# On the Duality of Wireless Systems with Multiple Cooperating Transmitters and Wireless Systems with Correlated Antennas

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***Abstract*— We analyze the error performance of distributed space-time codes in a wireless system with multiple cooperating transmitters and a single receiver. Due to the distributed nature of the system, the transmitted signals are subject to different average path losses. As our analysis shows, this effect leads to reduced diversity gains. We explain this result on the basis of an interesting duality between systems with distributed transmitters and systems with densely-packed transmitters: For the case of Rayleigh fading, we prove that any system with distributed transmitters can be transformed into an equivalent co-located multiple-antenna system with correlated antennas. Specifically, the case of equal average path losses corresponds to an uncorrelated system. In order to quantify the performance loss resulting from distributed transmitters, we propose a simple performance measure between zero and one, which is equivalent to a certain correlation measure recently proposed for co-located multiple-antenna systems.**

***Index Terms*—Wireless communications, cooperative networks, distributed space-time codes, diversity, correlation, BER analysis.**

## I. INTRODUCTION

WIRELESS systems are well known to suffer from fading effects. However, system performance can be improved significantly by exploiting some sort of diversity. By means of multiple antennas in conjunction with space-time coding techniques [1]-[6], *spatial* diversity can be exploited, provided that the individual transmission links from the transmit antennas to the receive antenna(s) fade independently. This yields significant gains compared to a system with just a single antenna at either end of the wireless link.

Spatial diversity can also be exploited in *cooperative* wireless networks, e.g. [7]-[13]. In such networks, multiple (single-antenna) nodes share their antennas, for example by using a *distributed* space-time coding scheme. By this means, a *virtual* multiple-antenna system is established (see Fig. 1). The concept of cooperative wireless networks has recently gained considerable attention, because cooperating nodes build the basis of any ad-hoc network and promise benefits also for other types of networks, e.g., cellular networks [12]. Examples for cooperative wireless networks include simulcast networks [7] and relay-assisted networks [8]-[13].

In this paper, focus is on simulcast networks that consist of multiple base stations using a distributed space-time coding scheme[1]. Simulcast networks are normally employed for broadcasting or for paging applications. Conventionally, simulcasting means that the base stations simultaneously transmit the *same* signal on the same carrier frequency. Mobile users within the intersection of the coverage areas are thus provided with a comparably small probability of shadowing (macroscopic spatial diversity). However, conventional simulcasting does not yield microscopic spatial diversity, i.e., diversity that is due to independently fading transmission links [7]. In this paper, we assume that the base stations use a distributed space-time code to provide an additional microscopic diversity gain.

[1]The results presented in this paper are also relevant for other types of cooperative wireless networks, such as relay-assisted networks. Specifically, in a relay-assisted network, the link from the relays to the destination node can often be regarded as a type of simulcast network.

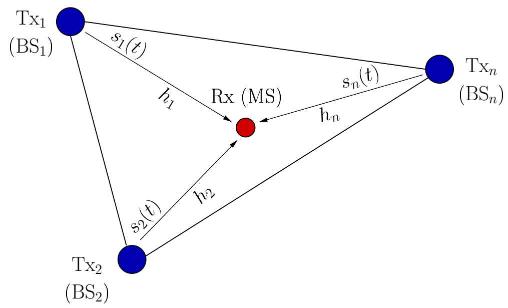


Fig. 1. Cooperating transmitters building a virtual multiple-antenna system.

In a system with distributed transmitters, specific differences arise compared to the co-located case. For example, in a distributed system the individual transmission links

- can have different fading statistics (shadowing, line-of-sight components, etc.)
- are normally characterized by different average path losses [13], due to different link lengths
- can exhibit non-negligible relative signal delays if the transmitters are spaced far apart [14],[15]
- can be subject to frequency offsets, due to independent local oscillators employed at the individual transmitters [16].

Since the above effects do only occur in distributed systems, they are usually not addressed in the standard literature on space-time codes. On the other hand, they are also neglected in most papers on cooperative wireless systems. In the present paper, we focus on the second item and its impact on the performance of distributed space-time codes. With focus on simulcast networks consisting of multiple cooperating base stations, relative signal delays can normally be neglected, because they are small for practical symbol durations and (urban) cell radii. However, the different average path losses are *not* negligible, even for small cells. Having broadcasting or paging applications in mind, link adaptation is not possible: In a broadcasting scenario, the base stations communicate with many mobile users at the same time. In a paging scenario, they communicate with a single mobile user with unknown position. Even if link adaptation is possible for a specific user, the problem of transmit power allocation is not trivial, as will be seen.

