Auctions Sequence as a New Spectrum Allocation Mechanism

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Abstract—**Based on the cognitive radio approach, auctions sequence as a potential spectrum allocation mechanism is introduced. The auctions will be periodically repeated in short-term within seconds or even milliseconds. Because of this hard time constraint, agents located in the MAC-layer have to arrange the auction automatically. A special class of auctions, the multi-unit sealed-bid auctions, saves signaling effort in comparison to the multi-unit open auction and the multi-unit sequential auction and thus is suitable for communication systems. This proposed auction sequence possesses two important advantages: First, an auction instantaneously reacts on the market and customers' demand. Second, as shown in a simulation, the operator's gain is almost always higher than for the billing mechanism in established communication systems. A simulative comparison between a Vickrey auction and a uniform-price auction shows both the economical and technical behavior of this highly dynamical stochastic auction process for an OFDMA/TDD system like IEEE 802.16. Furthermore, an optimal bidding strategy for both auctions is analytically deduced maximizing spectrum allocation.**

Keywords—**auction, spectrum allocation, OFDMA, MAC, IEEE 802.16**

I. Introduction

In communication systems the radio functionalities tend toward more and more intelligent algorithms. Their ability to react on different influences in an appropriate and sophisticate manner will increase more and more. In the future these radios will recognize their environment and learn about it, by gaining information from the environment. The learning process results in modified optimized actions and adaption to the environment. This cognition and the following execution can be realized in a cognitive radio [1]. One of the first steps toward a cognitive radio will be introduced in this paper by simultaneously allocating spectrum and determining the price per bandwidth at periodically repeated auctions.

In established billing systems the prices are fixed and the customers who demand first will be served first. Auctions allow the customers to incorporate their needs and demands instantaneously into the good allocation at the current market situation. Therefore, the price depends on the willingness of a customer to pay for goods offered at the moment. Clearly, the auctioneer has also the possibility to influence the market situation by announcing a reserved price. Each bid has to exceed this limitation in order to be accepted to the auction. Both sides, thus, have the opportunity to play a part in influencing the market progress in real time.

In this paper an auction mechanism will be applied to offer spectrum to users and allocate it after the auction. The spectrum is divided in discrete parts like subcarriers in Orthogonal Frequency Division Multiplexing (OFDM) [2], [3]. A multi-unit auction assigns these spectrum parts to users according to their bids and at the same time determines the price depending on the bids and the auctioneer's reserved price. Hence, the auction mechanism incorporates both spectrum allocation and billing.

Since the repetition duration should be very small like seconds or even milliseconds, signalling effort represents a very important constraint concerning the use of auctions for the ressource allocation in wireless communication systems. Auction types like open auctions and sequential auctions are not suitable, because the undetermined number of iterations leads to an unpredictable auction duration. On the other hand, the sealed-bid auctions are very fast. The auction duration is only a linear function of the amount of bidders. The signaling commands are short and can be piggybacked into some free space of the header or the data packets. Therefore, the sealed-bid auction can be implemented under a heavy time and signaling constraint.

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This paper is organized as follows: The reason for the use of auctions sequences as spectrum allocation mechanisms and a possible implementation within the MAC-layer will be highlighted in Section II. In Section III the auction will be described in general. In Section IV the multi-unit sealed-bid auctions are explained by considering the uniform-price auction and the Vickrey auction. The behavior of the auction sequences applied in an OFDMA/TDD scenario will be discussed concerning both the technical and the economical point of view in Section V. Finally, the important findings will be summarized in Section VI.

II. Auctions as MAC entities

In communication systems an auction can occur periodically in a local region. The operator represents the auctioneer's side and the users are the bidders. The entity responsible for the auction is located in the Medium Access Control layer (MAC) of a Radio Access Technology (RAT) in a specific cell and controls the bandwidth allocation by auctions. Its counterpart, the bidder, is located in the MAC-layer of the user terminal. The goods offered are bandwidths Δf leased for a certain time ΔT. This goods can also be data rate or an amount of data. Every duration ΔT an auction takes place in which the free spectrum is offered. At the auction both the currently served users and the new yet unserved users participate.

