# Time-Varying Autoregressive Process for Ultra-Wideband Indoor Channel Model

A. Taparugssanagorn[1], L. Hentilä[2], and S. Karhu[3]

*Abstract*—**In this paper, a time-varying autoregressive (AR) Ultra-Wideband indoor radio channel model based on frequency sweeping measurements in the 3.1-8.0 GHz is described. Firstly, the frequency domain measurements were carried out with a vector network analyzer at the 3.1-8.0 GHz band using vertically polarized omni-directional antennas. As a result, a complex transfer function of the radio channel was obtained. Subsequently a time-variant AR model is proposed in order to model the radio channel. This method is useful for a channel model that cannot be assumed to be stationary. The basis function, having the invariant basis coefficients approach, is used for estimating the time-variant AR parameters. Moreover, a path loss model is addressed in this work. In the end, the accuracy of the methods are analyzed by comparing the channel characteristics and the cumulative distribution functions of the RMS delay spread with that of the measurement results, the modified version of the IEEE 802.15.3a channel model and the conventional AR model. The results show that a fifth order time-varying autoregressive model is sufficient to represent the statistical characteristics of the channel where each pole identifies the arrival of a cluster of paths and is more accurate than the conventional AR model.**

*Index Terms*—**Ultra-wideband radio channel, non-stationary, basis time-varying functions, transfer function, RMS delay spread.**

## I. Introduction

The potential of Ultra-Wideband (UWB) technology has been well-known around for several years due to its great benefits, for instance, very high multipath resolution leading to huge frequency diversity, which makes the UWB signal resistant to severe multipath fading problems. Furthermore, the low probability of interception (LPI) and detection (LPD) features provided by Time Modulated Ultra-Wideband (TM-UWB) make it suitable for secure and military use [1].

To design a UWB high data rate communication system operating inside buildings, the indoor UWB channel must be investigated. The purpose of this paper is to introduce a frequency domain channel model using a time-varying autoregressive (AR) model based on the frequency domain channel measurement and to provide a path loss model, which is important for determining the radio coverage. The channel was measured in defined positions at the University of Oulu, Finland, and the data was stored for post-processing analysis. The post-processing consists of several steps leading to the final channel model.

In Section II, the frequency domain measurement system is described. The data post-processing composed of the conventional AR model and the time-varying AR model is presented in Section III. In Section IV, the path loss model is presented. The comparison of the simulation results is shown in Section V. The conclusions of this study are summarized in Section VI.

## II. Measurements: Setup And Environment

### A. Measurement Setup

The UWB radio channel measurement system used in this paper consists of an Agilent 8720ES vector network analyzer (VNA), an Agilent 83017A wideband amplifier, a wideband conical monopole antenna pair, 8 and 15 m coaxial cables, a stepped track for antenna position movement and a control computer with LabView™ 6i software. The measurement setup and analysis diagrams are shown in Figure 1 [2].

The network analyzer was operated in a transfer function measurement mode, where port 1 and port 2 were the transmitting and the receiving port respectively. An external amplifier was connected to port 1 to increase the transmission power level. The antennas are vertically polarized conical monopole antennas having typically omni-directional radiation patterns and constant phase centre. The sweep time of the VNA depends on the frequency points within the sweep band, being automatically adjusted by the VNA. Table I lists the main parameters of the measurements.

The frequency band used in the measurements is from 3.1 GHz to 8.0 GHz according to the FCC spectrum mask being from 3.1 GHz to 10.6 GHz for UWB transmission. The maximum number of frequency points per sweep is 1601, which can be used to calculate the maximum detectable delay of the channel as

$$\tau_{max} = \frac{(M-1)}{B}, \qquad (1)$$

where $M$ is number of points in the swept frequency band.

The floor plan of the room where the channel measurements were carried out is shown in Figure 2.