The outline of the paper is as follows: In Section II, the system model is introduced. In Section III, the error performance of a distributed space-time coding scheme is determined analytically, where a single receiver with fixed position is considered, and it is shown that unequal average path losses lead to reduced diversity gains. In Section IV, this result is explained on the basis of an interesting duality between systems with distributed transmitters and systems with densely-packed transmitters: For the case of Rayleigh fading it is proven that any system with distributed transmitters can be transformed into an equivalent co-located multiple-antenna system with *correlated* antennas. Specifically, the case of equal average path losses corresponds to an uncorrelated system. To the authors best knowledge, this duality has not yet been formulated in the literature. In order to quantify the performance loss resulting from distributed transmitters, a simple performance measure between zero and one is proposed, which is equivalent to a certain correlation measure recently proposed for co-located multiple-antenna systems [17].

## II. System Model and Basic Assumptions

Consider a simulcast network according to Fig. 1 consisting of $n$ base stations ($\mathrm{BS}_1$ to $\mathrm{BS}_n$) and a single mobile receiver (MS). For simplicity, we assume that the base stations and the mobile receiver are equipped with a single antenna. In order to provide a microscopic diversity gain, the base stations employ a distributed space-time block coding scheme. Due to the different link lengths, the transmitted signals $s_i(t)$ $(i = 1, ..., n)$ are subject to different average path losses.

Throughout this paper, the complex baseband representation is used. Assuming a frequency-flat block-fading channel model, we model the transmission link from the $i$-th base station to the mobile receiver by a single complex-valued channel coefficient $h_i := \alpha_i\, \mathrm{e}^{\mathrm{j}\varphi_i}$, which is constant over the duration of an entire data block. After each data block, the channel coefficient change randomly, where $h_i$ and $h_{i'}$ $(i \neq i')$ are statistically independent. Focus is on Rayleigh fading in the sequel, i.e., the channel coefficients $h_i$ are complex Gaussian random variables with zero mean and variance $\mathrm{E}\{|h_i|^2\} = \mathrm{E}\{\alpha_i^2\} =: \Omega_i$. The $k$-th received sample $y[k]$ is given by

$$y[k] \;=\; \sum_{i=1}^{n} h_i\, x_i[k] + n[k], \tag{1}$$

where $k$ denotes the discrete time index, $x_i[k] \in \mathbb{C}$ the transmitted symbol of base station $\mathrm{BS}_i$, and $n[k]$ a sample of a complex additive white Gaussian noise (AWGN) process with zero mean and variance $\sigma_n^2$, i.e., $n[k] \sim \mathcal{CN}(0, \sigma_n^2)$. The noise samples are assumed to be statistically independent of the data symbols and the channel coefficients. Note that the transmitted symbols $x_i[k]$ are coupled by the distributed space-time code (the underlying information symbols are denoted as $a[k]$ in the sequel). For the purpose of analysis, the normalization

$$\sum_{i=1}^{n} \Omega_i \;\stackrel{!}{=}\; n \tag{2}$$

is done. The individual base stations are assumed to use the same average transmit power $P_{\mathrm{Tx},i} := \mathrm{E}\{|x_i[k]|^2\} = P/n$. To provide a fair comparison with a single-antenna system, the overall transmit power $P$ is fixed. The received signal-to-noise ratio (SNR) for the $i$-th transmission link is given by

$$\gamma_i \;:=\; \frac{P}{n}\,\frac{\alpha_i^2}{\sigma_n^2}, \qquad \bar{\gamma}_i \;:=\; \mathrm{E}\{\gamma_i\} \;=\; \frac{P}{n}\,\frac{\Omega_i}{\sigma_n^2} \tag{3}$$

(instantaneous and average SNR, respectively). With the normalization (2), the overall average received SNR is $\bar{\gamma}_{\mathrm{ov}} := \sum_{i=1}^{n} \bar{\gamma}_i = P/\sigma_n^2$. The average received power for the $i$-th transmission link can be written as $P_{\mathrm{Rx},i} = a_i\, P_{\mathrm{Tx},i}$, where $a_i$ represents an unnormalized version of $\Omega_i$. The parameter $a_i$ is in essence a function of the distance $d_i$ between the base station $\mathrm{BS}_i$ and the mobile receiver ($a_i \propto 1/d_i^{\rho}$, where $\rho = 2...4$ is the path-loss exponent). Due to the normalization (2), $1/\Omega_i$ represents a *relative* average path loss, because it depends not only on the distance $d_i$, but also on the other distances $d_j$, $j \neq i$.

