

# Transform domain scrambling PAPR reduction technique for OFDM systems

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## Abstract

In this paper, we propose a novel scheme for reduction of the peak-to-average power ratio (PAPR) of Orthogonal Frequency Division Multiplexing (OFDM) transmitted signal by using Discrete Fourier Transform (DFT) as the preprocessing between data mapper and adding of pilot. The preprocessing is designed to decrease the correlation of original QAM constellation. Simulation results show that PAPR can be reduced about 4.3db at the symbol-clip probability of  $10^{-3}$  for 64-carriers QPSK without any side information. With a little extra computation, the proposed approach effectively reduce the PAPR for commercial OFDM systems

## I Introduction

Orthogonal frequency division multiplexing (OFDM) [1] has recently attracted considerable attention due to its high-bit-rate capability transmission over frequency selective fading channel. By serial-to-parallel converting  $N$  symbols and transmitting lower rate data streams simultaneously over  $N$  orthogonal carriers after IDFT processing, OFDM can decrease the relative amount of dispersion caused by multipath delay spread in wireless environment. The OFDM technique is already in use in many practical systems such as digital television broadcasting (DVB) [2] and wireless local area networks (e.g. IEEE 802.11a [3] and IEEE 802.16a [4]). However, it suffers from large peak-to-average power ratio (PAPR) that may cause inter-modulation and out-of-band radiation due to nonlinearity of power amplifier. The transmission amplifier must be operated in its linear region to prevent spectral distortion. An OFDM signal consists of a number of independently modulated subcarriers. When  $N$  modulated subcarriers are added with the same phase, the peak power is  $N$  times the average power of OFDM signal.

A number of methods have been proposed to solve the problem of PAPR that employ clipping the OFDM signal [5], coding techniques [6], Partial Transmit Sequence (PTS) [7] and Selected Mapping (SLM) [8]. Most of the methods are based on the same way of selecting the transmitted signal from a set of different representations with lower PAPR. Reference [9] [10] modify the transmitted signal constellation to combat large signal peaks. When the constellation shape is changed, the modifications will slightly effect BER performance.

In this paper, we introduce a new method to reduce the PAPR by using DFT kernels as the preprocessing between data mapper and adding of pilot. In this scheme, the OFDM signal with lower PAPR is achieved via a short length DFT kernel which decreases correlation of the original constellation symbols of used subcarriers. Because the specific transform kernel is used only for scrambling, no side information is required and there is no loss in data rate. Therefore, we show an easy way to constrain transmitted envelop of OFDM signal, and an average PAPR will be reduced greatly by the transform domain scrambling (TDS).

Rest of the paper is structures as follows. An introduction to OFDM system in Section I; Section II the definition of PAPR is given; Section III discusses the transform domain scrambling method to reduce the peak power of OFDM system; Section IV simulation environment and results are presented. The impact of transform domain scrambling on bit error rate (BER) performance is also investigated; and Section V draws the conclusions on the present work.

## II Definition of PAPR

An OFDM system is implemented by an IDFT of size  $N$  with  $K$  usable subcarriers. An input bits stream is mapped into QAM signal constellation to form a complex number  $\{X_n\}_{n=0,1,\dots,N-1}$ . The low pass equivalent transmitted signal  $\{s_l\}_{l=0,1,2,\dots,NL-1}$  after IDFT process is written as

$$s_l = \frac{1}{NL} \sum_{n=0}^{N-1} X_n \exp\left(\frac{j2\pi nl}{NL}\right) \quad (1)$$

In order to shape the transmit signal's power spectrum density, most applications of OFDM system just active central  $K$  subcarriers and the other subcarriers are set to zero.

The PAPR of the OFDM signal  $s_l$  in terms of power is defined as

$$\text{PAPR} = \frac{\max_{0 \leq l \leq NL-1} |s_l|^2}{E\{|s_l|^2\}} \quad (2)$$

Where  $E\{\bullet\}$  denotes the expectation operator. L is the over-sampling factor (L=4 is usually enough to estimate the peak power).

