A Novel Multi-User Interference Statistics Estimator for UWB-IR based on Gaussian Mixture Model

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***Abstract*— Ultra-Wideband Impulse Radio communications have been growing rapidly over the last few years, as a promising technique for high bit-rate and multi-user transmissions over the 3–10 GHz unlicensed spectrum. In literature, multi-user interference is often approximated with a white Gaussian process and embodied in the overall noise term. However, it has been found that, in a typical indoor environments, such approximation is inadequate. In this paper, we propose a new approach to characterize the interference, based on the Gaussian mixture model. We also derive an estimator for multi-user interference statistics, based on the iterative Expectation Maximization algorithm. The effectiveness of the proposed estimator is finally shown by means of numerical examples.**

I. INTRODUCTION

Ultra-Wideband (UWB) is an emerging communications technology, whose features are well suited for multi-user indoor environments, and is very attractive thanks to the possibility of exploiting the 3–10 GHz unlicensed spectrum [1].

The Impulse Radio (IR) modulation format we refer to was originally described in 1993 [2]. It consists of the transmission of ultra-short pulses with duration less than 1 ns, where the pulse pattern is built up using binary pulse position modulation (PPM) for data transmission with time-hopping (TH) sequences to provide multiple access capability.

When applying receivers with coherent single- or multi-user detection in fading channels, it is essential to provide an accurate estimate of the channel state, which has a major impact on the overall system performance. For this reason, several works have been devoted to channel estimation in IR systems, e.g. [3][4]. However, when deriving channel estimators, the multi-user interference (MUI) is always modeled as a Gaussian process, even though it has been shown that such approximation is often inadequate [5]. In fact, in a typical indoor scenario, MUI is due to the superposition of several interferers which may have different power levels. Thus, a strongly impulsive component may arise and then the central limit theorem hypotheses are no longer verified.

The aim of this work is to derive an estimator of MUI statistics suitable for further derivation of more reliable joint channel and interference estimator algorithms. The proposed solution is based on the Gaussian mixture model (GMM), which considers the interference as a stochastic process with probability density function (pdf) given by a weighted superposition of Gaussian pdfs, each having zero mean and assigned variance. Hence, the complete statistical description of the MUI is obtained by estimating the weights and the variances of the GMM.

It has been found that the GMM is appropriate for impulsive noise, which can be modeled as a train of randomly occurring narrow pulses in a background of Gaussian noise, [6], [7]. We show that the GMM is also well-suited to characterize the impulsive behavior of multi-user interference in IR systems and allows MUI statistics estimation via the Expectation Maximization (EM) algorithm [8].

The paper is organized as follows. In Section II, we introduce the system model, describing the IR signal format in an asynchronous multi-user scenario and the GMM. Afterwards, in Section III, we derive the MUI statistics estimator detailing the EM algorithm. Next, in Section IV convergence properties of the estimator are discussed, whereas Section V demonstrates the estimator performance. Finally, in Section VI some conclusions are drawn, addressing further investigation toward the realization of a GMM-based joint channel and interference estimator.

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II. SYSTEM MODEL

A. IR over fading channels

In an IR system, designed for a maximum number of N_c users, the signal transmitted by the u–th user is

$$s_u(t) = \sum_m w(t - mT_s - c_{u,m}T_c - b_{u,m}T) \qquad (1)$$

with a single pulse per slot, with duration T_s. With TH, each pulse is positioned within each slot according to a user-specific TH sequence $c_{u,m}$. Specifically, dividing each slot into N_c chips each of duration T_c, the u–th user TH code $c_{u,m} \in \{0, \dots, N_c\}$ corresponds to a time shift of $c_{u,m}T_c$ during the m–th slot. Furthermore, binary PPM maps the source stream $b_{u,m} \in \{0, 1\}$ of the u–th user into a further time shift of duration $b_{u,m}T$. Following [9], the duration of a slot is fixed to $T_s = 250$ ns, the chip period to $T_c = 0.44$ ns and the 2-PPM spacing to $T = 0.22$ ns. We assume periodic