In the sequel, we set $\bar{\gamma}_{\mathrm{ov}} =: E_{\mathrm{s}}/N_0$, where $E_{\mathrm{s}}$ denotes the average symbol energy and $N_0$ the single-sided noise power density. For the average energy per info bit we have $E_{\mathrm{b}} = E_{\mathrm{s}}/R_{\mathrm{t}}$, where $R_{\mathrm{t}}$ denotes the temporal rate of the distributed space-time code. Within the scope of this paper, the base stations are assumed to employ a distributed orthogonal space-time block code[2] (OSTBC) [2],[3]. OSTBCs yield full diversity in terms of the number $n$ of transmit antennas. In the flat-fading case, OSTBCs enable maximum-likelihood detection at the receiver by means of simple linear processing. However, a drawback of these schemes is that a temporal rate $R_{\mathrm{t}} = 1$ (full rate) is only accomplished for certain numbers of transmit antennas. Given a two-dimensional modulation scheme such as $M$-ary phase-shift keying (PSK), full-rate transmission is, in fact, only accomplished by Alamouti‘s OSTBC [2] for $n = 2$ transmitters. In the case of $n = 3$ and $n = 4$, for example, the maximum possible rate is $R_{\mathrm{t}} = 3/4$ [3].

## III. Error Performance of a Distributed OSTBC

In the following, a multiple-antenna system with $n_{\mathrm{T}}$ transmit and $n_{\mathrm{R}}$ receive antennas is denoted an ($n_{\mathrm{T}}$ x $n_{\mathrm{R}}$)-system. An ($n$x1)-system, which employs an OSTBC at the transmitter side and the corresponding linear detector at the receiver side, is equivalent to a (1x$n$)-system with identical fading statistics[3] and maximum-ratio combining (MRC) [2],[3] (apart from a possible rate loss due to the temporal rate of the OSTBC). In the equivalent (1x$n$)-system, the $k$-th received sample of transmission link $i$ is given by

$$y_i[k] \;=\; h_i\, a[k] + n_i[k]\,, \tag{4}$$

where $n_i[k]$ and $n_{i'}[k]$ $(i \neq i')$ are statistically independent ($a[k]$ is the $k$-th information symbol underlying the OSTBC).

In the sequel, we focus on binary transmission. If all links are characterized by Rayleigh fading, there are closed-form expressions for the average bit error probability $\bar{P}_{\mathrm{b}}$ [18]. For equal average SNRs $\bar{\gamma}_i = \bar{\gamma}_{\mathrm{ov}}/n$ one obtains:

$$\bar{P}_{\mathrm{b}} \;=\; \frac{1}{2^n}\,(1-\mu)^n \sum_{\nu=0}^{n-1} \binom{n-1+\nu}{\nu} \frac{1}{2^{\nu}}\,(1+\mu)^{\nu}\,, \tag{5}$$

where $\mu := \sqrt{\bar{\gamma}_{\mathrm{ov}}/(n+\bar{\gamma}_{\mathrm{ov}})}$. In the case of unequal SNRs $\bar{\gamma}_i$ one obtains

$$\bar{P}_{\mathrm{b}} \;=\; \frac{1}{2} \sum_{i=1}^{n} \left( \prod_{\substack{i'=1 \\ \bar{\gamma}_{i'} \neq \bar{\gamma}_i}}^{n} \frac{\bar{\gamma}_i}{\bar{\gamma}_i - \bar{\gamma}_{i'}} \right) \left( 1 - \sqrt{\frac{\bar{\gamma}_i}{1+\bar{\gamma}_i}} \right). \tag{6}$$

The bit error rate (BER) performance of a distributed OSTBC system with $n = 4$ transmitters and a single receive antenna is illustrated in Fig. 2, for the example of Rayleigh fading and various settings for the average SNRs $\bar{\gamma}_i = (P\,\Omega_i)/(n\,\sigma_{\mathrm{n}}^2)$. The exact analytical results for the average bit error probability according to (5) and (6) are plotted versus $E_{\mathrm{b}}/N_0$ in dB. For the OSTBC, a temporal rate of $R_{\mathrm{t}} = 3/4$ was assumed, i.e., $10\,\log_{10}(E_{\mathrm{b}}/N_0)\,\mathrm{dB} = 10\,\log_{10}(E_{\mathrm{s}}/N_0)\,\mathrm{dB} + 1.25\,\mathrm{dB}$. Consider first the case of equal average SNRs ($\Omega_i = 1$ for all $i$), which corresponds to a co-located system. The performance results in Fig. 2 show that significant gains over a (1x1)-system ($n = 1$) are obtained, especially for large values of $E_{\mathrm{b}}/N_0$. A high-SNR approximation of (5) yields [18]

$$\bar{P}_{\mathrm{b}} \;\approx\; \left(\frac{n}{4\,\bar{\gamma}_{\mathrm{ov}}}\right)^{n} \binom{2n-1}{n}\,. \tag{7}$$

[2]Although focus is on OSTBCs and frequency-flat fading, the results presented here are also relevant for frequency-selective channel models. For example, they can directly be applied to space-time coded orthogonal-frequency-division-multiplexing (OFDM) systems. They give also insight into the behavior of other distributed space-time coding schemes, such as (generalized) delay diversity [4],[5] or the time-reversal STBC scheme [6], which are both suited for frequency-selective fading.