### III Transform Domain Scrambling

The traditional OFDM system serial-to-parallel convert input bits stream and each bits is modulated onto central usable subcarriers. The transmitted frequency-domain complex symbols on usable subcarriers are given by

$$\mathbf{K} = [K_0, \dots, K_{K-1}]^T \quad (3)$$

For QPSK, only one of the four constellation points can be transmitted in the same time. When N points QPSK symbol are in the same phase, the peak power is N times the average power of OFDM signal. As shown in Fig. 1 in the transform domain scrambling scheme for each used subcarriers, a new constellation positions is generated by orthogonal transform with DFT kernel. It can be written as

$$\mathbf{M} = \mathbf{F}_{\text{TDS}} \mathbf{K} \quad (4)$$

where  $K \times 1$   $\mathbf{M} = [M_0, \dots, M_{K-1}]^T$  and  $K \times K$   $\mathbf{F}_{\text{TDS}}$  is given as :

$$\mathbf{F}_{\text{TDS}} = \frac{1}{\sqrt{K}} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{j2\pi/K} & \dots & e^{j2\pi(K-1)/K} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j2\pi(K-1)/K} & \dots & e^{j2\pi(K-1)^2/K} \end{bmatrix} \quad (5)$$

High peak power of the transmitted OFDM signal is generated for high correlation QAM symbol. The Karhunen-Loeve transform (KLT) [11] is known to be optimal in data compression in the sense that it compacts most of the signal energy into low-frequency components of the transform; it also generates uncorrelated eigenvalues. Due to lack of efficient algorithms to implement the KLT, we use suboptimal transform such as the DFT to decrease correlation of original QAM complex vector  $\mathbf{K}$ . DFT approximately diagonalize covariance matrices of the  $\rho^{|i-j|}$  type [12]. In addition to the property of unitary transform, DFT kernels can process complex data and can be used for all kinds' modulation mappers, no matter PSK or QAM. Besides, the DFT kernels we adopt are inherent in the typical OFDM system. Figure 2 plots the final constellation for all used subcarriers after transform domain scrambling with QPSK. For the OFDM system, the effect of using transform domain eigenvalue is to avoid adding additional sinusoidal and cosinusoidal in the same phase. The combination of these additional signals with low correlation amplitude/phase is effective in eliminating peaks of transmitted OFDM signal in time domain. Because

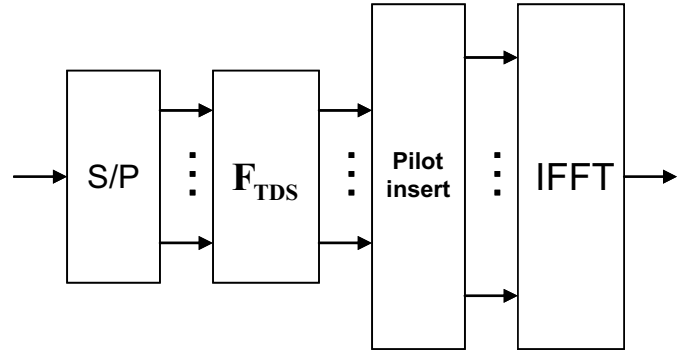


Fig. 1 Block diagram of TDS-OFDM transmitter with transform domain scrambling process

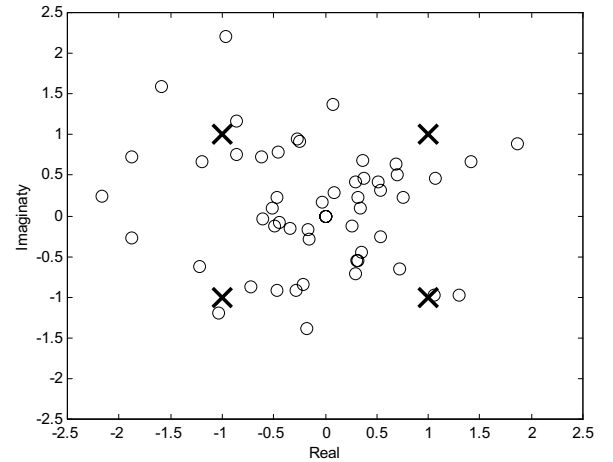


Fig. 2 Distribution of scrambled QPSK signals after TDS processing

the subcarriers are coherently modulated, pilot symbol-assisted channel estimation is also suggested in the receiver. In each OFDM symbol, subcarriers of number P are dedicated to pilot signals in order to detect frequency offsets and phase noise. After inserting pilot symbol and padding zeros, this discrete signal representing the scrambled constellation symbol before N-point IDFT process is a vector of N complex numbers given by