In such a highly dynamical process the user cannot bid by himself. An agent located in the MAC-layer should represent the bidder's behaviour. This behaviour has to be expressed in a suitable form, which an algorithm can handle. Therefore, users wishes, demands and behaviours should be expressable in a set of parameters or functions like QoS-graphs. Not only private information can influence the bidding strategy, but also the experience of past auctions. Since a user participates on serveral auctions, the information of past allocation and price development may be the input to a predictor in order to improve the bidding strategy.

On the operator's side an auctioneer's agent located in the proper MAC-layer represents the operator's behaviour. Its tasks include the announcement of an auction after every time ΔT, calculating and predicting the reserved price based on fixed costs and executing the auction mechanism.

Auctioneer's and bidders' agents act and react in a highly repeated auction sequence. In the past the focus in auction theory was mainly on auctions which occur once. This spectrum allocation approach needs a more sophisticated consideration of auctions. This sequence has to be considered as a dynamic process. The intention of maximizing expectations can be directly applied in this process, because a user takes part as often as his gain and his good allocation can be approximated by the expectations. Therefore, optimizing the bidding strategy with respect to the expectations makes sense.

From the economical point of view, this auction process is more suitable than established billing strategies concerning the revenue in a frequently visited cell.

III. Auction Theory

The auction mechanism can be organized into bidding, assignment and pricing as described below.

Bidding: In an auction an auctioneer a offers K goods to B bidders. Each bidder $b \in \{1, \dots, B\}$ values each good $g \in \{1, \dots, K\}$ by assuming his signal $\mathbf{x}_{b,g}$. All signal values $\mathbf{x}_{b,g}$ of bidder b are summarized in the signal vector $\mathbf{x}_b$. In other words, $\mathbf{x}_{b,g}$ represents the need and willingness of bidder

b to pay a certain amount of money for a good g. The signal vectors $\mathbf{x}_b = (\mathbf{x}_{b,1}, \ldots, \mathbf{x}_{b,K})$ are mapped to bidder's b bidding vector $\mathbf{b}_b$ by bidder's b bidding strategy $\beta_b(\mathbf{x}_1, \ldots, \mathbf{x}_B) = \mathbf{b}_b$. It is assumed that the bidder rationally bids, that is $\mathbf{b}_{bk} \leq \mathbf{x}_{bk}$, $k \in \{1, \ldots, K\}$. If the bidding strategy $\beta_b(\cdot)$ depends only on the bidder's signal vector $\mathbf{x}_b$, bidder b submits the bidding vector $\mathbf{b}_b$ only with respect to his *private information*. Generally, the bidding strategy β does not only incorporate bidder's b signal vector $\mathbf{x}_b$, but also other information. That is, the knowledge of price development in the past, the amount of other bidders and the quality of good may influence the bidding vector and therefore should be included in the bidding strategy. If a certain bidder knows the signaling vector or just some parts of it from other bidders, the bidder strategy is influenced by this *interdependent information*.

Assignment and Pricing: After all B bidders have submitted their B bidding vectors $\mathbf{b}_1, \ldots, \mathbf{b}_B$ to the auctioneer a, the bidding vectors $\mathbf{b}_b$ are summarized in the bidding matrix $\mathbf{B} = (\mathbf{b}_1, \ldots, \mathbf{b}_B)^T$. An assignment mechanism α exactly assigns one good to at most K bids $\mathbf{b}_{b,g}^{win}$, summarized in the winner set $\mathbf{b}_{b,g}^{win}$:

$$\alpha(\mathbf{B}) = (\alpha_{b,g}) = \begin{cases} \alpha_{b,g} = 1 & : \quad \mathbf{b}_{b,g} \in \mathbf{b}_{b,g}^{win} \\ \alpha_{b,g} = 0 & : \quad \mathbf{b}_{b,g} \notin \mathbf{b}_{b,g}^{win} \end{cases} \tag{1}$$

The matrix α consists of zeros and ones. An entry being one indicates that this bid wins a good. The first index b shows which bidder wins and the second index which good g he wins. Finally the billing mechanism computes the price $\mathbf{p}_{b,g}^{win}$ for each $\mathbf{b}_{b,g}^{win}$.

