

Advanced Channel Estimation Exploiting Second Order Statistics

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Abstract—MIMO transmission based on spatial multiplexing has been shown to offer great performance improvements and, therefore, will be included in future mobile radio systems. However, these very high performance gains require accurate channel state information either on the receiver [1] or on the transmitter side [2] or, even better, on both sides [3]. Unfortunately, the number of channel coefficients grows quadratically with the number of antennas on the transmitter and receiver sides whereas the channel capacity in the best case grows only linearly with the number of antennas [4]. Consequently, the performance improvement achievable by MIMO transmission will in practice be limited by the requirement to estimate the channel coefficients, which requires significant overhead for training signal transmission especially in high mobility scenarios. The present paper introduces new channel estimation techniques which can partially overcome this problem by exploiting second order channel statistics to improve the snapshot based channel estimates or equivalently to reduce the number of resources required for training signal transmission in order to achieve a certain channel estimation performance.

I. INTRODUCTION

State of the art channel estimation is based on the transmission of a priori known training signals, sometimes also referred to as pilots. In this case the received signal resulting from the training signal transmission is a known linear function of the unknown channel coefficients. These channel coefficients may be either thought of as samples of the channel impulse responses in the time domain, samples of the channel transfer functions in the frequency domain, or, more generally, as coefficients describing the channel properties with respect to a chosen set of basis functions. There is nothing special about the MIMO case except that in this case the channel coefficients stem from different physical channels. Training signals may be fed into all the inputs of the MIMO channel at the same time, although this does not offer significant improvements as compared to the case that the training signals are fed into the different inputs at disjoint time intervals, as the first case requires longer training signals in order to achieve the same channel estimation performance and consequently the total resource consumption for channel estimation is the same [5].

In the following the received vector $\underline{\mathbf{e}}(i)$ describes the received signal resulting from the i -th training signal transmission with respect to a set of basis functions, e.g., with respect to time shifted sinc-functions resulting in a vector of samples in the time domain. Using the system matrix $\underline{\mathbf{G}}$ and

the Gaussian noise vector $\underline{\mathbf{n}}(i)$, the received vector $\underline{\mathbf{e}}(i)$ can be written as a linear function of the vector $\underline{\mathbf{h}}(i)$ of channel coefficients valid at time instant i as follows:

$$\underline{\mathbf{e}}(i) = \underline{\mathbf{G}} \cdot \underline{\mathbf{h}}(i) + \underline{\mathbf{n}}(i). \quad (1)$$

For simplicity it is assumed that the same training signal is used for each training signal transmission i , i.e., the system matrix $\underline{\mathbf{G}}$ is independent of i .

With the covariance matrix

$$\underline{\mathbf{R}}_{\mathbf{n}} = \mathbf{E} \left\{ \underline{\mathbf{n}}(i) \underline{\mathbf{n}}(i)^{\mathbf{H}} \right\} \quad (2)$$

of the stationary Gaussian noise the optimum unbiased maximum likelihood snapshot based channel estimate is obtained as [6], [7]

$$\hat{\underline{\mathbf{h}}}(i) = \left(\underline{\mathbf{G}}^{\mathbf{H}} \underline{\mathbf{R}}_{\mathbf{n}}^{-1} \underline{\mathbf{G}} \right)^{-1} \underline{\mathbf{G}}^{\mathbf{H}} \underline{\mathbf{R}}_{\mathbf{n}}^{-1} \underline{\mathbf{e}}(i). \quad (3)$$

It should be noted that the maximum likelihood channel estimate only exists if the number of received values is at least as large as the number of unknown channel coefficients. Of course a luxury in the number of received values and thus in the length of the used training signals can improve channel estimation performance.

For the sake of simplicity we will focus our investigations on the estimation of the channel coefficients of one of the SISO subchannels of the MIMO channel. For the initial snapshot based channel estimation we will use unbiased maximum likelihood estimation, see (3). The results can be easily extended to MIMO channel estimation, which is mathematically the same as SISO channel estimation.