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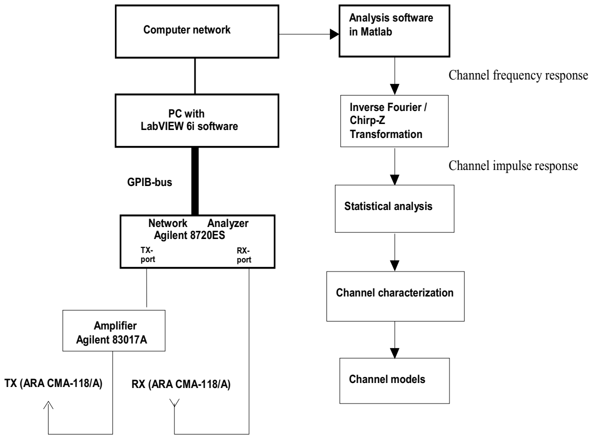


Figure 1. Measurement setup and analysis diagrams.

TABLE I
MEASUREMENT PARAMETERS SETUP

| Parameter | Value |
|---|---|
| Frequency band | 3.1 to 8.0 GHz |
| Bandwidth | 4.9 GHz |
| Number of points over the band | 1601 |
| Maximum detectable delay | 326.5 ns |
| Sweep time | 800 ms |
| Dynamic range | 90 dB |
| Transmission power | +5 dBm |
| Amplifier gain over the band (min/mean/max) | 35.3 / 35.9 / 36.9 dB |
| Antenna gain (typical/max @ 25° elevation) | 0 / 4.2 dBi |
| Antenna polarisation | Vertical |

## III. Data Post-Processing

Before any statistical channel modelling work can be carried out, the first step is to perform appropriate data analysis technique described as follows:

- all measurement data are calibrated with the calibration data measured in anechoic chamber to ensure only the propagation channel transfer function without the effect of amplifiers, cables and antennas, will be used for further analysis;
- the frequency domain models explained in the next section are processed. The regenerated channel transfer functions are transformed into the channel impulse responses through inverse Fourier transform;
- the channel impulse responses are then normalized such that the total power in each power delay profile (PDP) is equal to one.

### *A. Conventional Autoregressive (AR) Process*

The $N$ measured samples of the frequency response $H(f_n, x)$ at location $x$ can be interpreted as a random process. The autocorrelation function of this process calculated by

$$R(k,x) = \frac{1}{N}\sum_{i=1}^{N-k} H^*(f_i,x)H(f_{i-k},x),\ 0 \le k \le N-1, \qquad (2)$$

is used for the autoregressive modelling to solve the model parameter.

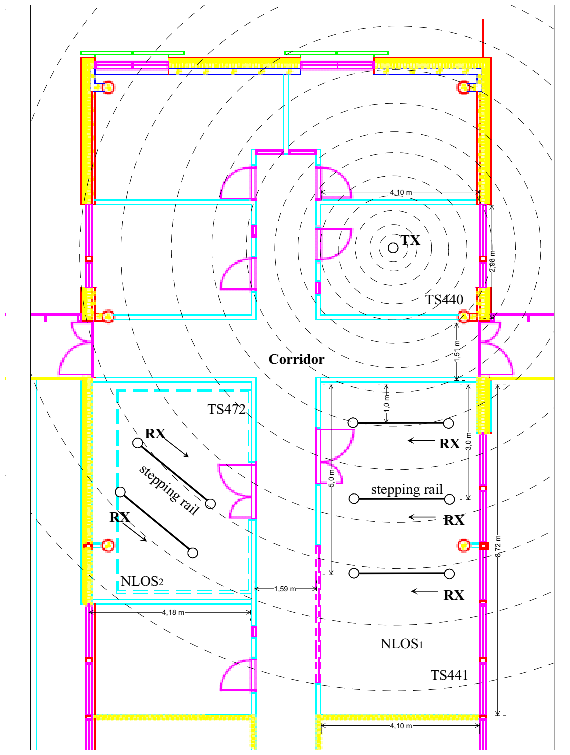


Figure2. Floor plan of the office building where the channel measurements were carried out.