$$\mathbf{X} = [X_0, \dots, X_{N-1}]^T = X_p[n] + X_d[n] \quad (6)$$

where  $n=0,1,\dots,(K+P)-1$  and

$X_p[n]$  subcarriers of sending pilot symbol

$X_d[n]$  subcarriers of sending data symbol

The vector  $\mathbf{X}$  is feed through the Inverse Fast Fourier Transform (IFFT) to acquire the transmitted signal in time domain. It can be written as

$$\mathbf{V} = \mathbf{F}_{\text{IFFT}} \mathbf{X} \quad (7)$$

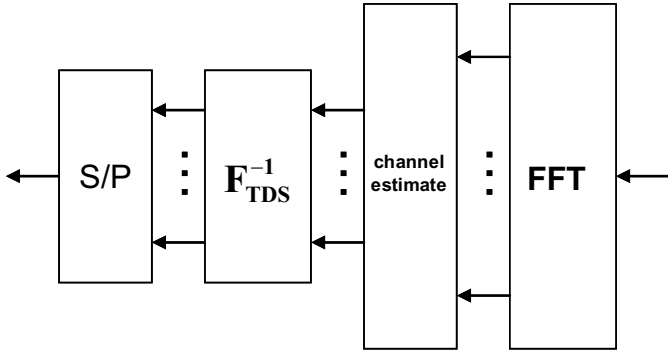


Fig. 3 Block diagram of TDS-OFDM receiver

where  $N \times 1$   $\mathbf{V} = [V_0, \dots, V_{N-1}]^T$  and  $N \times N$   $\mathbf{F}_{\text{IFFT}}$  is

$$\mathbf{F}_{\text{IFFT}} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{j2\pi/N} & \dots & e^{j2\pi(N-1)/N} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j2\pi(N-1)/N} & \dots & e^{j2\pi(N-1)^2/N} \end{bmatrix} \quad (8)$$

Firstly, at the receiver, the received signal is converted into frequency domain by the  $N$ -points FFT block. The Pilot Symbol-Assisted Modulation (PSAM) is employed in coherent OFDM for channel estimation and it is based on inserting known pilot symbol spreaded out throughout frequency domain. Once we know channel attenuations at pilot positions, channel compensation for each subcarrier can be done with an estimation algorithm. At a second step, the estimated data of used subcarriers is then fed in inverse transform coding matrix to decode into original QAM signal. Because the dedicated transform kernel is used only for scrambling amplitude/phase of original QAM signal, no side information is required and the received signal can be easily reconstructed by dedicated transform pair.

#### IV SIMULATION RESULT

The peak-reduction method of transform domain scrambling (TDS) is developed. For the ease of presentation, a 802.11a OFDM system is adopted by using an FFT size of  $N=64$  with  $K=52$  subcarriers used for data. To evaluate the performance, the simulations were conducted for 10,000 OFDM symbols.

Fig. 4 shows the complementary cumulative distribution function (CCDF) defined as the probability of  $\text{PAPR} > \text{PAPR}_0$  for TDS-OFDM. The oversampling factor is chosen as  $L=4$  for estimating the real PAPR of the transmitted signal. The results for 16-QAM and 64-QAM are also included for comparison. For QPSK, the original OFDM signal has a PAPR of 11.8dB and about 4.3dB reduction of TDS-OFDM over the original OFDM at the probability of  $10^{-3}$ . Fig. 5 shows the BER in an additive white Gaussian noise (AWGN)

