On the Design and Performance of LDPC Coded MC-CDMA Systems

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Abstract – **This paper presents and compares two different strategies for LDPC coded MC-CDMA system design. In general, Turbo multiuser detection is exploited by combining per-user MMSE-PIC detection and LDPC decoder together at the receiver end. First alternative, proposed earlier by the authors, is to apply the code *across* the substreams and transmit the coded data using MC-CDMA. The other approach, in turn, is based on applying the code(s) *within* each substream and form a MC-CDMA transmission symbol of these separately coded streams. The analysis shows some fundamental differences in the two alternatives, with the conclusion that the second system scenario has relatively better performance. This is confirmed using computer simulations in downlink channels. Furthermore, it is also shown that the second system scenario is easily applicable in the uplink case as well, and that in general relatively small spreading factors per MC-CDMA multiplex can be utilized to maintain reasonable detection and receiver complexity as a whole.**

Index Terms – **channel coding, code division multiple access, iterative decoding, multicarrier modulation, multiuser detection**

I. INTRODUCTION

Generally speaking, multicarrier modulation is considered as a key ingredient in future beyond 3G and 4G communication system developments. In particular, combining the advantages of OFDM (orthogonal frequency division multiplexing) and CDMA (code division multiple access), MC-CDMA (multicarrier CDMA) has received much attention among researchers [1]. However, it suffers from multiple access interference (MAI) in a multiuser setting, which leads to the decrease in the overall BER (bit error rate) performance abruptly with increasing number of users. Multiuser detection (MUD) techniques have been introduced to mitigate MAI in order to improve the system performance [2].

In recent years, channel coding has been studied in MC-CDMA context to improve the performance further. Correspondingly, a novel detection technique called Turbo multiuser detection [3] is applied at the receiver end. With reasonable complexity, Turbo multiuser detection can generally yield great improvements in performance over traditional receivers for digital communications. Convolutional and Turbo coded MC-CDMA systems have been investigated in [4]-[6]. In [7], we proposed a low density parity check (LDPC) coded MC-CDMA system. Turbo multiuser detection integrating per-user MMSE-PIC (minimum mean squared error - parallel interference cancellation) detector and LDPC decoding has been shown to provide impressive performance results. Even with considerably small spreading factors, such as 4 or 16, error propagation can essentially be avoided. Thus, good system performance can be obtained with reasonably low computational complexity. In this paper, this system is used as a reference and will be referred to as system A.

In this contribution, an alternative approach to LDPC coded MC-CDMA system design is presented. This system scenario will be termed system B. It will be shown that this new design strategy leads to yet improved performance compared to system A. Moreover, system B is basically more suitable for uplink channels and it also achieves satisfying performance with relatively small spreading factors.

The paper is organized as follows: Section II describes the two approaches for designing LDPC coded MC-CDMA systems, with natural emphasis on system B. Section III analyzes and compares the two approaches from the effective diversity point of view. Section IV provides some performance results and comparisons based on computer simulations. Section V focuses in more details on the performance of system B in uplink direction. Finally, conclusions are drawn in Section VI.

II. SYSTEM SCENARIOS

The basic idea in system A [7] is to take one source bit or symbol from each subchannel stream, encode these using the selected LDPC code, and transmit the coded data over the available subcarriers using MC-CDMA. To decrease the detection complexity as a whole, the total pool of subcarriers can be divided into several subgroups, and each MC-CDMA multiplex uses the subcarriers of one subgroup. As an example, assume the total number of subcarriers is 256. Then (512, 256) regular LDPC code could be used for channel coding. It encodes blocks of 256 information bits into 512 coded bits. These coded bits are then mapped into blocks of 256 coded symbols using, say, QPSK. Each block is transmitted using MC-CDMA with the total of 256 subcarriers. The number of subcarrier subgroups could be, e.g., 1 (plain MC-CDMA), 16, 64, or 256 (pure OFDM) such that a proper compromise between total detection complexity and performance is achieved.