[3]This means that the individual links of both systems are characterized by the same PDFs $p_{\Gamma_i}(\gamma_i)$ and specifically by the same average SNRs $\bar{\gamma}_i$.

Correspondingly, for large SNR values the BER decreases with $1/(E_\mathrm{b}/N_0)^n$. In the log-log plot shown in Fig. 2, this corresponds to a straight line with slope $-n$.

If the individual transmission links are characterized by identical fading conditions (e.g., by Rayleigh fading), unbalanced average SNRs $\bar{\gamma}_i$ lead to reduced performance gains[4], as can be seen in Fig. 2. A high-SNR approximation of (6) yields [18]:

$$\bar{P}_\mathrm{b} \approx \left(\frac{n}{4\,\bar{\gamma}_\mathrm{ov}}\right)^n \binom{2n-1}{n} \prod_{i=1}^{n} \frac{1}{\Omega_i} \tag{8}$$

($\Omega_i \neq 0$ for all $i$). Correspondingly, the asymptotic slope of the BER curve is still given by $-n$, which can also be observed in Fig. 2. However, the additional product term causes (asymptotically) an up-shift of the BER curves, which leads to a performance degradation[5]. As can be seen in Fig. 2, in the case $\Omega_{i_0} = n$ and $\Omega_i = 0$ for all $i \neq i_0$, the BER performance tends to that of the (1x1)-system, i.e., the asymptotic slope of the BER curve is $-1$. The shift of $1.25$ dB with respect to the BER curve of the (1x1)-system is due the temporal rate of $R_\mathrm{t} = 3/4$ that has been assumed for the OSTBC.

The behavior of a system with multiple distributed transmitters and unequal average SNRs $\bar{\gamma}_i$ resembles that of a co-located system with spatially *correlated* antennas. In the next section we will show that, in fact, for any system with multiple distributed transmitters, an equivalent co-located system with correlated antennas can be found. This insight reveals an astonishing duality between systems with distributed transmitters and systems with densely-packed transmit antennas.

## IV. Duality between Distributed Systems and Co-located Systems with Correlated Antennas

Consider an OSTBC system with $n$ distributed transmitters, a single receiver, and individual links characterized by Rayleigh fading with unequal average SNRs $\bar{\gamma}_i$ according to (2) and (3). In the equivalent (1x$n$)-MRC system (cf. (4)), the received samples $y_1[k], ..., y_n[k]$ before MRC can be written as

$$\mathbf{y}[k] = \mathbf{h}\,a[k] + \mathbf{n}[k]\,, \tag{9}$$

where $\mathbf{y}[k] := [y_1[k], ..., y_n[k]]^\mathrm{T}$, $\mathbf{h} := [h_1, ..., h_n]^\mathrm{T}$, and $\mathbf{n}[k] := [n_1[k], ..., n_n[k]]^\mathrm{T}$. The channel coefficients are assumed to be uncorrelated, i.e. $\mathrm{E}\{\mathbf{h}\mathbf{h}^\mathrm{H}\} = \mathrm{diag}([\Omega_1, ..., \Omega_n]) =: \mathbf{\Omega}$. Moreover, the noise is spatially white, i.e. $\mathrm{E}\{\mathbf{n}[k]\mathbf{n}^\mathrm{H}[k]\} = \sigma_\mathrm{n}^2\,\mathbf{I}_n$ for all $k$, and statistically independent of the channel vector $\mathbf{h}$ and the transmitted information symbols $a[k]$.

**Proposition 1:** The above (1x$n$)-system with unequal average SNRs $\bar{\gamma}_i$ can be transformed into an equivalent (1x$n$)-system with correlated receive antennas and equal average SNRs $\bar{\gamma}_i'$.

*Proof:* A unitary transform $\mathbf{y}'[k] := \mathbf{U}\,\mathbf{y}[k]$, where $\mathbf{U}$ denotes an arbitrary unitary ($n$x$n$)-matrix, does not change the statistical properties of the (1x$n$)-MRC system [19]. Specifically, the overall average SNR $\bar{\gamma}_\mathrm{ov} = \sum_{i=1}^{n} \bar{\gamma}_i$ (i.e., the average SNR at the combiner output) remains unchanged. Correspondingly, the transformation yields a (1x$n$)-system

$$\mathbf{y}'[k] = \mathbf{U}\mathbf{h}\,a[k] + \mathbf{U}\mathbf{n}[k] =: \mathbf{h}'\,a[k] + \mathbf{n}'[k] \tag{10}$$

[4] If the transmission links experience different fading conditions, unbalanced average SNRs can be beneficial: If those links with good fading conditions (e.g., Nakagami-$m$ fading with $m > 1$) also have large average SNRs, the BER performance will be better than in the case of equal $\bar{\gamma}_i$.