TH sequences with period $L_c = N_c = 509$, thus the signal (1) can be regarded as a cyclostationary process with period $L_c T_s$. Notice that the chip period T_c has been chosen such that $N_c T_c + T_g = T_s$, where the guard interval $T_g = 25$ ns is properly chosen to combat inter symbol interference (ISI) in typical UWB propagation conditions. The chosen pulse $w(t)$ is the second derivative of the Gaussian pulse

$$w(t) = (1 - x)\ e^{-\frac{x}{2}}\Big|_{x=\pi\left(\frac{t}{T_w}\right)^2} ,$$

with $T_w = 0.2$ ns.

The regarded channel model is detailed in [10], and the channel impulse response (CIR) writes

$$g_u(t) = \sum_{k=0}^{K} A_{u,k}\, \delta\left(t - \tau_{u,k} - \tau_u\right) \tag{2}$$

where $A_{u,k}$ and $\tau_{u,k}$ represent attenuation and delay of the k–th path for the u–th user, respectively. Without loss of generality we assume $\tau_{u,0} = 0$, $\forall u$. For the reference user, τ_u is the propagation delay, while for remaining interfering users τ_v, $v \neq u$, is modeled as a random variable uniformly distributed in the period of cyclostationarity. The received signal, pointing out both the contribution of MUI and thermal noise is:

$$r(t) = r_u(t) + \sum_{v \neq u} r_v(t) + \eta(t) = r_u(t) + n(t)$$

where $r_u(t)$ represents the signal component related to the reference user

$$r_u(t) = s_u * g_u(t) = \sum_{k=0}^{K} A_{u,k} s_u(t - \tau_{u,k}) \ .$$

The multi-user interference is collected in the signal $I_u(t) = \sum_{v \neq u} r_v(t)$, while $\eta(t)$ is additive white Gaussian noise (AWGN) with two-sided power spectral density $R_\eta(f) = N_0/2$. We include both MUI and thermal noise in a single term $n(t)$, hereafter referred to as noise. Before being further processed, the received signal $r(t)$ is sampled into $r_i = r(iT_0)$ with a proper sampling rate $F_0 = 1/T_0$.

In the following we are interested in the noisy component of the received signal, so that we assume data-aided transmission, i.e., the source stream $b_{u,m}$ is known to the receiver. Moreover, perfect knowledge of the attenuations $A_{u,k}$ and delays $\tau_{u,k}$ of the actual CIR (2) is assumed. Hence, at the receiver, a local copy of the signal component $r_{u,i} = r_u(iT_0)$ is generated and the noise component $n_i = n(iT_0)$ is obtained by subtraction, $n_i = r_i - r_{u,i}$.

B. The Gaussian Mixture Model

The receiver collects M samples of the noisy component $n(t)$ of the received signal into a row vector $\mathbf{n} = [n_0, \ldots, n_{M-1}]$. According to the Gaussian Mixture Model, each noisy entry n_i of the vector $\mathbf{n}$ is a realization of a random variable with pdf given by an L–term mixture of Gaussian pdfs, parameterized by a vector

$$\mathbf{N} = \left[\lambda_1, \ldots, \lambda_L, \sigma_1^2, \ldots, \sigma_L^2\right] \ , \tag{3}$$

that is

$$p(n_i|\mathbf{N}) = \sum_{\ell=1}^{L} \lambda_\ell\, p_\ell(n_i|\sigma_\ell^2)\,, \tag{4}$$

where the nonnegative coefficients λ_ℓ satisfy the normality condition $\sum_{\ell=1}^{L} \lambda_\ell = 1$ and $p_\ell(n_i|\sigma_\ell^2)$ is the pdf of a zero-mean Gaussian process with variance σ_ℓ^2, namely

$$p_\ell(n_i|\sigma_\ell^2) = \frac{1}{\sqrt{2\pi\, \sigma_\ell^2}}\, e^{-\frac{1}{2}\frac{n_i^2}{\sigma_\ell^2}} \ , \quad \ell = 1, \ldots, L\ . \tag{5}$$