The new proposed system B is different from system A in the coding of information streams. It divides the information stream into parallel substreams, as does system A, but then the substreams are encoded individually using identical (512, 256) regular LPDC codes. These encoded data are then mapped into symbols (using again, e.g., QPSK), and these symbol streams are transmitted using MC-CDMA, on a one symbol from each substream basis. Again, similar idea of grouping the total pool of available subcarriers into subgroups can be used to obtain proper compromise between detection complexity and performance. Notice, however, that here in system B, the signals in different subgroups are basically independent of each other (i.e., the different subgroups can be used and operated independently). The basic transmitter structure for system B is depicted at a principal level in Fig. 1. The corresponding iterative receiver structure based on per-user MMSE-PIC detection appears in Fig. 2. In general, for consistency, interleaving depth (if used) can be assumed identical in both systems A and B.

In both systems, the guard interval (GI) implemented as a cyclic prefix (CP) is assumed longer than the channel delay spread, such that the transmission channel is effectively flat within each subchannel. Moreover, all the user channels are identical in the downlink direction whereas different channels are more than likely in the uplink case. Assuming quasi-synchronous signals also in the uplink case, intersymbol interference is basically avoided, and then only one symbol from each user or spreading code channel is contributing to the observed data within one detection interval. After GI removal and FFT, a direct frequency domain signal model can thus be used in each subgroup of system A and/or in system B. Considering the downlink case for simplicity, the signal within one detection interval appears as (time index omitted)

$$\mathbf{r} = \begin{bmatrix} H_1c_{1,1} & H_1c_{2,1} & \cdots & H_1c_{K,1} \\ H_2c_{1,2} & H_2c_{2,2} & \cdots & H_2c_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ H_Nc_{1,N} & H_Nc_{2,N} & \cdots & H_Nc_{K,N} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_K \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix} \qquad (1)$$
$$= \mathbf{Sa} + \mathbf{n}$$

where $\mathbf{r} = [r(1), r(2), \ldots, r(N)]^T$ with $r(i)$ being the i-th subchannel observation. A_k is the transmitted coded data symbol for spreading code channel k and $\mathbf{n}$ denotes the noise vector. The code chips of spreading code k are denoted as $\mathbf{c}_k = [c_{k,1}, c_{k,2}, \ldots, c_{k,N}]^T$ and H_i denotes the i-th subchannel response. Furthermore, $\mathbf{S}$ can be written as $\mathbf{S} = \mathbf{\Lambda C}$, where

$$\mathbf{C} = \begin{bmatrix} c_{1,1} & c_{2,1} & \cdots & c_{K,1} \\ c_{1,2} & c_{2,2} & \cdots & c_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1,N} & c_{2,N} & \cdots & c_{K,N} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_K \end{bmatrix} \qquad (2)$$

and

$$\mathbf{\Lambda} = diag\{H_1, H_2, \cdots, H_N\} \qquad (3)$$

III. PERFORMANCE ANALYSIS

Assuming independently fading channels (in both time and frequency), it may seem at the first sight that the two coded MC-CDMA systems have no difference in performance. However, when analyzed from the available time and frequency diversity point of view, it can be shown that system B has higher total effective diversity order than system A. The regular LDPC code applied in the two systems is assumed to follow the D.J.C. MacKay's algorithm presented in [8]. In the following, independent fading between subcarriers and consecutive multicarrier symbols in time is assumed.

In system A, any specific information bit is transmitted over one OFDM symbol duration, thus the relative time diversity order is basically equal to 1. Based on the structure of the used LDPC code, one information or source bit has influence on around 130 coded bits. This means that at least 65 (and at most 130) coded QPSK symbols carry some information of any specific source bit. Take the "worst" case of 65 coded QPSK symbols as an example. With spreading factor of one (pure OFDM), the relative frequency-diversity order is then 65. Hence, the total effective diversity order is also 65. If instead of OFDM, we use actual MC-CDMA with e.g. spreading factor of 16 (meaning 16 groups of 16 subcarriers altogether), this gives also additional frequency-diversity of order 16. Nevertheless, as the total number of subcarriers is 256, some of the 65 symbols are definitely transmitted on the same group of subcarriers. Hence at the receiver end, the received symbols are correlated with each other. Thus, the total effective diversity order is basically greater than 65 but definitely always less than 65×16.