[5] As will seen in Section IV, the product term is always greater or equal to 1.

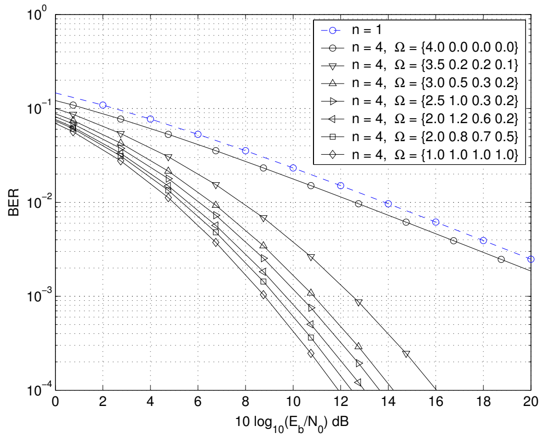


Fig. 2. BER performance of an OSTBC system with $n = 4$ distributed transmitters and a single receiver: Binary transmission, all links characterized by Rayleigh fading, unequal average SNRs $\bar{\gamma}_i = (P\Omega_i)/(n\sigma_\mathrm{n}^2)$.

that is equivalent to (9). The new noise vector $\mathbf{n}'[k]$ is still spatially white. However, the covariance matrix of the channel coefficients is changed. The new channel vector $\mathbf{h}'$ is in general characterized by a non-diagonal covariance matrix:

$$\mathrm{E}\{\mathbf{h}'\mathbf{h}'^\mathrm{H}\} = \mathbf{U}\,\mathrm{E}\{\mathbf{h}\mathbf{h}^\mathrm{H}\}\,\mathbf{U}^\mathrm{H} = \mathbf{U}\mathbf{\Omega}\mathbf{U}^\mathrm{H}\,. \tag{11}$$

By definition, the resulting covariance matrix $\mathbf{U}\mathbf{\Omega}\mathbf{U}^\mathrm{H}$ is a Hermitian matrix. In order to prove the proposition, a unitary matrix $\mathbf{U}$ must be found such that $\mathbf{U}\mathbf{\Omega}\mathbf{U}^\mathrm{H}$ is proportional or equal to a correlation matrix $\mathbf{R}$ with diagonal entries $r_{ii} = 1$ for all $i = 1, ..., n$ and non-diagonal entries $r_{ij}$ with $|r_{ij}| \leq 1$ for all $i \neq j$. Then, equal average SNRs $\bar{\gamma}_i' = P\,\mathrm{E}\{|h_i'|^2\}/(n\,\sigma_\mathrm{n}^2) = P/(n\,\sigma_\mathrm{n}^2)$ result for all $i$. Choosing $\mathbf{U}$ such that $|u_{ij}| = 1/\sqrt{n}$ for all $i, j = 1, ..., n$, one finds that

$$r_{ii} = \sum_{j=1}^{n} |u_{ij}|^2\,\Omega_j = \frac{1}{n}\sum_{j=1}^{n} \Omega_j = 1 \tag{12}$$

for all $i$. Moreover, since $\mathbf{U}$ is a unitary matrix, $|r_{ij}| \leq 1$ holds for all $i, j$. Thus, $\mathbf{U}\mathbf{\Omega}\mathbf{U}^\mathrm{H}$ is equal to a correlation matrix $\mathbf{R}$. In general one finds that $|r_{ij}| > 0$ ($i \neq j$), i.e., the receive antennas in the transformed (1x$n$)-system are correlated. Suitable unitary matrices $\mathbf{U}$ with $|u_{ij}| = 1/\sqrt{n}$ are, for example, the Fourier matrix $\mathcal{F}_n$ with entries $u_{ij} = \mathrm{e}^{\mathrm{j}2\pi(i-1)(j-1)/n}/\sqrt{n}$ (exists for any number $n$) or the Hadamard matrix $\mathcal{H}_n$ with entries $u_{ij} = \pm 1/\sqrt{n}$ (exists only for $n = 2^\nu$, where $\nu = 1, 2, ...$). □

Since the resulting (1x$n$)-MRC system with correlated receive antennas and equal average SNRs $\bar{\gamma}_i'$ is in turn equivalent to an ($n$x1)-OSTBC system (including an appropriate linear detector at the receiver) with correlated transmit antennas (same correlation matrix $\mathbf{R}$), and identical fading statistics (same average SNRs $\bar{\gamma}_i'$), we have the following theorem:

**Theorem 1:** For any OSTBC system with $n$ distributed transmitters, a single receiver, and individual links characterized by Rayleigh fading with unequal average SNRs $\bar{\gamma}_i$, we can find an equivalent co-located OSTBC system with correlated transmit antennas and equal average SNRs $\bar{\gamma}_i'$.