The rest of the paper is organized as follows: First the new concept of subspace based channel estimation is introduced. Basically it exploits second order channel statistics to determine the subspace in which the unknown vector $\underline{\mathbf{h}}(i)$ of channel coefficients lies. Alternatively one could exploit the knowledge of the second order statistics by the well known minimum mean square error channel estimator which is briefly recapitulated. The paper concludes with a performance comparison of the new subspace based channel estimation and the state of the art minimum mean square error channel estimation.

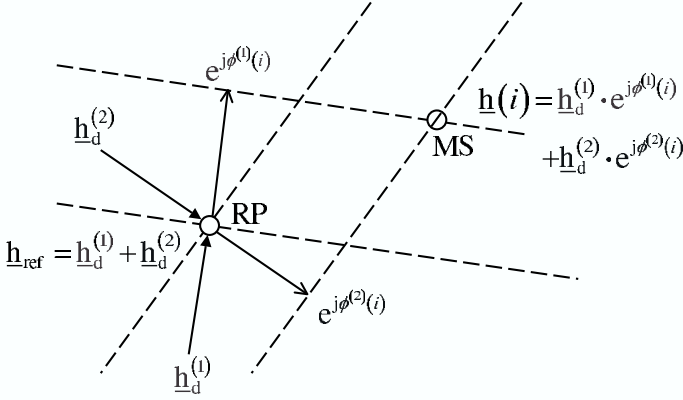


Fig. 1. Directional channels

II. SUBSPACE BASED CHANNEL ESTIMATION

A. Channel subspaces

1) *Time window*: In this example channel coefficients representing the time samples of the channel impulse response of a SISO channel are considered. Due to the a priori unknown access delay the dimension L of the channel impulse response $\underline{\mathbf{h}}(i)$ is usually chosen much larger than the excess delay W , i.e., the channel impulse response has the following structure:

$$\underline{\mathbf{h}}(i) = \begin{pmatrix} \underbrace{0 \dots 0}_{\text{access delay}} & \underbrace{h_W^{(1)}(i) \dots h_W^{(W)}(i)}_{\text{excess delay}} & 0 \dots 0 \end{pmatrix}^T. \quad (4)$$

In other words, the channel impulse response $\underline{\mathbf{h}}(i)$ lies in a W dimensional subspace of the L dimensional total space. With the shortened channel impulse response

$$\underline{\mathbf{h}}_W(i) = \left(h_W^{(1)}(i) \dots h_W^{(W)}(i) \right)^T \quad (5)$$

and the matrix $\underline{\mathbf{H}}_W(i)$ spanning the considered W -dimensional subspace one can write

$$\underline{\mathbf{h}}(i) = \underbrace{\begin{pmatrix} \mathbf{0} & & \\ 1 & & 0 \\ & \ddots & \\ 0 & & 1 \\ & \mathbf{0} & \end{pmatrix}}_{\underline{\mathbf{H}}_W(i)} \cdot \underline{\mathbf{h}}_W(i). \quad (6)$$

Interestingly, the subspace represented by $\underline{\mathbf{H}}_W(i)$ changes only slowly due to slow fading, i.e., changing propagation paths, if the transmitter or receiver moves, whereas the channel impulse response $\underline{\mathbf{h}}(i)$ and the shortened channel impulse response $\underline{\mathbf{h}}_W(i)$ change quickly due to fast fading.

2) *Directional channels*: Typically, the wavefronts impinge at the moving mobile station (MS) from rather few directions of arrival. In this example we consider the most simple case of two directions of arrival, see Fig. 1. The two radio channels corresponding to the two directions of arrival can be described by directional channel impulse responses $\underline{\mathbf{h}}_d^{(d)}$, $d = 1, 2$. At

the reference point (RP) the channel impulse response is just the sum

$$\underline{\mathbf{h}}_{\text{ref}} = \underline{\mathbf{h}}_d^{(1)} + \underline{\mathbf{h}}_d^{(2)} \quad (7)$$

of the two directional channel impulse responses. At a position not too far away from the reference point, i.e., at a position where the wavefronts are still the same as at the reference point, the channel impulse response is a superposition

$$\underline{\mathbf{h}}(i) = \underline{\mathbf{h}}_d^{(1)} \exp(j\phi^{(1)}(i)) + \underline{\mathbf{h}}_d^{(2)} \exp(j\phi^{(2)}(i)) \quad (8)$$

