Some General Properties of the Covariation Matrix for MIMO Communication Channels

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***Abstract* – It is proposed the use of the *prolate spheroidal wave* functions (PSWF) as a universal basis in order to represent the covariation matrix function for the Multiple Input – Multiple Output (MIMO) communication channel.**

Keywords – Channel covariation matrix, universal basis, eigenvalues and eigenfunctions.

I. Introduction.

The emerging interest for MIMO channel modeling and simulating has caused a strong attention to the problems of channel covariance matrix calculations. There are numerous papers already published where those problems are considered [1–5]. But to the best of our knowledge, the methodology, the calculus and the general properties of the covariation matrix for MIMO channel are still not well stated (see e. g. [6]) and are directly related to the interpretation of the problems of scatters reflection and refraction phenomenology of clusters of scatters and its influence to the properties of the received signal.

The way to represent the system characteristics of the channel by means of a finite set of artificial trajectories based on the use of the Karhunen–Loève integral Equation (KLE), was successfully applied for the scalar Single Input – Single Output (SISO) case (see [7–8] and references therein). The attempt to apply the same approach was presented by the authors at [9] for MIMO channels.

It was shown at [7] that for the scalar case (SISO) the effective way to avoid the solution of the KLE is to apply a *universal basis*[1] for the representation of the system characteristics of the channel, never mind if their covariation functions are separable or not by its arguments "t" and "τ". At [10–11] were proposed some basis for the representation of the MIMO channel matrix **H** in order to simplify the model for a large number of inputs and outputs.

Hereafter we will tackle the problem on how to calculate the correlation matrix for **H** and what can be a universal basis for the MIMO case. It can also be considered in the same sense as in [10–11]. A matrix–valued wavelet expansion [12] can be used as well.

It is worth to mention here that practically the same problems for the covariation matrix calculation exist at the radar theory (see e. g. [13–14]), where some basic properties for the covariance matrix were proposed, and is feasible to apply them for the MIMO channel covariance matrix calculation. The purpose of this paper is also to evoke those methods. Hereafter we do not consider polarization effects (only the horizontal plane, i. e., a bi–dimensional model), and all the assumptions made here are the traditional for the macro– and microcell wireless systems cases [1–2].

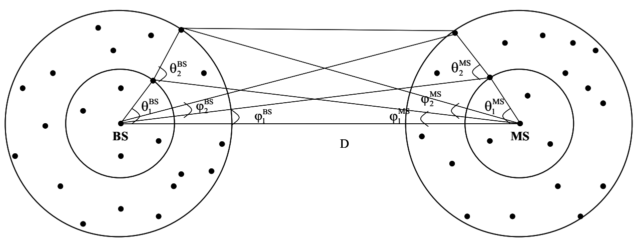

Fig. 1. The geometrically–based MIMO channel model.

We take into account the Geometrically–Based Wide–Sense Stationary Uncorrelated Scattering (GBWSSUS) MIMO channel model (see e. g., [15–17] and references therein), which is depicted for the general case at Fig. 1. In the model it is distinguished two kinds of scatters: the local scatters, which are confined into a circular region at the surrounding of the MS (BS); and the dominant scatters, which are in the circular ring regions[2] (as depicted in Fig. 1). The matrix impulse response for this channel is $\mathbf{H}(t, \tau, \boldsymbol{\theta})$ with scalar components [2, 18–19]:

$$\hat{h}_{m,n}(t,\tau,\theta) = \sum_{k=1}^{K_{m,n}} \hat{g}_k^{m,n}(t,\theta)\delta(\tau-\tau_k^{m,n}), \quad (1)$$

where $\hat{g}_k^{m,n}(t, \theta)$ are the complex amplitudes received from the k–th cluster and $K_{m,n}$ is the number of clusters.

Here we consider concretely one special case taken form Fig. 1, which is depicted at Fig. 2. Each summand in (1) can be represented in the following way, if at the receiver point one use the *l*–th element of the linear antenna array[3] (see Fig. 2) (here the δ(·) components are skipped):

[1] The idea behind a universal basis is to get a proper orthogonal representation of the channel impulse response, which is invariant to the channel covariation matrix (contrary to what happens with KLE). This is why we said that this basis is universal, but for sure it is not unique.

[2] For generality, we consider both types of scatters at both system link ends.

