLOADS AND ASYMPTOTIC GAINS FOR DIFFERENT CODE LENGTHS WITH ONE-LEVEL AND TWO-LEVEL PARTITIONING

N	q	M1	M2	OF1 (%)	OF2 (%)	G1 (dB)	G2 (dB)
8	3	4	—	50	—	0.8	—
16	4	11	12	69	75	1.3	1.25
32	5	26	32	81	100	1.6	1.8
64	6	57	79	89	123	1.8	2.3
128	7	120	184	94	144	1.9	2.7
256	8	247	410	96.5	160	2.0	3.0

We observe that the acceptable overload allowed by two-level partitioning becomes very significant with increasing *N*. However, this is accompanied by an increasing complexity of the trellis, causing an important decoding complexity.

IV. SUMMARY AND CONCLUSIONS

In this paper, we interpreted various channel overloading techniques as multilevel coding of an expanded signal constellation. The first technique examined, which uses a combination of TDMA and OCDMA, does not have any limitation in terms of overloading, but it can suffer from a decrease of the minimum Euclidean distance, causing some performance degradation. The second technique, described by Ross and Taylor, does not affect the minimum Euclidean distance, but the overload factor is limited to 33%. The interpretation of these techniques as multilevel codes enabled us to construct some new channel overloading schemes making use of one or two-level signal superposition. The proposed schemes offer the combined advantages of combined TDMA/OCDMA and of the construction technique of Ross and Taylor: They achieve high channel overloads while significantly improving asymptotic performance.

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