IV. Multi-unit sealed-bid Auction

Sealed-bid auctions are very fast and based on the revenue equivalence principle [4] comparable to the other auction types. In this section two well-known sealed-bid auctions the uniform-price auction and the Vickrey auction are described. Important variables and terms will be determined describing auctions as spectrum allocation mechanisms in communication systems.

A. Uniform-price Auction

The uniform-price auction belongs to the sealed-bid auction class. The bidder's b bids $\mathbf{b}_{b,g}$ are summarized in the bidding vector $\mathbf{b}_b$. No other bidder than bidder b knows the content of the bidding vector $\mathbf{b}_b$. This property characterizes a sealed-bid auction. Furthermore, in a uniform-price auction all winning bidders have to pay the same price per good.

The auctions mechanism starts by the announcement of the auction (see fig. 1). The auctioneer proclaims at least the amount of identical goods K and the reserved price r. Further information, such as the amount of bidders participating, is voluntary and serves as the input of a prediction mechanism or as the input of the bidding strategy. All bidders receive the announcement and submit a K-valued bidding vector $\mathbf{b}_b$ to the auctioneer. To simplify matters, it is assumed that the bidders have only private information and the auction is efficient.

The auctioneer collects all bids in the bidding matrix $\mathbf{B}$ and chooses the $m_{max} \leq K$ highest bids exceeding the reserved price r according to the following algorithm:

$$\begin{array}{l} L = \{(\mathbf{b}_{b,g}, b, g) | \mathbf{b}_{b,g} \geq r\}; \\ M = \emptyset; \\ \mathit{for} \quad j = 1 \ \mathit{to} \ \min\{|L|, K\} \\ \qquad (\mathbf{b}_{b,g}^{win,j}, b, g) = \max_1\{L \backslash M\} \\ \qquad M = \{(\mathbf{b}_{b,g}^{win,m}, b, g) | m = 1 \ldots j-1\} \\ \mathit{end} \end{array}$$

The maximum function $\max_1$ returns the triple $(b_{b,g}, b, g)$ with the biggest first component $\mathbf{b}_{b,g}$.

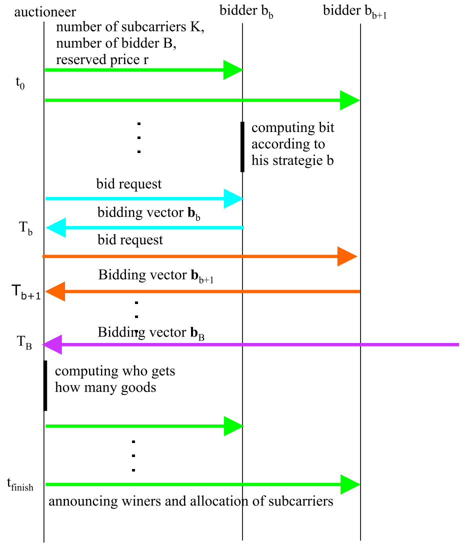

Fig. 1. Signaling in a sealed-bid auction

Another step of this algorithm calculates the market clearing uniform price p per good:

$$\begin{array}{l} M = \{(\mathbf{b}_{b,g}^{win,m}, b, g) | m = 1 \ldots m_{max}\} \\ (\mathbf{b}_{b,g}^{win,m_{max}+1}, b, g) = \max_1\{L \backslash M\} \\ p = \max\{r, \mathbf{b}_{b,g}^{win,m_{max}+1}\} \end{array}$$

The price p per good is the maximum of the reserved price and the highest loosing bid $\mathbf{b}_{b,g}^{win,m_{max}+1}$.

The bidding strategy is important to optimize both auctioneer's gain, bidder's gain and the good allocation efficiency. A good compromise between auctioneer's gain and bidder's gain should be found in order to satisfy both parties.