The frequency response can be interpreted as the output of an autoregressive process. Autoregressive modelling of time series data used for spectral estimation and the techniques for determination of the coefficients of the AR process are well known in the literature [3, 4]. With the assumption of an AR process and non-stationary channel, the frequency response at each location is a realization of an autoregressive process of order $p$ given by the equation

$$H(f_n,x) + \sum_{i=1}^{p} a_i H(f_{n-i},x) = e(f_n), \qquad (3)$$

where $H(f_n,x)$ is the $n$ th sample of the complex frequency domain measurement at location $x$ and $\{e(f_n)\}$ is a complex white noise process [5-7]. The parameters of the model are the complex constants $a_i$ and $p$ is the order of the process. Taking the z-transformation of (3), the AR process can be depicted as the output of a linear filter with transfer function

$$G(z) = \frac{1}{1+\sum_{i=1}^{p} a_i z^{-1}} = \prod_{i=1}^{p}\frac{1}{(1-p_i z^{-1})}, \qquad (4)$$

where $p_i$ is the $i^{th}$ pole of the transfer function.

Using AR model, the channel frequency response which is represented by the $N$ samples will be identified with the $p$ AR parameters, which also mean the $p$ poles of the $G(z)$. The AR parameters $\{a_i\}$ are the solution of the Yule-Walker equation [5]:

$$R(-l)+\sum_{i=1}^{p}a_i R(i-l)=0, \qquad (5)$$

in which $R(k,x)=R(k)$ is the frequency correlation function defined in (2) and $R(-l)$ is equal to $R^*(l)$. The variance of the zero mean white noise $\{e(f_n)\}$ is the same as the minimum mean error of the predictor, which is given by

$$\sigma_v^2 = R(0)+\sum_{i=1}^{p}a_i R(i). \qquad (6)$$

The modelling can be explained that a pole close to the unit circle represents significant power at the delay related to the angle of the pole. The delay is calculated as

$$\tau_i = \arg(p_i)/2\pi f_s, \qquad (7)$$

where $f_s$ is the stepping frequency.

To determine the estimate of the order of the process, Akaike Information-theoretic Criterion (AIC) [8] and Minimum Description Length Criterion (MDL) [9] are introduced. Consider an AR system characterized above by (3), the criteria are formulated as

$$AIC(k)=\log(\hat{\sigma}_k^2)+2\frac{k}{N}, \qquad (8)$$

$$MDL(k)=\log(\hat{\sigma}_k^2)+\frac{k}{N}\log(N), \qquad (9)$$

where $\hat{\sigma}_k^2$ is the estimate of the minimum mean error of the predictor done under the assumption that $k$ is the true order.

### *B. Time-Varying Autoregressive (AR) Process*

Modelling non-stationary signals with autoregressive (AR) models having time-varying coefficients is well studied [10, 11]. No general mathematical framework for dealing with non-stationary signals exists, and in practice, the problem of time dependency is circumvented by presuming that the process is locally stationary over a relatively short time interval but globally non-stationary. This idea is proposed for modelling a UWB channel without the static assumption. Let us further assume that the measured data, which is non-stationary in space, is also non-stationary in time.

Unlike in the conventional AR model, the coefficients are now time-variant as

$$H(f_n,x)+\sum_{i=1}^{p}a_i(n)H(f_{n-i},x)=e(f_n), \qquad (10)$$

where $a_i(n)$ is the function representing the *i*th time-varying AR coefficient, and $e(f_n)$ is a zero-mean, stationary Gaussian white noise process. If it is assumed that the function $a_i(n)$ can be represented by a family of M time-varying basis functions as [10, 11]

$$a_i(n)=\sum_{j=0}^{m}a_{ij}g_j(n), \qquad (11)$$

the process can be written as

$$H(f_n,x)+\sum_{i=1}^{p}\sum_{j=0}^{m}a_{ij}g_j(n)H(f_{n-i},x)=e(f_n), \qquad (12)$$

where $i=1,...,p$ and $j=1,...,m$.