*Proof:* Follows directly from Proposition 1, via the equivalent (1x$n$)-MRC systems. □

*Remark:* It is straightforward to generalize the above theorem to more than one receive antenna. Moreover, the theorem can also be formulated for Rician fading. The equivalent co-located

OSTBC system will, however, be characterized by different Rice factors $K_i$, compared to the original system [19]. •

**Corollary:** For an OSTBC system with $n$ distributed transmitters, a single receiver, and individual links characterized by Rayleigh fading with average SNRs $\{\bar{\gamma}_{\text{ov}}, 0, ..., 0\}$, we find an equivalent co-located OSTBC system that is *fully* correlated, i.e. $|r_{ij}| = 1$ for all $i, j = 1, ..., n$.

*Proof:* Again, the equivalent (1x$n$)-MRC systems are considered. The weight matrix $\mathbf{\Omega}$ of the (1x$n$)-system with unequal average SNRs is in this case given by

$$\mathbf{\Omega} = \text{diag}([0, ..., 0, \Omega_{i_0}, 0, ..., 0]) \,,$$

where $\Omega_{i_0} = n$. The corresponding correlation matrix of the co-located system results as

$$\mathbf{R} \;=\; n \begin{bmatrix} u_{1i_0} u_{1i_0}^* & \cdots & u_{1i_0} u_{ni_0}^* \\ \vdots & \ddots & \vdots \\ u_{ni_0} u_{1i_0}^* & \cdots & u_{ni_0} u_{ni_0}^* \end{bmatrix} . \qquad (13)$$

Since $|u_{ij}| = 1/\sqrt{n}$ for all $i, j$, $|r_{ij}|$ is always equal to one. □

*Remark:* The corollary states that the set $\{\bar{\gamma}_{\text{ov}}, 0, ..., 0\}$ constitutes the worst case among all possible sets of unequal average SNRs (provided that all transmitters use the same average transmit power), i.e., it leads to the worst possible BER performance (cf. Fig. 2). Obviously, the set $\{\bar{\gamma}_{\text{ov}}/n, ..., \bar{\gamma}_{\text{ov}}/n\}$ of equal average SNRs constitutes the best case, because it corresponds to the uncorrelated case ($\mathbf{\Omega} = \mathbf{I}_n = \mathbf{R}$). This can also be seen when regarding (7) and (8). The BER in the case of unequal average SNRs can, asymptotically ($\bar{\gamma}_{\text{ov}} \to \infty$), never be smaller than in the case of equal average SNRs, because the product term $\prod_{i=1}^{n} 1/\Omega_i$ in (8) is always greater or equal to one: Rewriting the product term as the inverse of the determinant of the weight matrix $\mathbf{\Omega}$ and using the fact that $\mathbf{U}\mathbf{\Omega}\mathbf{U}^{\text{H}} = \mathbf{R}$, one finds that

$$\prod_{i=1}^{n} \frac{1}{\Omega_i} \;=\; \frac{1}{\det(\mathbf{\Omega})} \;=\; \frac{1}{\det(\mathbf{U}^{\text{H}}\mathbf{R}\mathbf{U})} \;=\; \frac{1}{\det(\mathbf{R})} \;\geq\; 1\,, \quad (14)$$

where we have used the fact that the determinant is invariant under a unitary transform, and that the determinant of any correlation matrix $\mathbf{R}$ (i.e. $r_{ii} = 1$ for all $i$, $|r_{ij}| \leq 1$ for all $i, j$, and $\mathbf{R} = \mathbf{R}^{\text{H}}$) is always smaller or equal to one. Although the above result has been derived for large average SNRs, it also applies for practicable SNR values, cf. Fig. 2. •

*Discussion:* The unitary transform $\mathbf{\Omega} \mapsto \mathbf{R}$ is not unique. For example, in the case $n = 4$ we can either use the Hadamard matrix $\mathcal{H}_4$ or the Fourier matrix $\mathcal{F}_4$ for the transformation, which leads to equivalent co-located OSTBC systems with different correlation matrices. The weights $\Omega_i$ can be interpreted as the eigenvalues of the resulting correlation matrix $\mathbf{R}$, and the column vectors of $\mathbf{U}$ as the corresponding eigenvectors (by definition, $\mathbf{R}\mathbf{U} = \mathbf{U}\mathbf{\Omega}$). The transform $\mathbf{\Omega} \mapsto \mathbf{R}$ is thus somewhat dual to the Karhunen-Loève transform (KLT), which is often used to analyze correlated transmission systems (see e.g. [20]). However, the entries of the unitary transformation matrix used in the KLT are not subject to special constraints, as opposed to the entries of the matrix $\mathbf{U}$. •