A.1 Expected gains, revenue and efficiency

Consider a bidder b wins the good g, bidding $\mathbf{b}_{b,g}$ and evaluating g with the signal $\mathbf{x}_{b,g}$; this bidder gets the gain $\mathbf{g}_{b,g}$:

$$\mathbf{g}_{b,g} = \begin{cases} \mathbf{x}_{b,g} - p & , \quad \mathbf{b}_{b,g} \in M \\ 0 & , \quad \text{otherwise} \end{cases} \tag{2}$$

Summing up all $\mathbf{g}_{b,g}$ results in the bidder's total gain $\mathbf{g}_b$:

$$\mathbf{g}_b = \sum_{g=1}^{K} \mathbf{g}_{b,g} \tag{3}$$

$\mathbf{g}_b$ is the gain of an 'experiment', that is, an auction occurs and the auctioneer allocated the goods g. In order to predict the bidder's gain $\mathbf{g}_b$ and auctioneer's gain g_a and to design a judicious strategy, their expectations Γ should be determined relating to a proper auction model:

$$\mathbf{\Gamma_b} = E\{\mathbf{g}_b\} = E\left\{\sum_{k=1}^{K} \mathbf{g}_{b,g}\right\} \tag{4}$$

In the same way the auctioneer's gain can be determined by subtracting all prices paid per good won from the signal $\mathbf{x}_{a,g}$ the auctioneer values the good:

$$g_a = \sum_{m=1}^{m_{max}} (p - \mathbf{x}_{a,m}) \tag{5}$$

Creating the expectation, it must be taken into account that all bids winning with a proper probability.

$$\Gamma_a = E\left\{g_a\right\} \tag{6}$$

All money an auctioneer gets from bidders is expressed by the auctioneer's revenue R:

$$R = m_{max} \cdot p \tag{7}$$

and its expectation P:

$$P = E\left\{m_{max} \cdot p\right\} \tag{8}$$

Another important criterion concerning the selling process is the number of goods allocated. This can be described by the good allocation efficiency ξ:

$$\xi = \frac{m_{max}}{K}, \tag{9}$$

which can be described as the quotient of the goods allocated to the number of goods offered. Maximizing allocation by auction sequences leads to maximizing the expectation of (9):

$$\Xi = E\{\xi\}. \tag{10}$$

A.2 Bidding Strategy

In order to evaluate different approaches of bidding strategies, a stochastical bidder model is introduced.

The bidder b has a stochastic signal vector $\mathbf{X}_b = (\mathbf{X}_{b,1}, \ldots, \mathbf{X}_{b,K})^T$ possessing K components, represented by random variables, if K goods are offered. Concerning identical goods and standard auctions, the probability variables can be assumed to be sorted in a decreasing manner:

$$\mathbf{X}_{b,1} \geq \mathbf{X}_{b,2} \geq \ldots \geq \mathbf{X}_{b,K} \tag{11}$$

To get the stochastic characteristics of the signal vector $\mathbf{X}_b$, first, a K-dimensional stochastic vector $\theta = (\theta_1, \ldots, \theta_K)^T$ is assumed whose components are identically and independently distributed in the interval $[0, \omega]$. Using a function which sorts the components θ_g of θ leads to (11). The theory of order statistics allows to describe the probability behavior of $\mathbf{X}_b$.

If θ_g has the probability density function (pdf) $f_\Theta(\theta)$ and the cumulative distribution function (cdf) $F_\Theta(\theta)$, the probability density $f_{\mathbf{X}_{b,g}}(\mathbf{x}_{b,g})$ and the cumulative distribution function $F_{\mathbf{X}_{b,g}}(\mathbf{x}_{b,g})$ can be described as:

$$F^K_{X_{b,g}}(c) = \sum_{l=0}^{g-1} \binom{K}{l} \left(F_\Theta(c)\right)^{K-l} \left(1 - F_\Theta(c)\right)^l \tag{12}$$

and

$$\begin{aligned} f^K_{X_{b,g}}(c) &= K f_\Theta(c) \binom{K-1}{g-1} \left(F_\Theta(c)\right)^{K-g} \\ &\quad \cdot \left(1 - F_\Theta(c)\right)^{g-1}, \end{aligned} \tag{13}$$

respectively. In other words, (12) describes the probability that the g^{th} signal $\mathbf{X}_g$ is at most c, implying $g-1$ signals are greater than X_g. A realization of X_g shows the bidder's appreciation if he gains the g^{th} good.