In system B, in turn, the parallel substreams are coded separately with identical LDPC codes. Then these symbol streams are transmitted using MC-CDMA, on a one symbol from each substream basis. Like in system A, after LDPC coding, one information or source bit has influence on around 130 coded bits. Again, at least 65 and at most 130 coded QPSK symbols are affected by a specific source bit but now these symbols are transmitted over consecutive OFDM symbol durations. For comparison, we again take the case of 65 coded QPSK symbols carrying the information of a source bit as an example. Suppose first that we use, instead of MC-CDMA, traditional OFDM (spreading factor one) to transmit the coded substreams. Then the relative time-diversity order is 65 and thus also the total effective diversity order is 65. Now, instead of OFDM, use MC-CDMA with spreading factor of e.g. 16, yielding additional relative frequency-diversity of order 16. Thus, the total effective diversity order is now 65×16.

The above analysis shows that for pure OFDM, the total effective diversity of system A equals that of system B. However, for MC-CDMA case, the total effective diversity order (total "time-frequency window" used to transmit any information bit) of system B is greater than that of system A. Thus, system B should have better performance than system A in general. The following simulation results will back up these conclusions.

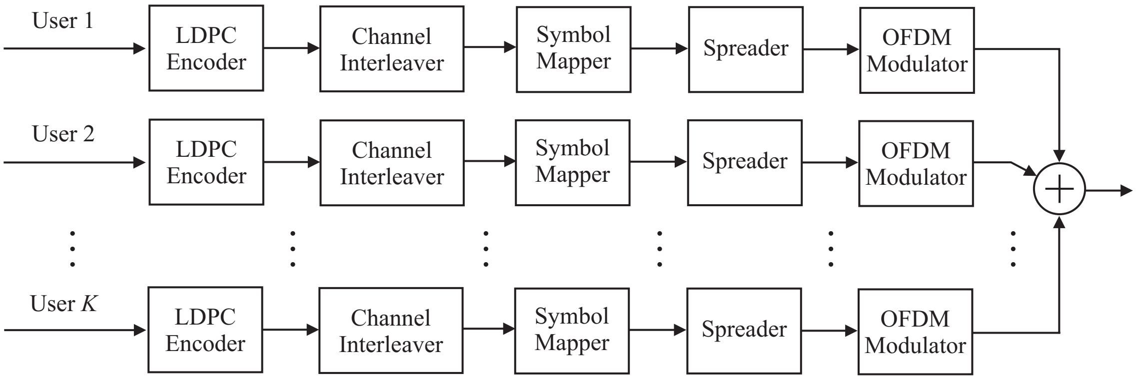

Figure 1. LDPC coded MC-CDMA system transmitter (system design *B*). “User” is here referring to a spreading code channel.

IV. PERFORMANCE SIMULATIONS

The performance comparison of the two system designs in downlink channel is presented in this section. The MC-CDMA system models described in Section II are used. In system *A*, the spreading is done by Walsh-Hadamard codes of the same length as the number of subchannels used in the subgroup. In system *B*, the spreading factor per MC-CDMA multiplex is set equal to that of system *A*, and the spreading is also done by Walsh-Hadamard codes. Subchannels are assumed to be independently Rayleigh fading and non-frequency-selective. QPSK modulation is used for all the users in the simulation. Fully loaded cases are considered in the simulation. (512,256) regular LDPC code with column weight 3 is used for channel coding. In the LDPC decoder, the maximum number of iterations is set to 10.

At the receiver end, turbo multiuser detection is applied by integrating per-user MMSE-PIC detector and LDPC decoding. The detailed turbo multiuser detection algorithm for system *A* has been presented in [7]. In this case, the receiver can apply the per-user MMSE-PIC detector and LDPC decoding iteratively whenever one MC-CDMA symbol is received. For system *B*, on the other hand, the receiver applies the per-user MMSE detector first whenever one MC-CDMA symbol is available. Then, when a block of 256 MC-CDMA symbols are all received, the receiver applies LDPC decoding and PIC detector repeatedly to improve the system performance. In comparison with system *A*, system *B* has thus relatively longer detection delay. The BER performances after 5-th PIC iterations in fully loaded cases are used for comparison.

Fig. 3 compares the performance of the two systems with different spreading factors (per MC-CDMA multiplex). According to the obtained results, system *B* has a gain of around 0.6-0.8 dB over the system *A* when measured at coded BER level of 10^{-4}. The used spreading factors per MC-CDMA multiplex are 16, 8 and 4. Thus, the simulation results clearly indicate that the system *B* has consistently better performance than system *A*, which supports the analysis in Section 3. Naturally, in case of pure OFDM, the two systems have exactly the same performance which is also verified by the simulations.