Then, the pdf of the noise samples results

$$p(n_i|\mathbf{N}) = \sum_{\ell=1}^{L} \frac{\lambda_\ell}{\sqrt{2\pi\sigma_\ell^2}}\, e^{-\frac{1}{2}\frac{n_i^2}{\sigma_\ell^2}} \ . \tag{6}$$

We call L the order of the GMM. Notice that, setting $L = 1$, the L–term Gaussian mixture model reduces to the Gaussian model. This proves that the proposed model includes the Gaussian approximation (GA), typically encountered in literature, as a particular case.

III. Multi-user Interference Statistics Estimation

Given the GMM introduced above, an estimate of the MUI pdf, which is a complete statistical description, may be obtained from the maximum likelihood (ML) estimate of the vector $\mathbf{N}$. However, direct application of the ML algorithm to this problem requires the maximization of the likelihood function

$$L(\mathbf{n}|\mathbf{N}) = \prod_{i=0}^{M-1} p\left(n_i|\mathbf{N}\right) \tag{7}$$

with respect to the vector $\mathbf{N}$. The resulting search space has dimension $2L$, therefore the ML estimation becomes impracticable in terms of computational effort.

A. Expectation Maximization estimation with GMM

We propose a lower complexity solution based on the EM algorithm [8], which is a broadly applicable approach to the iterative computation of ML estimates. The derivation of the EM algorithm relies on the two key notions of the complete (unobservable) and incomplete (observable) data. Following [11], the complete data set $\mathcal{N}_c$ is obtained assuming that each observed component n_i is modeled by only one term of the GMM (6), that is

$$\mathcal{N}_c = \{y_i \mid i = 0, \ldots, M-1\} \ ,$$

where $y_i = (n_i, c_i)$ and $c_i \in \{1, \ldots, L\}$ is the variable identifying the term of the GMM (6) that models the noise sample n_i. The likelihood function of the vector $\mathbf{y}$, parameterized by $\mathbf{N}$, is

$$L(\mathbf{y}|\mathbf{N}) = \prod_{i=0}^{M-1} p(n_i|\mathbf{N}_{c_i}) \tag{8}$$

where

$$\mathbf{N}_{c_i} = [0, \ldots, 0, \lambda_{c_i}, 0, \ldots, 0, \sigma_{c_i}^2, 0, \ldots, 0]$$

is obtained from $\mathbf{N}$ by setting all its components to zero, except λ_{c_i} and $\sigma^2_{c_i}$. Since the vector $\mathbf{y} = [y_0, \ldots, y_{M-1}]'$ is not observable, the estimation is obtained iteratively from the observation $\mathbf{n}$ of the incomplete data set

$$\mathcal{N} = \{n_i \mid i = 0, \ldots, M-1\} ,$$

and from the previous estimate of $\mathbf{N}$. The algorithm starts with an arbitrary initial guess $\mathbf{N}^{(1)}$, where

$$\mathbf{N}^{(k)} = \left[\lambda_1^{(k)}, \ldots, \lambda_L^{(k)}, \sigma_1^{2^{(k)}}, \ldots, \sigma_L^{2^{(k)}}\right]$$

denotes the current estimate of $\mathbf{N}$ after k iterations of the algorithm. Then, each iteration cycle can be divided into two steps, namely, the expectation step (E-step) and the maximization step (M-step).