Coming back to the original question and looking for the optimal bidding strategy, the optimization focuses on maximization of the expected amount of goods a certain bidder gets. The use of the expectation is mandatory, because based on the assumption of private information, the bidder can only guess how the other bidding vectors look like. Nevertheless, the intention holds even if the bidder does not know the cdf of the other bidders' bids.

Considering the strategy for a certain bidder, there are $B-1$ other bidders. Their $(B-1) \cdot K$ bids can be sorted into the components of a vector called $\mathbf{C}$ (comp. (11)). The component $\mathbf{C}_i$ of $\mathbf{C}$ possesses a cdf $F_{\mathbf{C}_i}(\mathbf{c}_i)$ according to the order statistic.

Without loss of generality the reserved price equals zero and bidder b_1's bidding strategy is the one to be optimized. The bids do not exceed the signals because a bidder does not want a negative gain. In a standard auction the K highest bids win. Therefore, the probability that bidder b_1 wins at least one good is $F^{(B-1)\cdot K}_{\mathbf{C}_K}(\mathbf{b}_{1,1})$, being the probability that bid $\mathbf{b}_{1,1}$ is larger than the K^{th} bid of the other bidders. Consequently, the probability that he wins at least two goods is $F^{(B-1)\cdot K}_{\mathbf{C}_{K-1}}(\mathbf{b}_{1,2})$. Combining both by subtracting the probability of gaining at least one good from the probability of gaining at least two goods results in the probability to get exactly one good. Generalizing this fact, the probability that bidder b_1 wins exactly k goods is:

$$F^{(B-1)\cdot K}_{\mathbf{C}_{K-k+1}}(\mathbf{b}_{1,k}) - F^{(B-1)\cdot K}_{\mathbf{C}_{K-k}}(\mathbf{b}_{1,k+1}). \tag{14}$$

Let κ be the number of goods bidder b_1 wins, then the expectation of the goods allocated, $E\{\kappa\}$, is the sum of the probability to get k goods multiplied by k:

$$\begin{aligned} E\{\kappa\} &= \sum_{k=1}^{K} k \left(F^{(B-1)\cdot K}_{C_{K-k+1}}(b_{1,k}) - F^{(B-1)\cdot K}_{C_{K-k}}(b_{1,k+1})\right) \\ &= \sum_{k=1}^{K} F^{(B-1)\cdot K}_{C_{K-k+1}}(b_{1,k}). \end{aligned} \tag{15}$$

The expectation sums up K CDFs. Each CDF has a different bid $\mathbf{b}_{1,k}$ as an input argument. A CDF is a non-decreasing function. That is, the higher the bids, the higher the expectation. Therefore, with respect that a bid does not exceed its signal the expectation of the goods allocated is maximized if the bidder bids his signal,

$$\mathbf{b}_b = \beta_b(\mathbf{x}_b) = \mathbf{x}_b. \tag{16}$$

This strategy is called *true bidding*. The result assumes neither asymmetric nor symmetric bidder nor interdependent values among the other bidders. That is, the result (16) can be used in a very general manner.

Optimizing other parameters like gain or revenue may lead to other bidding strategies. If the optimization focuses on expected bidder's gain $\mathbf{\Gamma}_b$, it can be shown that a proper solution can only be found for the highest bid $\mathbf{b}_{1,1}$. This bid has to equal the highest signal $\mathbf{x}_{1,1}$. Concerning the other components a general statement is not possible. Generally, in an efficient uniform-price auction, an optimal bidding strategy optimizing bidder's gain does not exist. For special cases of the probability characteristics of $\mathbf{C}$, an optimal bidding strategy can be specified.

B. Vickrey Auction

The Vickrey auction is another type of multi-unit sealed-bid auction and can be considered as the generalization of the single-unit second-price auction. It possesses the same procedure as described in the uniform-price auction. The two auctions differ only in the pricing strategy.

The auctioneer offers K goods and announces the reserved price r. Each bidder submits a bidding vector $\mathbf{b}_b$ the components are arranged in decreasing order. The auctioneer chooses the K highest bids and if they exceed the reserved price, a good is assigned to each winning bid.

If a bidder wins k goods, he has to pay the sum of the k highest loosing bids of the other bidders if these bids exceed the reserved price, otherwise the reserved price.