V. SYSTEM *B* AND UPLINK CHANNELS

In system *B*, the substreams are encoded separately and the mapped QPSK symbol streams are transmitted using MC-CDMA, on a one symbol from each substream basis. Thus, unlike system *A*, the new proposed LDPC coded MC-CDMA system is directly applicable also in uplink scenarios. A substream is corresponding to a mobile station. The detection and decoding are performed at the base station. In this section, the performance of system *B* in uplink channel will be demonstrated, using the traditional assumption of quasi-synchronized transmitters [9]. Like in downlink channel, the guard interval (GI) implemented as a cyclic prefix (CP) is assumed longer than the channel delay spread to guarantee that the transmission channel is effectively flat within each subchannel. After GI removal and FFT, a direct frequency domain signal model similar to (1) can be used, except that different users have different channel responses at each subchannel.

In [10], the error propagation phenomenon in per-user MMSE-PIC detector has been studied in uncoded MC-CDMA system. Only when the number of subchannels is sufficiently large, the 5-th PIC iteration is consistently the best as expected from theoretical results. Otherwise, error propagation prevents BER from dropping with more iterations. When turbo multiuser detection is used in the system *B*, error propagation is avoided, even with very small numbers of subcarriers, in the uplink channel. The BER performances after 5-th PIC iteration in fully loaded cases are presented for comparison. Fig. 4 presents the simulation results of the system *B* for the cases with the number of subcarriers being 16, 8, and 4 in uplink direction. In these cases, there is only slight performance degradation when the number of subcarrier is decreased. At coded BER of 10^{-4}, there is a 0.13 dB gap between 16 and 8 subcarriers cases, and a 0.44 dB gap between 16 and 4 subcarriers cases after 5-th PIC iteration. The performance of per-user MMSE-PIC detector for uncoded MC-CDMA system in uplink direction is also provided for reference with the number of subcarriers being 256. Compared with the uncoded case, the figure shows that more than 4 dB coding gain can be obtained at BER $= 10^{-4}$ at iteration 5 when the number of subcarriers is 16 per multiplex in the coded system.