The E-step is carried out by evaluating the expectation of the log-likelihood function (8) of the complete data $\mathbf{y}$ conditioned upon the incomplete data $\mathbf{n}$ and the current estimate $\mathbf{N}^{(k)}$. Exploiting the Independence of the observed samples n_i, we obtain

$$\begin{aligned} Q\left(\mathbf{N}|\mathbf{N}^{(k)}\right) &= \mathrm{E}\left[\log \prod_{i=0}^{M-1} p(n_i|\mathbf{N}_{c_i}) \,\middle|\, \mathbf{n}, \mathbf{N}^{(k)}\right] \\ &= \sum_{c=1}^{L} \sum_{i=0}^{M-1} \log\ p(n_i|\mathbf{N}_c) p\left(y_i|n_i, \mathbf{N}^{(k)}\right) \end{aligned} \quad (9)$$

where

$$p(y_i|n_i, \mathbf{N}^{(k)}) = \frac{p\left(n_i|\mathbf{N}_c^{(k)}\right)}{p\left(n_i|\mathbf{N}^{(k)}\right)} \quad (10)$$

represents the pdf of the complete data sample y_i conditioned upon n_i and $\mathbf{N}^{(k)}$. Finally, by substitution of (10) and (6) into (9), and after some straightforward manipulation, we obtain

$$\begin{aligned} &Q\left(\mathbf{N}|\mathbf{N}^{(k)}\right) = \\ &= \sum_c \sum_i \log \frac{\lambda_c}{\sqrt{2\pi\sigma_c^2}}\, e^{-\frac{1}{2}\frac{n_i^2}{\sigma_c^2}}\, \frac{p\left(n_i|\mathbf{N}_c^{(k)}\right)}{p\left(n_i|\mathbf{N}^{(k)}\right)} \\ &= \sum_c \sum_i \left[\log \lambda_c - A - \frac{1}{2}\log \sigma_c^2 - \frac{n_i^2}{2\sigma_c^2}\right] \frac{p\left(n_i|\mathbf{N}_c^{(k)}\right)}{p\left(n_i|\mathbf{N}^{(k)}\right)} , \end{aligned}$$

where $A = \log\sqrt{2\pi}$ is a term independent of $\mathbf{N}$.

During the M-step (maximization), we search for the maximum of the function Q with respect to the parameters of the vector $\mathbf{N}$. Solving $\partial Q/\partial\lambda_c = 0$, with $c = 1, \ldots, L$, subject to the constraint $\sum_c \lambda_c = 1$, we obtain updated estimates for the coefficients λ_c, whereas for the variances σ_c^2 the update equation follows from $\partial Q/\partial\sigma_c^2 = 0$, $c = 1, \ldots, L$. Thus, the parameter estimates are updated by means of the following

$$\lambda_c^{(k+1)} = \frac{1}{M}\sum_i p(y_i|n_i, \mathbf{N}^{(k)}) , \quad c = 1, \ldots, L , \quad (11a)$$

and

$$\sigma_c^{2^{(k+1)}} = \frac{\sum_i n_i^2\, p(y_i|n_i, \mathbf{N}^{(k)})}{\sum_i p(y_i|n_i, \mathbf{N}^{(k)})} , \quad c = 1, \ldots, L , \quad (11b)$$

to obtain $\mathbf{N}^{(k+1)}$.

In short, at the first iteration, the initial estimates of the unknown parameters $\lambda_c^{(1)}$ and $\sigma_c^{2^{(1)}}$ are arbitrarily chosen, then $p(y_i|n_i, \mathbf{N}^{(1)})$ is evaluated and, finally, parameter estimates are updated by means of (11). The E-step and M-step are iteratively repeated until convergence.