of the two directional channel impulse responses. The factors $\exp(j\phi^{(d)}(i))$, $d = 1, 2$, correspond to the steering factors well known from the theory of array antennas. For the more general case of D directions of arrival the superposition of (8) can be equivalently written as

$$\underline{\mathbf{h}}(i) = \underbrace{\left(\underline{\mathbf{h}}_d^{(1)} \dots \underline{\mathbf{h}}_d^{(D)} \right)}_{\underline{\mathbf{H}}_d} \cdot \underbrace{\begin{pmatrix} \exp(j\phi^{(1)}(i)) \\ \vdots \\ \exp(j\phi^{(D)}(i)) \end{pmatrix}}_{\underline{\mathbf{a}}(i)}. \quad (9)$$

It is important to notice that the matrix $\underline{\mathbf{H}}_d$ made up of the directional channel impulse responses $\underline{\mathbf{h}}_d^{(d)}$, $d = 1 \dots D$, and representing the subspace does not change if the mobile station only moves in a small area where the wavefronts do not change, whereas the channel impulse response $\underline{\mathbf{h}}(i)$ and the steering vector $\underline{\mathbf{a}}(i)$ change quickly due to fast fading.

3) *Generalized mathematical model*: The previous examples show that the channel vector $\underline{\mathbf{h}}(i)$ typically lies in a rather low dimensional subspace. In general, using the tall $L \times D$ matrix $\underline{\mathbf{H}}_S(i)$ spanning the subspace, one can write the channel vector $\underline{\mathbf{h}}(i)$ of dimension L as a function of the subspace based channel vector $\underline{\mathbf{h}}_S(i)$ of dimension D as follows:

$$\underline{\mathbf{h}}(i) = \underline{\mathbf{H}}_S(i) \cdot \underline{\mathbf{h}}_S(i). \quad (10)$$

This general model includes the two previous examples as special cases. Substituting (10) into (1) one obtains the subspace based linear system model

$$\underline{\mathbf{e}}(i) = \underline{\mathbf{G}} \cdot \underline{\mathbf{H}}_S(i) \cdot \underline{\mathbf{h}}_S(i) + \underline{\mathbf{n}}(i). \quad (11)$$

It is important to notice that typically the matrix $\underline{\mathbf{H}}_S(i)$ changes only slowly over the time. Typically, L is much larger than D .

B. Concept of subspace based channel estimation

The basic idea of subspace based channel estimation is that in a certain snapshot i only the subspace based channel vector $\underline{\mathbf{h}}_S(i)$ needs to be estimated, which typically results in a significant reduction of the number of unknowns to be estimated as compared to the conventional snapshot based channel estimator which directly estimates the channel vector $\underline{\mathbf{h}}(i)$.

Fig. 2 shows the resulting block diagram of a subspace based channel estimator. In a first step a low quality initial snapshot based channel estimate

$$\hat{\underline{\mathbf{h}}}(i) = \left(\underline{\mathbf{G}}^H \underline{\mathbf{R}}_n^{-1} \underline{\mathbf{G}} \right)^{-1} \underline{\mathbf{G}}^H \underline{\mathbf{R}}_n^{-1} \underline{\mathbf{e}}(i) \quad (12)$$