The bidding strategy, mapping the signals to bids, should optimize both the expectation of goods allocated and the expectation of bidder's gain $\mathbf{\Gamma}_b$. Firstly,(15) holds for Vickrey auctions as well. That is, true bidding (16) maximizes the expectation of goods allocated. Secondly, as shown in [4], the expectation of bidder's gain gets a maximum if the bidder also follows the true bidding strategy. Summarizing this issues, true bidding maximizes both expectation of goods allocated and expectation of bidder's gain $\mathbf{\Gamma}_b$.

V. SIMULATION OF AN OFDMA/TDD SCENARIO

To get more familiar with the mechanism which allocates spectrum and regulates pricing, an Orthogonal Frequency Division Multiplex Access/ Time Division Duplex (OFDMA/TDD) scenario is simulated. A well-known representative of this access mechanism is the Metropolitan Area Network (MAN) standard IEEE 802.16a [5] for 2-11 GHz. The Base Station (BS) automatically periodically assigns the Service Station (SS) a certain number of subcarriers for a certain time with respect to QoS classes [6].

Keeping this system in mind and abstracting this to investigate auctions as a periodically repeated spectrum algorithm, one cell is considered in the following simulation. A BS offers 50 OFDM subcarriers to user within a cell. The subcarriers can be rented within a multi-unit sealed-bid auction occuring every $\Delta T = 1s$ (see fig. 2). That is, a periodically repeated spectrum allocation mechanism process is considered. The auction duration is negligible in comparison to ΔT because the simulation focuses on gain, revenue and efficiency considerations.

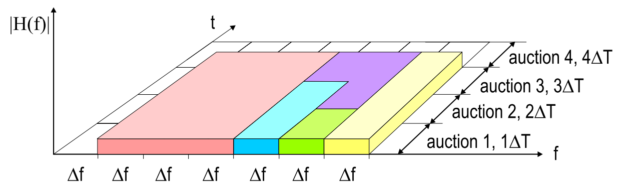

Fig. 2. Auctioning of subbands Δf in OFDMA/TDD

The users enter the cell according to Poisson proccesses with arrival rates $\lambda_1 = \frac{1}{30s}$, $\lambda_2 = \frac{1}{60s}$ and $\lambda_3 = \frac{1}{90s}$. Each user wants to do a video conference (VC) and to set up an FTP session. The duration of VC is Gaussian distributed with mean $\mu_{VC} = 300s$ and variance $\sigma^2_{VC} = 60s$. In the same way, the amount of data of the FTP session is uniformly distributed with mean $\mu_{FTP} = 128Mbit$ and variance $\sigma^2_{FTP} = 54,6Mbit$. Depending on the duration of the VC, the amount of data of FTP and their QoS, the user wants to transmit a certain amount of data using a minimum data rate. To generate different bidders for simulation, the evaluation of the subcarriers follows the order statistics and the signals $\mathbf{x}_{b,g}$ are normalized to be between 0 and 1. To maximize his data rate (see (9) and (15)) involving higher QoS, each bidder uses true bidding.

Coming into the cell a bidder tries $180s$ to get access to the network. If he does not succeed, he leaves the cell. After a user has won subcarriers his communication can start. During this communication he participates in further auctions in order to get a higher data rate in order to increase the QoS.

The spectrum, which was won by a bidder, can be used exclusively and is not offered at auctions until a certain time limit is reached or the user finishes his communication, i.e. if both VC and FTP session are closed.

In this simulation the auctioneer's gain g_a, the sum of bidder's gains $\sum \mathbf{g}_b$ and the unused subcarriers $s_l = K - m_{max}$ are investigated depending on a variation of the reserved price r. The reserved price r ranges between 0 and 1. That is the same range as the signals. It is assumed that the reserved price equals the fixed costs. This is reasonable when assuming that the auctioneer wants the fixed costs to be covered per subcarrier. The difference between the market clearing price and the fixed costs