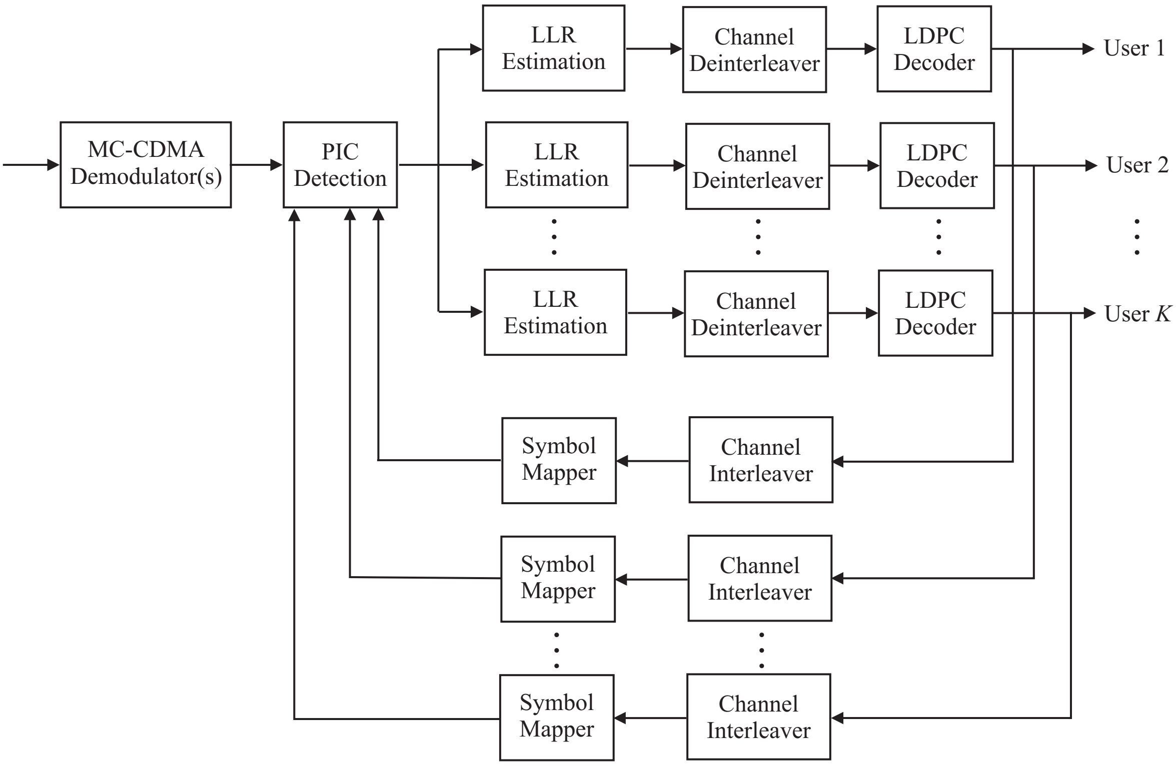

Figure 2. Iterative receiver structure for LDPC coded MC-CDMA system (system design B).

VI. CONCLUSIONS

In this contribution, a new LDPC coded MC-CDMA system, termed system B, was proposed. This system design was compared with the previously proposed system from the relative time and frequency diversity point of view. Assuming independent fading, it was shown that the total effective diversity order in the new design is fundamentally larger. This was confirmed by the presented simulation results in downlink channel. Moreover, system B is by design more suitable for uplink channels. Impressive detection performance was demonstrated, both in the uplink and downlink cases, with relatively small spreading factors (4, 8, 16) per MC-CDMA multiplex, implying that good detection performance can in general be obtained with reasonable computational complexity.

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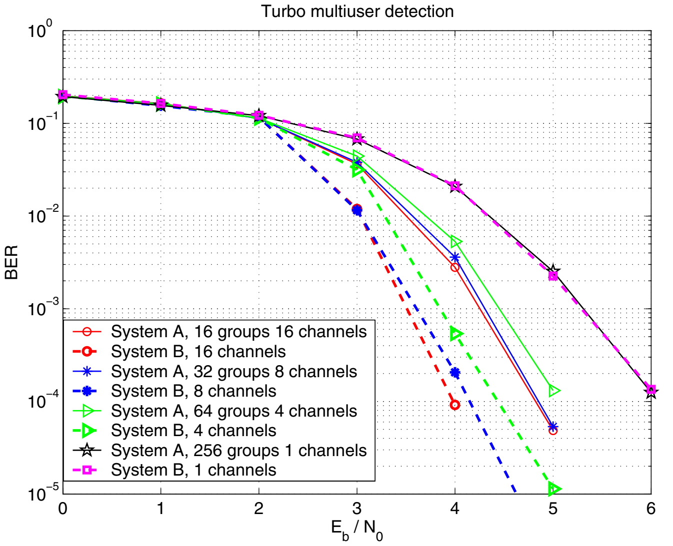

Figure 3. Performance comparison of the two LDPC coded MC-CDMA systems in i.i.d. Rayleigh fading downlink.

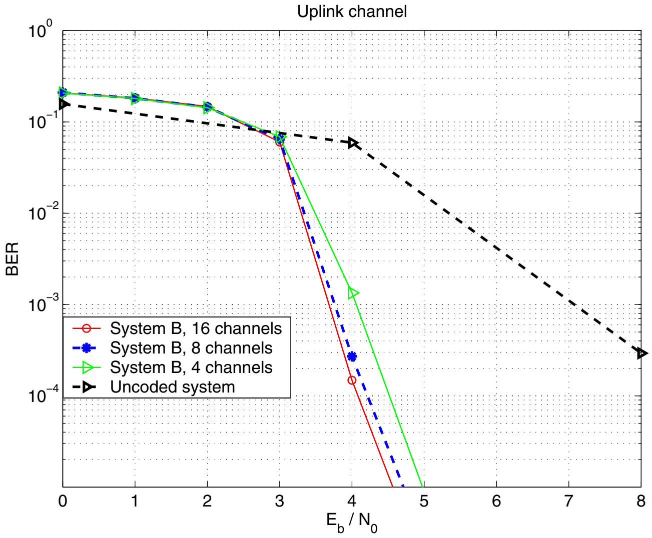

Figure 4. Performance of LDPC coded MC-CDMA system *B* and uncoded MC-CDMA in i.i.d. Rayleigh fading uplink.