[3] The generalization for other types of antennas can be done as well.

$$\hat{h}^{k}_{m,n}(t,\boldsymbol{\eta}_l) = \frac{\sqrt{P_0}}{4\pi|\mathbf{r}_k|^{\alpha}} G_T(\varphi_k - \varphi_0) G_R(\theta_k - \theta_0)\cdot$$
$$\cdot\,\hat{g}^{m,n}_{k}(t)\, e^{2\pi j (x_l/\lambda)\sin\theta_k} e^{j\nu^{m,n}_{k} t}, \qquad (2)$$

where P_0 is the average power at the transmitter, $1 \le \alpha \le 3$ is the propagation exponent, φ_0 and θ_0 are the steering Angle of Departure (AoD) and Angle of Arrival (AoA) for the transmitter and receiver antennas, respectively. $G_T(\cdot)$ and $G_R(\cdot)$ are the antenna patterns for the transmitter and receiver, respectively. $\hat{g}_k^{m,n}(t)$ is the received complex amplitude, $\mathbf{r}_k$ is the distance vector for the k–th cluster, x_l is the position of the l–th antenna element referred to the geometrical center of the linear antenna array, $\boldsymbol{\eta}_l$ is its radii–vector, $\nu_k^{m,n}$ is the Doppler shift provided by the MS movement. Note that if $|\varphi_k - \varphi_0|$ are rather small for every k, then $G_T(0) \approx$ *const*, and (2) can be simplified in the following way:

$$\hat{h}^{k}_{m,n}(t,\boldsymbol{\eta}_l) = \mathrm{const}\cdot\sqrt{P_k}\, G_R(\theta_k)\hat{g}^{m,n}_{k}(t)\, e^{2\pi j (x_l/\lambda)\sin\theta_k} e^{j\nu^{m,n}_{k} t}, \qquad (3)$$

where, without any lost in generality, we have defined $\theta_k = \theta_k - \theta_0$, and $P_k = P_0 / (4\pi|\mathbf{r}_k|)^{\alpha}$.

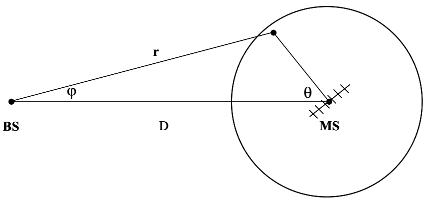

Fig. 2. A single bounce geometrical scenario.

The covariance function $b^k_{l,\rho}$ of $\hat{h}^k_{m,n}$ for two antenna elements l and ρ, is $b^k_{l,\rho} = \langle(\hat{h}^k_{m,n})\,(\hat{h}^k_{m,n})^*\rangle$, where l, ρ = 1, 2, …, K, and K is the number of antenna elements.

II. Structure of the Covariation Matrix.

Let us consider two elements "l" and "ρ" for a linear antenna, then the matrix covariance element $b^k_{l,\rho}$ will stand only for space components[4] as:

$$b^k_{l,\rho} = \mathrm{const}\cdot \int_{-\frac{1}{2}\Delta}^{\frac{1}{2}\Delta} |G_R(\theta)|^2 W_\theta(\theta) e^{(2\pi j/\lambda)(x_l - x_\rho)\sin\theta} d(\sin\theta). \qquad (4)$$

From (4) it follows that $b^k_{l,\rho}$ are the Fourier coefficients for $|G_R(\theta)|^2\, W_\theta(\theta)$, which is defined for $\theta_{min} \le \theta \le \theta_{max}$, and $W_\theta(\theta)$ is the AoA Probability Density Function (PDF), and $\Delta\theta = |\theta_{max} - \theta_{min}|$.

The structure of the matrix $\mathbf{B} = [b^k_{l,\rho}]$ depends on the selection of the basis of element representation[5], but taking into account that Δθ is a limited and rather narrow angle, the eigenvalues for this matrix has an *isolated group of large values* [13]. Although **B** can be actually approximated in a subspace which dimension is equal to the one of this isolated group. Hence the *effective rank* (r_{eff}) of the matrix **B** can be evaluated from [14], where this problem was tackled by applying functions with *double orthogonality*, concretely the PSWF as the basis (see [20]). At [13] it was shown that:

$$r_{eff} = \lfloor c_0/(2\pi)\rfloor + 1, \qquad (5)$$

where $\lfloor\cdot\rfloor$ is the integer part operator, and c_0 is a parameter which can be found from the antenna aperture's normalization: $\Delta z_{l,\rho} = 2\,(x_\rho - x_l)\,/\,D_a$, then [13] $c_0 = (\pi\, D_a / \lambda)\cdot(\varphi_j - \varphi_0)$, and

$$r_{eff} = \lfloor(\varphi_j - \varphi_0)D_a/(2\lambda)\rfloor + 1, \qquad (6)$$

where D_a is the dimension of the linear antenna array.