B. Maximum Likelihood estimation with GA

As already mentioned in Section II.B, the GMM includes the Gaussian approximation as a particular case, when the order L equals 1. Thus, the EM estimation derived above reduces to the conventional ML estimation of the variance of a Gaussian process. In fact, letting $L = 1$ in the E-step (9), the summation over c vanishes yielding the log-likelihood function (7) of the observed data $\mathbf{n}$ and then the estimate is obtained at the first iteration. Furthermore, from the M-step (11), $\lambda_1 = 1$ according to the normality condition, cf. (11a), and the variance estimate (11b) becomes

$$\sigma_1^2 = \frac{1}{M}\sum_i n_i^2 .$$

IV. CONVERGENCE PROPERTIES

It is known that the incomplete data log-likelihood function (7) increases monotonically on the sequence of estimates generated by the EM algorithm. Hence, convergence toward a local maximum of the log-likelihood function is assured. However,the drawback of the EM algorithm, in combination with mixture densities, is the high number of iterations required for convergence [11]. In order to characterize convergence properties, in terms of the number of iterations, we split the parameter vector $\mathbf{N}$ into two halves, namely

$$\boldsymbol{\lambda}^{(k)} = \left[\lambda_1^{(k)}, \ldots, \lambda_L^{(k)}\right] \text{ and } \boldsymbol{\sigma}^{(k)} = \left[\sigma_1^{2^{(k)}}, \ldots, \sigma_L^{2^{(k)}}\right] . \quad (12)$$

The rate of variations experienced by estimates after each iteration is measured by means of the following convergence indexes

$$r_\lambda(k) = \frac{||\boldsymbol{\lambda}^{(k)} - \boldsymbol{\lambda}^*||^2}{||\boldsymbol{\lambda}^* - \boldsymbol{\lambda}^{(1)}||^2} , \; r_\sigma(k) = \frac{||\boldsymbol{\sigma}^{(k)} - \boldsymbol{\sigma}^*||^2}{||\boldsymbol{\sigma}^* - \boldsymbol{\sigma}^{(1)}||^2} \quad (13)$$

where $\boldsymbol{\lambda}^*$ and $\boldsymbol{\sigma}^*$ represent respectively the vectors of estimates to which the sequences $\{\boldsymbol{\lambda}^{(k)}\}$ and $\{\boldsymbol{\sigma}^{(k)}\}$ tend as $k \to \infty$. The convergence indexes (13) quantify the rate of convergence of the estimates toward the actual values, including a normalization factor which takes into account the distances between the initial setting $\mathbf{N}^{(1)}$ and the final result. Observe that, due to the normality condition on $\boldsymbol{\lambda}$, weights and variances have different orders of magnitude, thus the choice of evaluating two different rate indexes.

To evaluate the convergence properties of the derived EM algorithm, an IR system typical for indoor scenarios is defined. We simulate the transmission of data packets, made up of 509

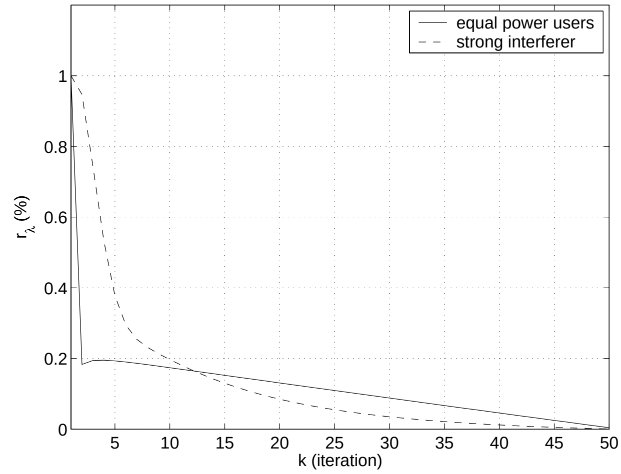

Fig. 1. Percentage variation of the coefficients.