Let us define the vectors:

$$A=\left[a_{10},\cdots,a_{1m},\cdots,a_{p0},\cdots,a_{pm}\right]^T. \qquad (13)$$

$$H(f_{n-i},x)=\begin{bmatrix} H(f_{n-1},x)g_0(n),\cdots,H(f_{n-1},x)g_m(n), \\ \cdots,H(f_{n-p},x)g_0(n),\cdots,H(f_{n-p},x)g_m(n) \end{bmatrix}. \qquad (14)$$

Then (10) becomes

$$H(f_n,x)+H(f_{n-i},x)A=e(f_n). \qquad (15)$$

In practice, the goodness of fit achieved using this technique is dependent on the subspace spanned by the chosen basis functions. Standard choices for these basis functions include

- time polynomial function;
- Taylor expansion of the variant parameters;
- Fourier base;
- Legendre polynomial function.

Each basis function has its own unique accuracy. Legendre polynomials perform quite well if the coefficients are smoothly changing with time [11]. In this paper, Legendre function was used to simulate the time-varying AR coefficients.

## IV. PATH LOSS MODEL

A signal propagated through the radio channel undergoes attenuation, which is called path loss. It is usually presented in decibels as

$$PL(d)=PL_{FS}(d_0)+10\gamma\log_{10}(d/d_0)+X_\sigma, \qquad (16)$$

where

$d$ is the distance between the TX and the RX,

$d_0$ is a reference distance, which is a far field distance of the antenna,

$\gamma$ is a path loss exponent,

$X_\sigma$ is a zero-mean Gaussian random variable with standard deviation $\sigma$, both given in decibels and free space loss defined in decibels as

$$PL_{FS}(d)=20\log_{10}\left(\frac{4\pi d}{\lambda}\right). \qquad (17)$$

A general path loss formula (16) contains a variable $X_\sigma$, which gives the statistical variability of the path loss values. In order to obtain reliable statistics of the model by measurements, a large amount of measurement positions in different environments must be included in the study.

In this paper, the path loss was calculated in all the measured environments by averaging the transfer functions over the frequency band as a function of distance, as [12].

$$PL(d)=-10\log_{10}\left(\frac{1}{1601}\sum_{i=1}^{1601}\left|H(d,f_i)\right|^2\right), \qquad (18)$$

where $H(f_i)$ is the channel transfer function.

## V. RESULTS

The model order selection criteria previously explained are examined to determine the estimate of the order of the process. By applying these criteria to many measured frequency response, it is concluded that a fifth order process is sufficient to represent the statistic of the channel model. For a fifth order model of the room TS441 ($NLOS_1$) at the first stepping rail, the magnitude of the largest pole $(p_1)$ was 0.8184, the second pole $(p_2)$ was 0.8008 and the magnitudes of the remaining poles $(p_{3-5})$ were around 0.5, which are not so significant. Figure 3 illustrates the complex z-plane scatter plot of the time-varying AR model of order 5.

A pole close to the unit circle represents significant power at the delay related to the angle of pole according to (7). The arrivals of the significant paths are then 0.04 and 0.07 $\mu s$. respectively, which corresponds to the angular range of -7π/25 and -10π/25 respectively. Figure 4 and 5 show the channel impulse response obtained by taking the inverse Fourier transform of the frequency response regenerated from the AR model and the time-varying AR model respectively. These figures reproduce the multipath propagation characteristics observed in the measurements. Table II lists the comparison of the channel model characteristics in the $NLOS_1$ and $NLOS_2$ cases. Moreover, figure 6 shows the CDF of the RMS delay spreads from 235 samples to confirm that the regenerated models are close statistical fits to the measurement data as well as the time-varying AR model is more accurate than the conventional AR model due to its less variance.