When the argument of $\lfloor\cdot\rfloor$ is a small value, then $r_{eff} \approx 1$, and it happens for many practical cases. It is well known (see [14]), that when $\Delta\theta/\delta\theta \gg 1$, where $\delta\theta \approx 1/D_a$, the PSWF can be approximated by *sinc*(·) functions, and r_{eff} represents the number of uncorrelated samples in the antenna aperture space domain. With these considerations, asymptotically form (4) one gets:

$$b^k_{l,\rho} = \mathrm{const}\cdot \mathrm{sinc}\left[(x_l - x_\rho)\cos\theta\right] e^{(2\pi j/\lambda)(x_l - x_\rho)\sin\theta}, \qquad (7)$$

which naturally coincides with [14, 21]. It can be seen from (4) that for Δθ small, $[b^k_{l,\rho}]$ can be represented only by one eigenfunction with $\theta \approx \langle\theta\rangle$. Here *const* means the product of all other parameters multiplied by Δθ.

Though in the general case, the matrix **B** in its own basis can be represented in a subspace with the span r_{eff} as in [13]:

$$\mathbf{B} \approx \mathbf{V}\Lambda\mathbf{V}^H, \qquad (8)$$

where $[\mathbf{V}_1]$, $[\mathbf{V}_2]$, …, $[\mathbf{V}_r]$ are the column matrix (K x r_{eff}) of eigenvectors, Λ is a diagonal matrix of the isolated group of eigenvalues with rank r_{eff}. The properties of the PSWF depend only on the antenna parameters and qualitatively, the representation of **B** in the PSWF basis in the form given by (8) can be considered as a universal basis. For scalar time–dependent channels only, these universal bases were proposed in [7, 22].

When the *sinc*(·) function in (7) is approximately the unity (i. e., $(x_\rho - x_l)\, cos(\theta) \to 0$), then:

$$b^k_{l,\rho} = \mathrm{const}\cdot e^{(2\pi j/\lambda)(x_l - x_\rho)\sin\theta}, \text{ and} \qquad (9)$$

$$B = \mathrm{const}\cdot \mathbf{V}(\theta_0)\mathbf{V}^H(\theta_0), \qquad (10)$$

where $\mathbf{V}(\theta_0)$ is the steering linear array antenna vector with components . Certainly (9) and (10) are well known and can be generalized as it is done in [4] for any type of antenna array, with the appropriate form of the steering vector. It happens in the asymptotic case that for any argument of the PSWF, which depends on the antenna geometry, the *sinc*(·) function tendency behavior preserves (as it was shown beforehand) and formula (10) will be valid, but the components for the (K+1) steering vector at (10) will be obviously different.

Beforehand we assume that the AoDs are relatively narrow, what it is not always the real scenario. For example, for the scenario depicted in Fig. 3, the AoD

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[4] Here are usually taken the assumptions for factorization of space–time covariation matrixes (see [1–2, 13]). The part of the component of $b^k_{l,\rho}$ which depends on the time variation can be found at [14, 29].

[5] The simplest way for its representation is by doing it in its proper basis, but it is a complex problem [13], e. g., from [4] it can be seen that a representation can be done in a Bessel functions basis, but a series expansion in this basis converges very slowly (see [30]). Meanwhile when $|G_R(\theta)|^2 \approx$ *const* and $W_\theta(\theta) = 1/2\pi$, from (4) follows the well know Jakes' model.

which is $\Delta\theta = \theta_1^{BS} + \theta_2^{BS}$ is rather large as illumination is provided for both rings of scatters, $G_T(\theta_k - \theta_0)$ have to be considered and the integral in (4) can be simplified in the way:

$$b^k_{l,\rho} \approx const \cdot \int_{\Theta} |G_T(\theta)|^2 W_\theta(\theta) d\theta \cdot$$
$$\cdot Re\left\{ \int_{\Psi} |G_R(\varphi)|^2 W_\varphi(\varphi) e^{(2\pi j/\lambda)(x_l - x_\rho)\sin\varphi} d(\sin\varphi) \right\}. \quad (11)$$

As the first term is a constant, which does not depends on φ, one can see that the main properties of $[b^k_{l,\rho}]$ discussed at section II, as still valid here as well.