*Practical relevance:* Theorem 1 states an interesting duality between systems with distributed transmitters and systems with densely-packed transmit antennas. This duality is useful, in order to analyze the performance of virtual antenna arrays (VAAs) [9]. The concept of VAAs was first proposed for the downlink of cellular networks, in order to provide spatial diversity at the receiver side: It is argued in [9] that due to space limitation at the mobile receiver, multiple co-located antennas will cause strong spatial correlations and are therefore often not practicable. In order to circumvent the problem of correlation, it is proposed to establish VAAs by means of multiple adjacent single-antenna receivers that mutually relay their received signals. However, according to Proposition 1 this use of VAAs merely trades one form of correlation for another: Depending on the path-loss exponent $\rho$ and the distances between the base station and the individual cooperating receivers, unequal average link SNRs can arise that correspond to significant correlation values $r_{ij}$.

The duality stated by Theorem 1 is also useful, in order to reuse existing transmitter and receiver strategies for spatially correlated multiple-antenna systems. For example, in distributed systems where link adaptation is possible, existing transmit power allocation strategies for correlated systems [21] can be reused without loss of optimality[6]. •

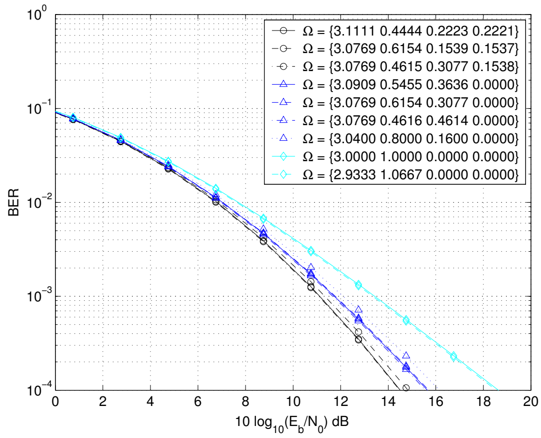


Fig. 3. BER performance of various distributed OSTBC systems with $n = 4$ transmitters, a single receiver, and a performance measure of $\Delta(\mathbf{\Omega}) \approx 0.7$: All links characterized by Rayleigh fading with unequal average SNRs $\bar{\gamma}_i$, where $\bar{\gamma}_i = (P\Omega_i)/(n\sigma_{\text{n}}^2)$.

The weight matrix $\mathbf{\Omega}$ does not directly reflect the BER performance of the corresponding distributed OSTBC system. Specifically, it is not immediately clear, how two different distributed OSTBC systems with weight matrices $\mathbf{\Omega}_1$ and $\mathbf{\Omega}_2$ compare. For this purpose, a simple performance measure $\Delta(\mathbf{\Omega})$ between zero and one is introduced in the sequel, which allows for a classification of distributed OSTBC systems. As will be seen, different systems with the same value $\Delta(\mathbf{\Omega})$ exhibit a very similar BER performance. Let

$$\Delta(\mathbf{\Omega}) \;:=\; \frac{1}{\sqrt{n(n-1)}} \sqrt{\sum_{i=1}^{n} (\Omega_i - 1)^2}\,, \qquad (15)$$

where $\mathbf{\Omega} = \text{diag}([\Omega_1, ..., \Omega_n])$. The measure $\Delta(\mathbf{\Omega})$ quantifies the degree of SNR unbalance. For $\mathbf{\Omega} = \mathbf{I}_n$, which represents the best case with regard to BER performance, one obtains $\Delta(\mathbf{\Omega}) = 0$. For $\mathbf{\Omega} = \text{diag}([0, ..., 0, n, 0, ..., 0])$, which represents the worst case, one obtains $\Delta(\mathbf{\Omega}) = 1$.

Fig. 3 shows the BER performance of various distributed OSTBC systems with $n = 4$ transmitters, a single receiver, and different weight matrices $\mathbf{\Omega}$. All systems are characterized by a performance measure of approximately $\Delta(\mathbf{\Omega}) = 0.7$. As can be

[6] Given a fixed overall transmit power $P$ it is, for example, clearly suboptimal to simply equalize the unequal average SNRs by using average transmit powers $\propto 1/\bar{\gamma}_i$ at the individual transmitters.

seen, for low to moderate SNR values all systems exhibit a very similar BER performance. In the high SNR regime, the slope of the BER curves is determined by the number of weights $\Omega_i$ that are unequal to zero. Correspondingly, when classifying the BER performance of distributed OSTBC systems for large SNR values, one should not only consider the resulting performance measure $\Delta(\mathbf{\Omega})$. As can be seen in Fig. 3, those systems with the same number of weights $\Omega_i \neq 0$ exhibit a very similar BER performance, also for large SNR values.