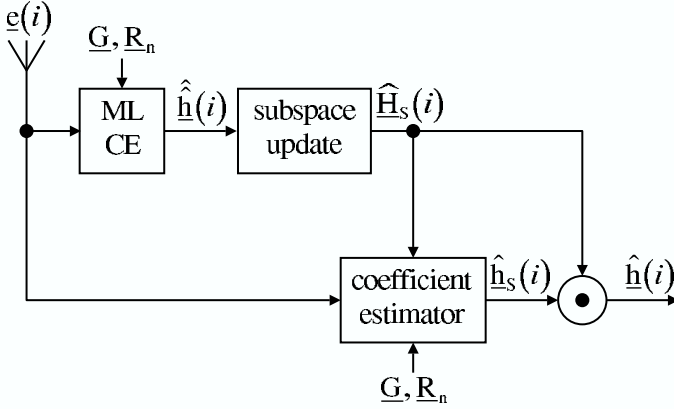


Fig. 2. Block diagram of subspace based channel estimator

is obtained from the received signal $\underline{e}(i)$ at time instant i by conventional maximum likelihood channel estimation. By observing these initial channel estimates $\hat{\underline{h}}(i)$ over a longer time one can estimate the subspace represented by the matrix $\underline{\mathbf{H}}_S(i)$, more precisely the subspace estimate $\hat{\underline{\mathbf{H}}}_S(i)$ is updated at each time instant i using the past I initial channel estimates $\hat{\underline{h}}(j)$, $j = i - I + 1 \dots i$. In the following I is also called the observation duration. This allows a tracking of slowly time variant subspaces. Due to the possibility to observe the channel for a longer time for subspace estimation the quality of the initial channel estimates $\hat{\underline{h}}(i)$ is not that critical. Using the subspace estimate $\hat{\underline{\mathbf{H}}}_S(i)$ one can estimate the subspace based channel vector $\hat{\underline{\mathbf{h}}}_S(i)$ at each time instant i . Following the maximum likelihood rationale [8],

$$\hat{\underline{\mathbf{h}}}_S(i) = \left(\hat{\underline{\mathbf{H}}}_S^H(i) \underline{\mathbf{G}}^H \underline{\mathbf{R}}_n^{-1} \underline{\mathbf{G}} \hat{\underline{\mathbf{H}}}_S(i) \right)^{-1} \hat{\underline{\mathbf{H}}}_S^H(i) \underline{\mathbf{G}}^H \underline{\mathbf{R}}_n^{-1} \underline{\mathbf{e}}(i) \quad (13)$$

holds. Finally, the estimate

$$\hat{\underline{\mathbf{h}}}(i) = \hat{\underline{\mathbf{H}}}_S(i) \cdot \hat{\underline{\mathbf{h}}}_S(i) \quad (14)$$

of the channel vector $\underline{\mathbf{h}}(i)$ at time instant i is obtained.

C. Subspace estimation

For the covariance matrix of the channel vector $\underline{\mathbf{h}}(i)$

$$\begin{aligned} \underline{\mathbf{R}}_h(i) &= \mathbb{E} \left\{ \underline{\mathbf{h}}(i) \underline{\mathbf{h}}(i)^H \right\} \\ &= \underline{\mathbf{H}}_S(i) \mathbb{E} \left\{ \underline{\mathbf{h}}_S(i) \underline{\mathbf{h}}_S(i)^H \right\} \underline{\mathbf{H}}_S(i)^H \end{aligned} \quad (15)$$

holds, i.e., the subspace spanned by the columns of the matrix $\underline{\mathbf{H}}_S(i)$ is equal to the subspace spanned by the covariance matrix $\underline{\mathbf{R}}_h(i)$. As we are only interested in any set of basis vectors of the subspace it is sufficient to find one orthonormal basis of the subspace spanned by the covariance matrix $\underline{\mathbf{R}}_h(i)$. In practical applications the covariance matrix $\underline{\mathbf{R}}_h(i)$ is not known but has to be estimated based on the initial channel

estimates $\hat{\underline{\mathbf{h}}}(i)$ as follows:

$$\begin{aligned} \underline{\mathbf{R}}_h(i) &\approx \mathbb{E} \left\{ \hat{\underline{\mathbf{h}}}(i) \hat{\underline{\mathbf{h}}}(i)^H \right\} \\ &= \mathbb{E} \left\{ \underline{\mathbf{h}}(i) \underline{\mathbf{h}}(i)^H \right\} + \left(\underline{\mathbf{G}}^H \underline{\mathbf{R}}_n^{-1} \underline{\mathbf{G}} \right)^{-1} \\ &\approx \frac{1}{I} \sum_{j=i-I+1}^i \hat{\underline{\mathbf{h}}}(j) \hat{\underline{\mathbf{h}}}(j)^H. \end{aligned} \quad (16)$$

One can clearly see that the noise $\underline{\mathbf{n}}(i)$ is not really disturbing the subspace estimation as its influence can be eliminated, provided that its covariance matrix $\underline{\mathbf{R}}_n$ is known and that one can average over a sufficiently large number I of initial channel estimates $\hat{\underline{\mathbf{h}}}(i)$.