Averaging over frequencies in (18) can be explained by the fact that path loss is relatively insensitive to frequency, as depicted in figure 7. Figure 8 depicts the path losses in the both measurement cases compared with the ones from the regenerated models. The path loss exponent is calculated from the slope of the linear regression line, which is shown in the path loss figures.

## VI. Conclusion

The aim of the paper was to model an indoor UWB radio channel using the frequency domain model, time-varying AR model. A statistical AR model and time-varying AR model for

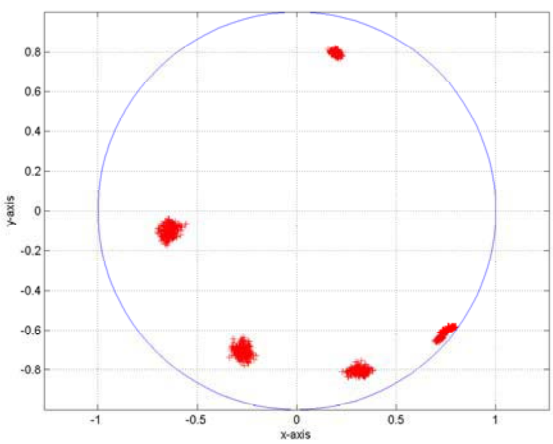


Figure 3. The complex z-plane scatter plot of a time-varying AR model of order 5 in TS441 ($NLOS_1$) at the first stepping rail.

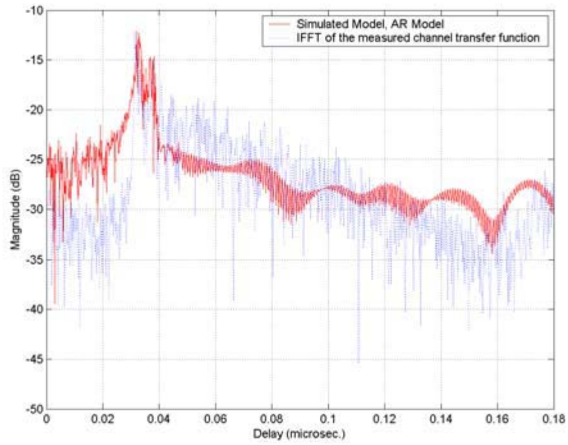


Figure 4. The channel impulse response obtained by taking the inverse Fourier transform of the frequency response regenerated from the AR model in TS441 ($NLOS_1$) at the first stepping rail.

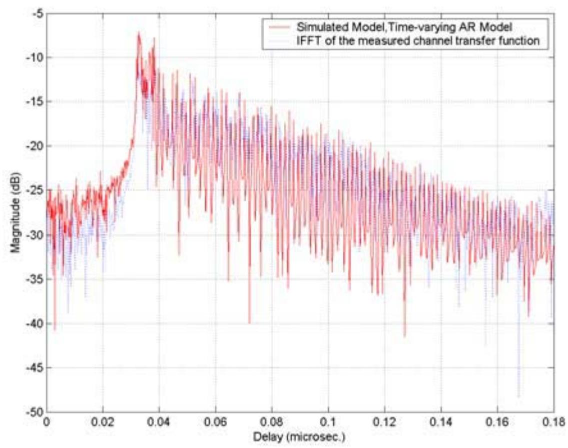


Figure 5. The channel impulse response obtained by taking the inverse Fourier transform of the frequency response regenerated from the time-varying AR model in TS441 ($NLOS_1$) at the first stepping rail.