Recently, a similar performance measure has been introduced for spatially correlated multiple-antenna systems [17]. Given an $(n\text{x}n)$ correlation matrix $\mathbf{R}$, the corresponding performance measure is calculated according to

$$\Psi(\mathbf{R}) \;=\; \frac{1}{\sqrt{n(n-1)}} \sqrt{\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} |r_{ij}|^2}\,. \qquad (16)$$

Similar to the measure $\Delta(\mathbf{\Omega})$, the value of $\Psi(\mathbf{R})$ is between zero and one: $\Psi(\mathbf{R}) = 0$ results for an uncorrelated system (i.e., $\mathbf{R} = \mathbf{I}_n$) and $\Psi(\mathbf{R}) = 1$ for a fully correlated system (i.e., $|r_{ij}| = 1$ for all $i, j$). An interesting question is: Given a distributed OSTBC system with performance measure $\Delta(\mathbf{\Omega})$, what is the performance measure $\Psi(\mathbf{R})$ of the corresponding equivalent co-located OSTBC system? The following theorem states that $\Delta(\mathbf{\Omega})$ and $\Psi(\mathbf{R})$ are, in fact, identical.

**Theorem 2:** Let $\mathbf{\Omega}$ be the weight matrix of an OSTBC system with $n$ distributed transmitters, a single receiver, and individual links characterized by Rayleigh fading with unequal average SNRs $\bar{\gamma}_i$. Furthermore, let $\mathbf{R}$ be the correlation matrix of an equivalent co-located OSTBC system with correlated transmit antennas. Then the two performance measures $\Delta(\mathbf{\Omega})$ and $\Psi(\mathbf{R})$ according to (15) and (16) are identical.

*Proof:* The performance measure $\Delta(\mathbf{\Omega})$ can be written as

$$\Delta(\mathbf{\Omega}) \;=\; \frac{1}{\sqrt{n(n-1)}} \sqrt{\sum_{i=1}^{n} \xi_i^2} \;=\; \frac{||\Xi||_{\mathrm{F}}}{\sqrt{n(n-1)}}\,, \qquad (17)$$

where $\xi_i := (\Omega_i - 1)$, $\;\Xi := \mathrm{diag}([\xi_1, ..., \xi_n])$ and $||.||_{\mathrm{F}}$ denotes the Frobenius norm. On the other hand, the performance measure $\Psi(\mathbf{R})$ can be written as

$$\Psi(\mathbf{R}) \;=\; \frac{||\mathbf{R} - \mathbf{I}_n||_{\mathrm{F}}}{\sqrt{n(n-1)}}\,. \qquad (18)$$

Moreover, $\mathbf{R} = \mathbf{U}\mathbf{\Omega}\mathbf{U}^{\mathrm{H}} = \mathbf{U}\,(\mathbf{I}_n + \Xi)\,\mathbf{U}^{\mathrm{H}} = \mathbf{I}_n + \mathbf{U}\,\Xi\,\mathbf{U}^{\mathrm{H}}$ holds. Correspondingly,

$$||\mathbf{R} - \mathbf{I}_n||_{\mathrm{F}} \;=\; ||\mathbf{U}\,\Xi\,\mathbf{U}^{\mathrm{H}}||_{\mathrm{F}} \;=\; ||\Xi||_{\mathrm{F}}\,. \qquad (19)$$

In the last step, we have used that the Frobenius norm is invariant under a unitary transform. □

## V. Conclusions

In this paper, we have analyzed the error performance of distributed space-time codes in a mobile broadcasting system with multiple cooperating base stations. Since the distances between the individual transmitters and the mobile receiver are typically different, unequal average signal-to-noise ratios result for the individual links. We have shown that this effect can lead to significantly reduced diversity gains. For the case of Rayleigh fading and orthogonal space-time block codes it has been proven that systems with distributed transmitters and unbalanced average signal-to-noise ratios are, in fact, equivalent to co-located systems with correlated transmit antennas. A simple performance measure has also been proposed, in order to classify the performance of space-time coded systems with distributed transmitters. An interesting consequence of our findings is that transmitter techniques (e.g. power allocation schemes) originally developed for spatially correlated systems can be reused for systems with distributed transmitters. Although focus has been on Rayleigh fading, the main results of the paper apply also for other types of fading, such as Rician fading or Nakagami-$m$ fading.

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