III. MINIMUM MEAN SQUARE ERROR CHANNEL ESTIMATION

Subspace based channel estimation basically exploits the knowledge of the covariance matrix $\underline{\mathbf{R}}_h(i)$ of (15) to first estimate the subspace and then estimate the channel vector $\underline{\mathbf{h}}(i)$. Alternatively the knowledge of the covariance matrix $\underline{\mathbf{R}}_h(i)$ of the channel vector could be exploited by minimum mean square error channel estimation. Using the covariance matrix $\underline{\mathbf{R}}_h(i)$ of the channel coefficients one obtains the minimum mean square error channel estimate as [8]

$$\hat{\underline{\mathbf{h}}}(i) = \underline{\mathbf{R}}_h \underline{\mathbf{G}}^H \left(\underline{\mathbf{G}} \underline{\mathbf{R}}_h \underline{\mathbf{G}}^H + \underline{\mathbf{R}}_n \right)^{-1} \underline{\mathbf{e}}(i). \quad (17)$$

IV. PERFORMANCE INVESTIGATIONS

A. Performance of conventional channel estimation

Before going into the details of the performance investigation of subspace based channel estimation, one should briefly review the performance of conventional maximum likelihood channel estimation of (3), which constitutes the reference. For the sake of simplicity, white Gaussian noise is assumed in the following, i.e., the covariance matrix of (2) reads

$$\underline{\mathbf{R}}_n = \sigma^2 \mathbf{I}. \quad (18)$$

As conventional maximum likelihood channel estimation yields unbiased channel estimates $\hat{\underline{\mathbf{h}}}(i)$, the only disturbances result from the Gaussian noise $\underline{\mathbf{n}}(i)$ and are themselves Gaussian distributed. The mean square error of the estimate of the l -th channel coefficient $\hat{h}^{(l)}(i)$ in the case of maximum likelihood channel estimation reads

$$\mathbb{E} \left\{ \left| \hat{h}^{(l)}(i) - h^{(l)}(i) \right|^2 \right\} = \sigma^2 \left[\left(\underline{\mathbf{G}}^H \underline{\mathbf{G}} \right)^{-1} \right]_{l,l}. \quad (19)$$

The signal-to-noise-ratio (SNR) of the maximum likelihood channel estimate of the l -th channel coefficient $\hat{h}^{(l)}(i)$ reads

$$\gamma_{\text{ML}}^{(l)}(i) = \frac{\left| h^{(l)}(i) \right|^2}{\sigma^2 \left[\left(\underline{\mathbf{G}}^H \underline{\mathbf{G}} \right)^{-1} \right]_{l,l}}, \quad (20)$$

which is smaller by the SNR-degradation [6]

$$\delta_{\text{ML}}^{(l)} = \left[\underline{\mathbf{G}}^H \underline{\mathbf{G}} \right]_{l,l} \left[\left(\underline{\mathbf{G}}^H \underline{\mathbf{G}} \right)^{-1} \right]_{l,l} \quad (21)$$

than the optimum SNR

$$\gamma_{\text{MF}}^{(l)}(i) = \frac{|\hat{h}^{(l)}(i)|^2}{\sigma^2} [\mathbf{G}^H \mathbf{G}]_{l,l}, \quad (22)$$

which could be obtained by biased matched filter channel estimation. By proper training signal design the performance losses $\delta_{\text{ML}}^{(l)}$, $l = 1 \dots L$, due to the need to eliminate biases can be avoided [6]. For optimized training signals

$$\delta_{\text{ML}}^{(l)} = 1, \quad l = 1 \dots L \quad (23)$$

holds, i.e., $\mathbf{G}^H \mathbf{G}$ is a diagonal matrix. For reasons of fairness we will furthermore require that the same transmitted energy, i.e., energy one, is available for the estimation of each channel coefficient $\hat{h}^{(l)}(i)$, $l = 1 \dots L$. In the following we will use the term optimum training signals for training signals satisfying

$$\mathbf{G}^H \mathbf{G} = \mathbf{I}. \quad (24)$$

B. Performance of subspace based channel estimation

Here again white Gaussian noise is assumed, see (18). In the case of perfectly known subspace the channel estimates $\hat{\mathbf{h}}(i)$ obtained by subspace based channel estimation are unbiased. The mean square error of the estimate of the l -th channel coefficient $\hat{h}^{(l)}(i)$ in the case of subspace based channel estimation reads

$$\begin{aligned} \mathbb{E} \left\{ \left| \hat{h}^{(l)}(i) - h^{(l)}(i) \right|^2 \right\} &= \sigma^2 \\ &\cdot \left[\mathbf{H}_S(i) \left(\mathbf{H}_S(i)^H \mathbf{G}^H \mathbf{G} \mathbf{H}_S(i) \right)^{-1} \mathbf{H}_S(i)^H \right]_{l,l}. \end{aligned} \quad (25)$$