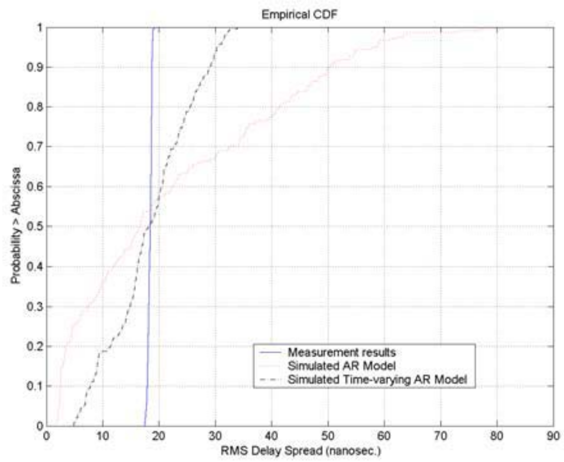


Figure 6. The CDF of the RMS delay spreads from 235 samples for $NLOS_1$.

TABLE II
THE COMPARISON OF THE CHANNEL MODEL CHARACTERISTICS IN THE BOTH CASES, $NLOS_1$ and $NLOS_2$

| **Model Characteristics** | $\tau_{RMS}$ **[ns]** | $NP_{10\ dB}$ |
|---|---|---|
| **$NLOS_1$ (Measurement)** | 18.19 | 26 |
| **$NLOS_1$ (Modified IEEE 802.15.3a)** | 18 | 16 |
| **$NLOS_1$ (AR model)** | 16.17 | 17 |
| **$NLOS_1$ (Time-Varying AR model)** | 17 | 30 |
| **$NLOS_2$ (Measurement)** | 17 | 23 |
| **$NLOS_2$ (Modified IEEE 802.15.3a)** | 21 | 27 |
| **$NLOS_2$ (AR model)** | 15 | 10 |
| **$NLOS_2$ (Time-Varying AR model)** | 19 | 32 |

$\tau_{RMS}$ is the RMS delay spread and $NP_{10\ dB}$ is the number of multipath components within 10 dB of the peak.

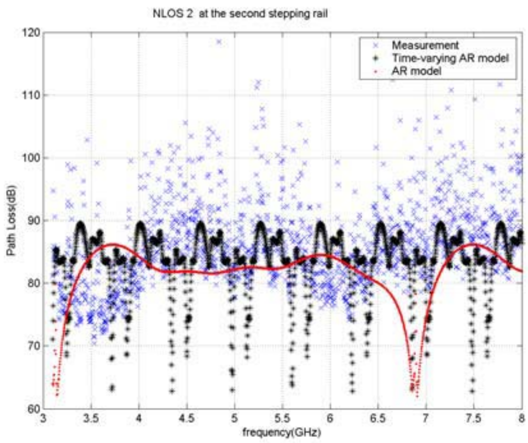


Figure 7. Example of path loss as a function of frequency of $NLOS_2$ at the second stepping rail.

the simulation of the indoor UWB radio channel was described.

These AR models are simpler than the existing time domain models due to their fewer parameters. The location of each pole of the AR model represented the arrival of a cluster of paths where the angle of the pole represented the arrival time and the closeness to the unit circle indicated the strength of the

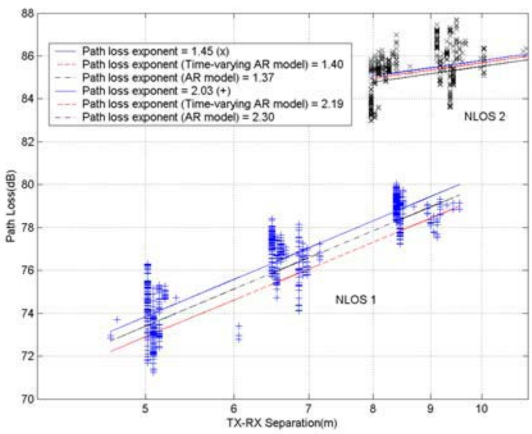


Figure 8. Path loss in TS441 ($NLOS_1$) and TS472 ($NLOS_2$).

cluster. A five pole model was shown to be sufficient to regenerate the statistical characteristics of the radio channels. Furthermore, a time-varying AR model can obtain the more accurate results.

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