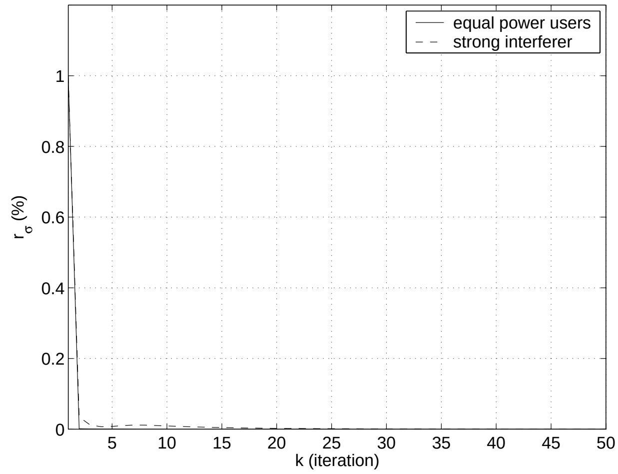

Fig. 2. Percentage variation of the variances.

bits over a noisy LOS channel specified by the model CM1, in [10]. The noise level corresponds to a SNR of 18 dB. We choose two different scenarios, with 8 transmitting users. In the first, here denoted as *equal power users*, all the transmitted signals reach the receiver with the same power level. Hence, all the interferers give the same contribution to the overall MUI I_u. The second scenario differs in the presence of a *strong interferer* with a power level 20 dB higher than the other users. We expect the conventional Gaussian approximation to be effective in the former scenario, unlike in the latter, where the resulting impulsive MUI is more reliably modeled by the GMM.

In the following all the presented results are obtained by averaging over 100 independent realizations of the channel. The natural choice for the estimator initialization would be the ML estimate, namely $\boldsymbol{\lambda}_1^{(1)} = [1, 0, \ldots, 0]$ and $\sigma_1^{2^{(1)}} = \frac{1}{M}\sum_{i=0}^{M-1} n_i^2$, with arbitrary $\sigma_\ell^{2^{(1)}}$, $\ell = 2, \ldots, L$. However, with this setting the EM estimator collapses into the ML one (see Section III.B). Thus, we modified initialization conditions posing $\boldsymbol{\lambda}_1^{(1)} = [0.99, \alpha, \ldots, \alpha]$, with $\alpha = 0.01/(L-1)$ and $[\sigma_\ell^{2^{(1)}}]_\ell$ with $\sigma_\ell^{2^{(1)}} = \gamma\sigma_{\ell-1}^{2^{(1)}}$, $\gamma = 50$.

As a first result, simulations confirm that the proposed EM algorithm actually increases the likelihood of observations in each iteration, as expected. The convergence index $r_\lambda(k)$ and $r_\sigma(k)$ are shown in Figs. 1 and 2, where we have chosen $\boldsymbol{\lambda}^* = \boldsymbol{\lambda}^{(50)}$, which corresponds to assume complete convergence after 50 iterations.

Aiming to a reduced complexity, we have set the number L of pdfs involved in the mixture model to 3. This choice is motivated from the results in [12], where it has been shown that increasing L does not provide an improve in the reliability of the GMM.

We observe that the rate of convergence is different for r_λ and r_σ. In particular, r_λ converges to 0 after an higher number of iterations than r_σ. Furthermore, in both the equal power users and strong interferer scenarios, $k = 8$ iterations is a good trade-off between convergence and complexity. In fact, about 80% of the variations of r_λ occurs in the first 8 iterations.

We also remark that further simulations, not reported here, demonstrate that the rates of convergence, depicted in Figs. 1 and 2, are affected by the choice of the initial setting $\mathbf{N}^{(1)}$. However, limiting the number of iterations to $k = 8$ always results in a good trade-off.

V. Performance Analysis

In this section, the performance of the proposed MUI statistics estimator are demonstrated by means of the cumulative distribution function (cdf) of the noise.

Figure 3 shows the actual and estimated cdf, in the equal power users scenario. In particular, four estimates of the cdf are obtained by setting $L = 1, 2, 3, 4$. Since in the equal power users scenario the Gaussian approximation is reasonable, it is not surprising that increasing the order L of the GMM does not improve the estimator performance.