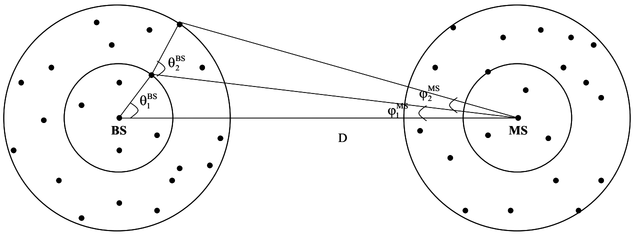

Fig. 3. Local and dominant scatters reflection scenario.

Regarding the case of $r_{eff} \approx 1$, it is necessary to dedicate a few words to the maximum eigenvalue λ_{max}. It is worth to mention that formally, $\mathbf{B}(\mathbf{Q}) \leftrightarrow \mathbf{G}(\omega)$ by means of a Fourier transform and if $\Lambda_0 = [0, T] \times [0, T] \times [0, R]$ is the space of consideration, then asymptotically [14] :

$$\lambda = \int_{-\infty}^{\infty} \mathbf{B}(\mathbf{Q}) e^{2\pi j \mathbf{Q}\omega} d\mathbf{Q} = \mathbf{G}(\omega), \text{ and}$$
$$\lambda_{max} = \max_{\omega} \mathbf{G}(\omega). \quad (12)$$

here $\mathbf{Q}$ is a vector argument for $\mathbf{B}(\cdot)$ and ω is a vector argument for its Fourier transform. Other methods for the estimation of λ_{max} can be found at the theory of matrixes (see e. g. [23]). (For the proof of (12) see Appendix A.)

The expression (12) shows how the maximum eigenvalue of $\mathbf{B}(\cdot)$ can be evaluated through the Power Density Spectrum (PDS), useful when $r_{eff} \approx 1$, and that asymptotically the covariation matrix $\mathbf{B}$ has a continuum spectrum of eigenfunctions and eigenvalues, and not the discrete ones.

III. More about the Structure of the Covariation Matrix.

Let us consider the scenario depicted at Fig. 3, supposing that the MS occupy the position of the BS and viceversa. This assumption gives us an opportunity to introduce the mobility of the MS together with a more general case of the scattering scenario: the MS mobility was considered at [24], assuming only the movement of the geometrical center of the MS scattering ring.

Here we will consider a different model: we will neglect the changes in $\mathbf{r}_k$ and the instant changes in the Doppler shift ν_k, but we will take into account the changes in the cluster scenario with the MS movement. The attempt to take all above mentioned phenomena into account is done in [25] and is partially investigated in [26] as a Spatial Channel Model (SCM). Here we make some further simplifications to the SCM to get an analytical solution.

The movement of the MS will include not only changes in the number of active clusters, but changes of the types of clusters: disappearing dominant (local) clusters and appearing new local (dominant) ones. It can be modeled by assuming that the MS randomly encounter both of them, being in different states and applying only local scatters or only dominant ones with a *fast change*[6] of these states. In the first state the BS will see the angle φ_1^{BS} and in the second $\varphi_1^{BS} + \varphi_2^{BS}$ (see Fig. 3)[7].

Taking into account the MS movement, both of those angles are stochastic processes. Such kind of scenarios can be successfully modeled by the models with random structure (see [27, chp. 6]). Supposing that both angles have approximately Gaussian distributions in the linear aperture, one can use the results of example [27, pp. 312].

The essence of the modeling of the above mentioned scenario with high intensity of the changes of those two states is to create a new stochastic process $\varphi_\infty^{BS}(t)$ which has two components: $\varphi_1^{BS}(t)$ and $\varphi_2^{BS}(t)$ with known one–dimensional distributions $W_\varphi(\varphi_1^{BS})$ and $W_\varphi(\varphi_2^{BS})$ (Gaussian in our case). Using the results of the example in [27, chp. 6], it is easy to obtain the two–dimensional PDF for $\varphi_\infty^{BS}(t)$ as:

$$W(\varphi_\infty^{BS}, \varphi_{\infty_\tau}^{BS}) = W_{\varphi_\infty^{BS}}(\varphi_\infty^{BS}) W_{\varphi_{\infty_\tau}^{BS}}(\varphi_{\infty_\tau}^{BS}) \cdot$$
$$\sum_{n=0}^{\infty} \frac{1}{2^n n!} H_n\left(\frac{\varphi_\infty^{BS}}{\sqrt{2}\sigma_\infty}\right) H_n\left(\frac{\varphi_{\infty_\tau}^{BS}}{\sqrt{2}\sigma_{\infty_\tau}}\right) e^{-\alpha_\infty n|\tau|}, \quad (13)$$

where $\sigma_\infty^2 = ½ (\sigma_1^2 + \sigma_2^2)$, $\alpha_\infty = 2\nu p_1$, and $p_1 = p_2$ are the final probabilities for the states *one* and *two*, where $W(\varphi_\infty^{BS})$ is a zero mean, and σ_∞^2 variance normal distribution, and $H_n(x)$ are the *Hermite polynomials*.