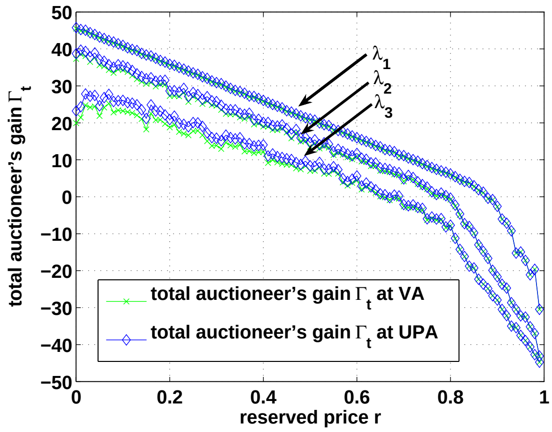

Fig. 3. Averaged total auctioneer's gain Γ_t depending on the reserved price r

per subcarrier represents his gain per subcarrier. Therefore, this market model can be more convenient for the auctioneer compared with the established one. Furthermore, the Uniform-Price Auction (UPA) and the Vickrey Auction (VA) are compared and the differences between auctioneer's gain in the proposed auction sequences and the normal established market are shown.

An economical question arises over the auctioneer's overall gain after subtracting the additional fixed costs of the unused spectrum. This averaged auctioneer's total gain as a function of the reserved price is depicted in Fig. 3. For the following discussion, we assume that the reserved price equals the fixed costs per subcarrier.

The averaged total auctioneer's gain per auction Γ_t decreases if the reserved price r increases (as shown in Fig. 3), since the probability decreases that the last unserved bid at the uniform-price auction and the relevant last unserved bids at the Vickrey auction are higher than the reserved price. The smaller the arrival rate λ, the lower the auctioneer's gain, because a smaller amount of bidders results in a smaller probability that the relevant last unserved bids are higher than the reserved price.

The total auctioneer's gain at a Vickrey auction is always smaller than the one at a uniform-price auction. The reason for this is as follows, by assuming that a bidder wins k subcarriers, a bidder at the Vickrey auction has to pay the sum of the k highest unserved bids of the other ones what is always smaller than k times the highest unserved bid. Consequently, an auctioneer would favour the uniform-price auction over the Vickrey auction

Considering r closed to 1 in comparison to small r values, the graphs decrease faster when the reserved price increases, since a higher reserved price results in a lower probability that enough bids exceed the reserved price in order to fully assign the spectrum to users. That is, the probability increases that subcarriers are unused and their fixed costs are not covered. Each total gain graph has a zero crossing. Considering the total gain characterized by λ_3, the operator has to push his fixed costs per subcarrier under 0.7 of the maximum bidder's signal in order to gain money. The lower the relative fixed costs are in comparison to the maximum bidder's signal, the higher his total gain is.

Leaving the auctioneer's point of view and turning over to the averaged sum of bidder's gain per auction, the graphs show a contrary behavior as expected. The averaged sum of bidder's gain per auction $\mathbf{\Gamma}_b$ depending on the reserved price is shown in Fig. 4. The higher the reserved price is, the lower the probability becomes that the relevant last unserved bids exceed the reserved price, this causes the sum of bidder's gain to get lower.

Comparing the graphs with respect to different arrival rates λ_i two main influences have to be taken into account. For small reserved prices the probability that the reserved price determines the market clearing price is very small and second, the probability that enough bids are higher than the reserved price is very high. Consequently, the more bidders take part at the auction,

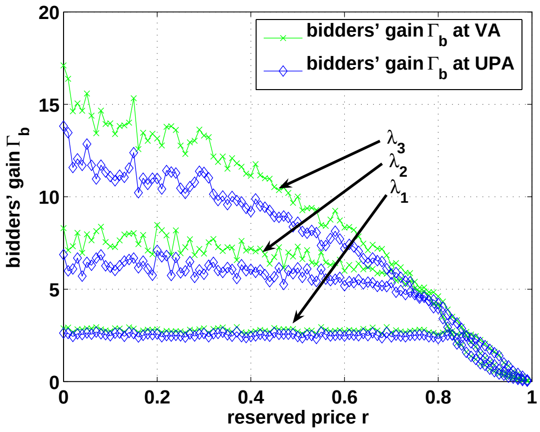

Fig. 4. Averaged sum of bidders' gain Γ_b depending on reserved price r

the higher the price. This is expressed by the higher arrival rate. Therefore the sum of bidders' gain for λ_1 is smaller than that λ_2.