Nevertheless, in typical indoor environments the presence of strong interferers is highly probable. Focusing on the actual cdf curves depicted in Fig. 4 (continuous line) it is evident that in the strong interferer scenario the MUI has an impulsive behavior, since the cdf tail is lower compared with that of the equal power users scenario, confirming a well-known result.

Fig. 4 also compares the performance of the ML estimator with the GA $(\mathrm{ML} : L = 1)$ and the EM estimator with GMM $(\mathrm{EM} : L = 2, 3)$ in the strong interferer scenario. Due to the symmetry of cdfs, only a portion of the cdfs is depicted, thus allows easier curves comparison.

It is evident that the ML estimate is far from the actual cdf, since it has been derived with the GA, which, with the presence of a strong interferer, is no longer effective. On the other hand, the derived EM estimator obtains reliable estimates because the GMM fits the MUI statistics. Furthermore, satisfactory performances are achieved limiting the GMM order L to 3. This fact validates the choice of the number of iterations

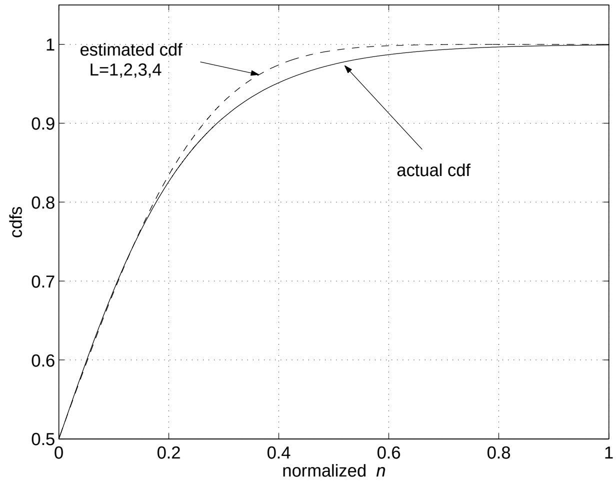

Fig. 3. Actual and estimated noise cdfs: *equal power users* scenario.

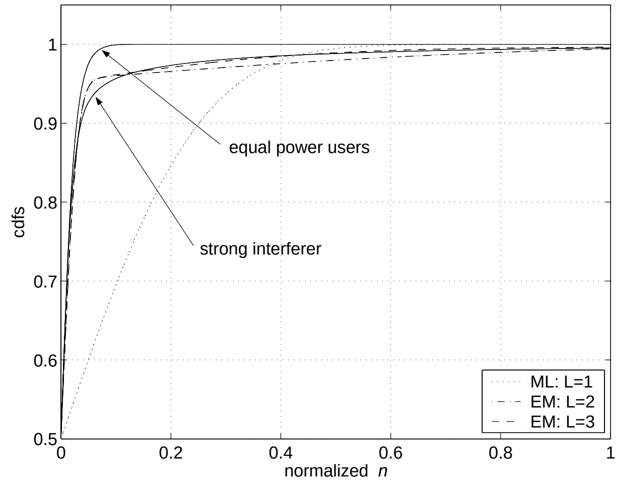

Fig. 4. Actual and estimated noise cdfs for different GMM order L.

$k = 8$, carried out in Section IV.

VI. Conclusions

In this paper, we have shown that the impulsive nature of multi-user interference is well characterized by the Gaussian mixture model, making the usually accepted Gaussian approximation no longer valid. Starting from this result, we have proposed an iterative multi-user interference statistics estimator for IR systems, based on the Gaussian mixture model and the Expectation Maximization algorithm. The derived estimator significantly outperforms conventional estimators based on the maximum likelihood approach, at the expense of a slight increase in the complexity.

The results presented in this paper may be exploited for the derivation of channel estimation and synchronization algorithms, which, relaxing the Gaussian approximation, promise a performance improvement compared to existing solutions. Presently, research is focused on an iterative joint channel and noise statistic estimator based on the Space Alternating Generalized Expectation Maximization (SAGE) algorithm.

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