Now applying (13) for the calculus of the covariation term $<(\hat{h}^k_{m,n}(t_1, \eta_l))(\hat{h}^k_{m,n}(t_2, \eta_\rho))^*>$ for the antenna elements l and ρ (see sections I and II), one can obtain:

$$b^k_{l,\rho} = const \cdot b^k_{l,\rho_\tau} \sum_{n=0}^{\infty} \frac{e^{-\alpha_\infty n|\tau|}}{2^n n!} \int_{\varphi_{\infty_\tau}^{BS}} W_{\varphi_{\infty_\tau}^{BS}}(\varphi_{\infty_\tau}^{BS}) H_n(\varphi_{\infty_\tau}^{BS}) d\varphi_{\infty_\tau}^{BS} \cdot$$
$$\int_{\varphi_\infty^{BS}} |G_R(\varphi_\infty^{BS})|^2 H_n(\varphi_\infty^{BS}) e^{(2\pi j/\lambda)(x_l - x_\rho)\sin\varphi_\infty^{BS}} \cos\varphi_\infty^{BS} d\varphi_\infty^{BS}. \quad (14)$$

The last integral is equal to one for $n = 0$ and for $n > 0$ is zero, though,

$$b^k_{l,\rho} = const \cdot b^k_{l,\rho_\tau} \int_{\varphi_\infty^{BS}} |G_R(\varphi_\infty^{BS})|^2 e^{(2\pi j/\lambda)(x_l - x_\rho)\sin\varphi_\infty^{BS}} \cos\varphi_\infty^{BS} d\varphi_\infty^{BS}. \quad \ldots (15)$$

Formula (15) (see Appendix B for details) shows that the elements $b^k_{l,\rho}$ are completely separable. From the structure for $b^k_{l,\rho}$ in (15), one can also see that the material of section II is totally valid for this case as well.

[6] Here we apply the word fast change in the same context as it was considered in [27], i. e., the intensities of changes of the states are sufficiently higher than $\nu \approx 1/\tau_c$, being τ_c the covariation time in each state.

[7] Please note that the etiquettes are already changed.

IV. Comparison of the Universal PSWF Eigenbasis with other Proposals for Channel Orthogonalizations.

Here we will compare the PSWF Eigenbasis with the so–called *virtual MIMO channel representation* [10] and other basis, already used by some authors for the orthogonal representation (see e. g. [30])[8].

a) Comparison with [10]. For the lack of space, we can not make a detailed review of [10], but we would like to mention that actually the virtual channel representation is a partitioning procedure of the spatial propagation environment, into M·N virtual AoD–AoA *artificial trajectories*. Those trajectories are predefined by a uniform quantization of the angles.

But considering, e. g., equations (1), (21), etc. from [10], one can see that the functions,

$$f_Q(\theta) = \frac{e^{-2\pi j\theta Q}}{Q}\frac{\sin \pi Q\theta}{\sin \pi\theta}, \tag{16}$$

which are the basis for $H_V(q, p)$, (for definitions see the original material at [10]), has a module,

$$\left|f_Q(\theta)\right| = \frac{1}{Q}\left|\frac{\sin \pi Q\theta}{\sin \pi\theta}\right|, \tag{17}$$

and it is nothing else but a beam pattern for a uniform weighted linear antenna. It was shown that this basis can be represented by PSWF (see pp. 384–385 [14]).

b) At [30] it was proposed a Salz–Winters model with the Bessel series expansion for $b_{l,\rho}$ which converges very slowly. At [21] this model was modified for the case of $\Delta\theta$ small and shown that for this case, $b_{l,\rho}$ can be represented by only one *sinc*(·) function, but this is exactly an asymptotic case for PSWF [13, 28].

V. Conclusions.

The paper provides a unique methodology for the calculation of the principal element of the covariation matrix for the MIMO channel, and introduces the idea of a universal basis, by means of the PSWF, which has double orthogonality properties. The PSWF are a complete basis that does not depend on the channel properties, but it does on the antenna parameters, as stated above

The calculation of the distribution functions for $W_\varphi(\varphi)$ and $W_\theta(\theta)$ at (4) and (11) follows from the previous work of the authors at [9, 18].