The SNR can be easily calculated as

$$\begin{aligned} \gamma_S^{(l)}(i) &= \frac{|\hat{h}^{(l)}(i)|^2}{\sigma^2} \\ &\cdot \frac{1}{\left[\mathbf{H}_S(i) \left(\mathbf{H}_S(i)^H \mathbf{G}^H \mathbf{G} \mathbf{H}_S(i) \right)^{-1} \mathbf{H}_S(i)^H \right]_{l,l}}. \end{aligned} \quad (26)$$

As can be easily verified this SNR is invariant to the choice of the basis vectors of the subspace included in the matrix $\mathbf{H}_S(i)$. The resulting SNR-degradation reads

$$\begin{aligned} \delta_S^{(l)}(i) &= [\mathbf{G}^H \mathbf{G}]_{l,l} \\ &\cdot \left[\mathbf{H}_S(i) \left(\mathbf{H}_S(i)^H \mathbf{G}^H \mathbf{G} \mathbf{H}_S(i) \right)^{-1} \mathbf{H}_S(i)^H \right]_{l,l}, \end{aligned} \quad (27)$$

which now can also be smaller than one, i.e., the SNR of the channel estimates $\hat{\mathbf{h}}(i)$ obtained by subspace based channel estimation can be higher than the SNR of the channel estimates $\hat{\mathbf{h}}(i)$ obtained by matched filter estimation. In the special case

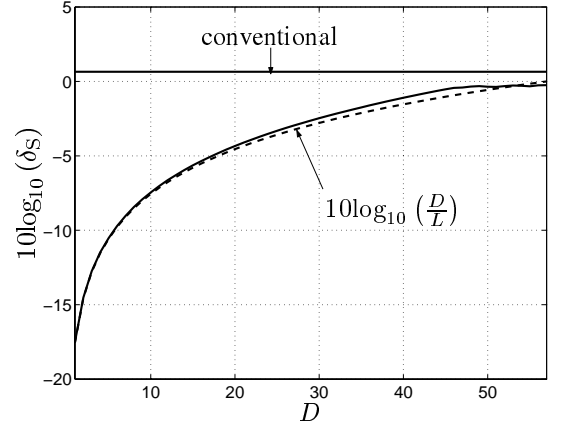


Fig. 3. Performance with perfectly known subspace

of optimum training signals (24), the SNR-degradation of (27) reads

$$\delta_S^{(l)}(i) = \left[\mathbf{H}_S(i) \left(\mathbf{H}_S(i)^H \mathbf{H}_S(i) \right)^{-1} \mathbf{H}_S(i)^H \right]_{l,l}, \quad (28)$$

i.e., it is a function of the matrix $\mathbf{H}_S(i)$ describing the subspace alone.

C. Performance of minimum mean square error channel estimation

Minimum mean square error channel estimation yields biased channel estimates even in case of perfect knowledge of the covariance matrix $\mathbf{R}_h(i)$ of the channel vector. Thus its performance can only be characterized by the mean square error $\mathbb{E} \left\{ \left| \hat{h}^{(l)}(i) - h^{(l)}(i) \right|^2 \right\}$ of the estimate of the l -th channel coefficient $\hat{h}^{(l)}(i)$ and a performance measure like the SNR-degradation which is independent of the actual noise variance and SNR does not exist.