Toward higher reserved prices another influence becomes dominant. This influence leads to an inversion of the set of curves in Fig. 4. The higher the reserved price is, the lower the probability gets that enough bids exceed the reserved price and the smaller the sum of bidder's gain. The lower the arrival rate is, the smaller the amount of bids becomes and the smaller the probability that enough bids exceed the reserved price. Therefore, the sum of bidders's gain gets smaller when the arrival rate decreases for high reserved prices.

The comparison of the two types of auctions shows that true bidding leads to a higher bidder's gain in Vickrey auction than in uniform-price auction. One reason is that at Vickrey auction true bidding is also a weakly dominant strategy to optimize bidder's gain. This fact cannot be confirmed for uniform-price auction.

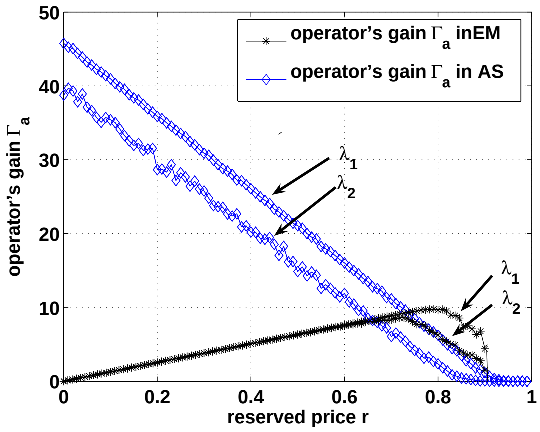

Fig. 5. Comparison between Auction Sequence market model (AS) and fixed-cost Established Market model (EM) concerning averaged auctioneer's gain Γ_a depending on reserved price r

An interesting question is still open, concerning the comparison between the auction sequence market model and the established market model. As mentioned before by the discussion of Fig. 3 the reserved price equals the fixed costs and the auctioneer's gain per subcarrier is the substraction of the fixed costs per subcarrier from the market clearing price per subcarrier. The gain of the established market model is usually chosen to be 0.3 times the fixed costs. That is, the price is calculated multiplying the fixed costs by 1.3 in order to get a 30 % gain. The graphs of the auctioneer's gain using this two models are depicted in Fig. 5. The gain of the established market model is only greater than the one of the auction sequence model in a short intervall. That is, from the seller's point of view the auction sequence model reacts more efficiently to the market and consequently gets higher gain. It adapts the price instantaneously if the customers want to pay more in order to get spectrum.

The lower the arrival rate is, the bigger the interval becomes in which the established market should be preferred concerning the seller's gain. Therefore, the auction sequence market model is more suitable than the established market model in a well-attended cell like hotspots or pico cells in downtown.

VI. Conclusion

An auction is a special mechanism including both billing strategy and spectrum allocation. The new proposed common allocation and billing mechanism consists of a fast periodically repeated auction sequence. This mechanism is executed decentralized, e.g, in a WLAN for each hotspot. Because of the high repetition rate and the limited channel capacity in communication systems, multi-unit sealed-bid auctions are suitable, reducing signaling tremendously in contrast to multi-unit open auctions and multi-unit sequential auctions. The duration of a multi-unit sealed-bid auction is only a linear function of the number of bidders and, therefore, predictable.

In contrast to fixed-price billing in established communication systems, this kind of medium access control instantaneously reacts to the current market situation and customers' demands. Furthermore, the customers can influence the allocation by mapping their evaluation of data rate, QoS, amount of data and urgency of connection to their bidding vector. Thus, the flexibility and the freedom concerning spectrum allocation is increased and the functionalities can be incorporated in a cognitive radio in future.

This new medium access control increases the operator's gain in most cases based on the instantaneous reaction on the market and the competition among the customers. The operator can also influence the market behaviour by varying the reserved price.

In future work the signaling effort for different system concepts, the improvement of the repetition time, the reduction of control traffic and the investigation of new auction mechanisms with respect to communication systems and signaling effort should be further investigated in order to optimize operator's and customer's economical gain, spectrum efficiency and customer's QoS.

VII. Acknowledgement

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