With the symmetry of the channel for the transmitter and receiver terminals, the formulas (4), (11) and (15) are valid for the covariation matrix at the transmission end, as well.

VI. Appendixes.

Appendix A.

Let us consider the KLE [28]:

$$\lambda\boldsymbol{\Psi}(t,\tau,\mathbf{r}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B(t-t',\tau-\tau',\mathbf{r}-\mathbf{r}')\boldsymbol{\Psi}(t',\tau',\mathbf{r}')dt'd\tau'dr',$$

… (A.1)

this integral equation has a solution for the scalar case as [28],

$$\boldsymbol{\Psi}(t,\tau,r) = e^{j(\omega_t t+\omega_\tau \tau+\omega_r r)}, \quad -\infty < \omega_t t, \omega_\tau \tau, \omega_r r < \infty. \tag{A.2}$$

By substituting (A.2) into (A.1) one can get the first equation in (12).

Appendix B.

Let us consider the product $<(\hat{h}^k_{m,n})\ (\hat{h}^k_{m,n})^*>$ in the way:

$$\left\langle \left(h^k_{m,n}(t_1,\boldsymbol{\eta}_l)\right)\left(h^k_{m,n}(t_2,\boldsymbol{\eta}_\rho)\right)^*\right\rangle = \frac{P_0\cos\varphi(t_1)}{(4\pi)^2\left|\mathbf{r}_k\right|^2}\cdot$$
$$\mathrm{Re}\left\{\left|G_T(\theta)G_R(\varphi)\right|^2 \hat{g}_\rho(t_1)\hat{g}_l(t_2)e^{-j\nu_k^{m,n}(t_2-t_1)}e^{(2\pi j/\lambda)\left[x_l\sin\varphi(t_1)-x_\rho\sin\varphi(t_2)\right]}\right\},$$

… (C.1)

then,

$$\left\langle \left(h^k_{m,n}(t_1,\boldsymbol{\eta}_l)\right)\left(h^k_{m,n}(t_2,\boldsymbol{\eta}_\rho)\right)^*\right\rangle = \frac{P_0 e^{-j\nu_k^{m,n}\tau}}{(4\pi)^2\left|\mathbf{r}_k\right|^2}\left\langle \hat{g}_\rho(t_1)\hat{g}_l^{\,*}(t_2)\right\rangle\cdot$$
$$\mathrm{Re}\left\{\left\langle\left|G_T(\theta)\right|^2\right\rangle_\theta \int_\varphi\int_{\varphi_\tau} G_R(\varphi)W(\varphi,\varphi_\tau)e^{(2\pi j/\lambda)\left[x_l\sin\varphi(t_1)-x_\rho\sin\varphi(t_2)\right]}\cos\varphi\, d\varphi\, d\varphi_\tau\right\};$$

… (C.2)

Here we apply $\tau = t_2 - t_1$ and assume that *cos*(φ) does not change sufficiently during the small interval τ, because the changes of the states are fast comparing with $\tau_c \approx 1/\nu$; and for $\tau \to \infty$, all the components of the covariation matrix tends to zero.

Though the exponential term for small τ can be represented in the way $e^{(2\pi j/\lambda)\left\lfloor x_l - x_\rho\right\rfloor\sin\varphi}e^{(2\pi j/\lambda)x_\rho\tau\sin 2\varphi}$, the module of the second term for $\tau \to 0$ tends to one, hence approximately its influence can be neglected and one can easily obtain (15); here $\varphi = \varphi_\infty$ and $\varphi_\tau = \varphi_{\infty\tau}$:

$$b^k_{l,\rho} = \mathrm{const}\cdot b^k_{l,\rho_\tau}\int_{\varphi_\infty^{BS}}\left|G_R(\varphi_\infty^{BS})\right|^2 e^{(2\pi j/\lambda)(x_l-x_\rho)\sin\varphi_\infty^{BS}}\cos\varphi_\infty^{BS}d\varphi_\infty^{BS}.$$

where $b^k_{l,\rho_\tau} = \left\langle \hat{g}_\rho(t_1)\hat{g}_l^*(t_2)\right\rangle e^{-j\nu_k^{m,n}\tau}$ and $\mathrm{const} = \left\langle G_T(\theta)\right\rangle_\theta\cdot$

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