D. Numerical results

The SNR-degradation of (27) can be numerically evaluated for given scenarios. In the following a UMTS like scenario characterized by

- $K = 8$ transmitters and one receiver, i.e., a MISO subchannel of the MIMO channel,
- chip duration 244ns,
- channel impulse response length $L = 57$,
- D directions of arrival per transmitter,
- constant envelope low SNR-degradation training signals derived from the basic midamble code of hexadecimal representation

C482462CA7846266060D21688BA00B72E1EC84
A3D5B7194C8DA39E21A3CE12BF512C8AAB6A70
79F73C0D3E4F40AC555A4BCC453F1DFE3F6C82

by the method described in [6], and

- random directional channel impulse responses $\mathbf{h}_d^{(d)}$, $d = 1 \dots D$, with power delay spectrum according to the COST 207 typical urban model [9]

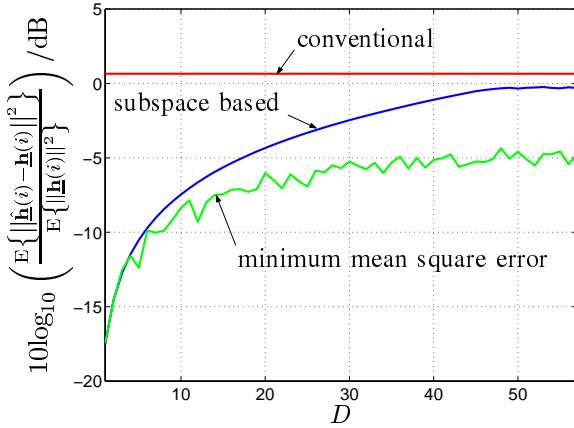


Fig. 4. Performance comparison of subspace based channel estimation and minimum mean square error channel estimation, $E\{\|\underline{h}(i)\|^2\} = 1$, $\sigma^2 = 1$

is investigated. The constant envelope training signals are optimized in such a way that the SNR-degradations $\delta_{ML}^{(l)}$, $l = 1 \dots L$, of (21) of maximum likelihood channel estimation are all equal and take a small value of $\delta_{ML} = 1.1613$. The directional channel impulse responses $\underline{h}_d^{(d)}$, $d = 1 \dots D$, span the subspace. The dimension of the subspace will be smaller than D if the directional channel impulse responses $\underline{h}_d^{(d)}$, $d = 1 \dots D$, are linearly dependent. Fig. 3 shows the SNR-degradation

$$\delta_S = E\{\delta_S^{(l)}(i)\} \quad (29)$$

averaged over the channel coefficients $l = 1 \dots L$ and the random channel as a function of the number D of directions of arrival. The smaller the number D of directions of arrival, the smaller the SNR-degradation δ_S will be due to the fact that the dimension of the subspace is at most D . As D goes to infinity the SNR-degradation δ_S converges to the SNR-degradation of maximum likelihood channel estimation. Interestingly, with the number L of the channel coefficients $\underline{h}^{(l)}(i)$

$$\delta_S \approx \frac{D}{L} \quad (30)$$

holds in good approximation for the average SNR-degradation of subspace based channel estimation, i.e., the improvement as compared to conventional maximum likelihood channel estimation is roughly $\frac{L}{D}$. This is just approximately the factor by which the dimension D of the subspace is smaller than the dimension L of the full space.

The performances of subspace based channel estimation and minimum mean square error channel estimation are compared using the same UMTS like scenario as described above. Additionally the random channel impulse responses $\underline{h}(i)$ are normalized in such a way that their average energy is one. Fig. 4 depicts the average mean square error $E\{\|\underline{\hat{h}}(i) - \underline{h}(i)\|^2\}$ of the estimated channel impulse responses as a function of the number D of directions of arrival for a noise variance

$$\sigma^2 = 1. \quad (31)$$

The price to be paid for the unbiasedness of the subspace based estimates is an increase of the mean square error $E\{\|\underline{\hat{h}}(i) - \underline{h}(i)\|^2\}$ as compared to the minimum mean square error estimate. Especially in the typical case of small numbers D of directions of arrival this price to be paid for the unbiasedness is rather small.

V. CONCLUSION

The paper presents the basic principles of a novel subspace based channel estimation technique which can improve the performance of channel estimation significantly as compared to conventional maximum likelihood channel estimation. The performance gains stem from the exploitation of long term channel properties in form of the subspace in which the channel vector lies. In contrast to minimum mean square error estimation the novel subspace based channel estimation technique delivers unbiased estimates. Future work may focus on subspace estimation and the influence of improved channel estimation techniques on the system performance.

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