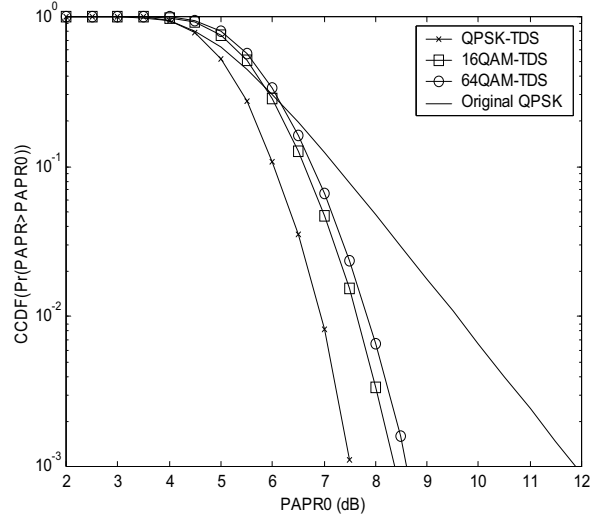


Fig. 4 Comparison of CCDF with 52 used subcarriers using 64-points FFT

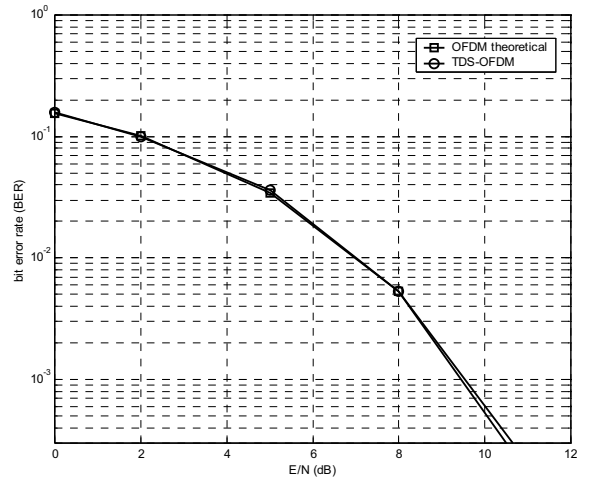


Fig. 5 BER for TDS-OFDM in AWGN channel

channel for TDS-OFDM. The theoretical BER for traditional OFDM is also shown. For QPSK the scrambling noise causes negligible degradation in BER. Because of scrambling process using the property of orthogonal transform, the proposed method does not change the total energy of original OFDM symbol.

Fig. 6 and 7 shows the system bit error rate performances for the Rayleigh fading channel. The channel has two and seven independent path respectively and the Doppler frequency  $f_d$  is 20Hz. All the simulated system performance was compared with theoretic AWGN channel. From Fig. 6 and 7, it can be observed that TDS method introduce little performance degradation since the AWGN dominates the channel estimation in low SNR environment. When SNR is high, however, the frequency selected fading due to multipath environment dominates the system performance; hence the performance of TDS-OFDM is improved as to distribute

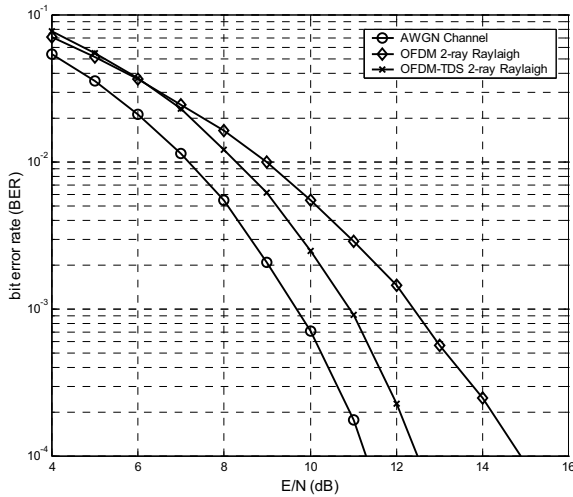


Fig. 6 BER for TDS-OFDM system with 64 subcarriers and 52 used subcarriers. The channel has 2-rays and  $f_d = 20\text{Hz}$

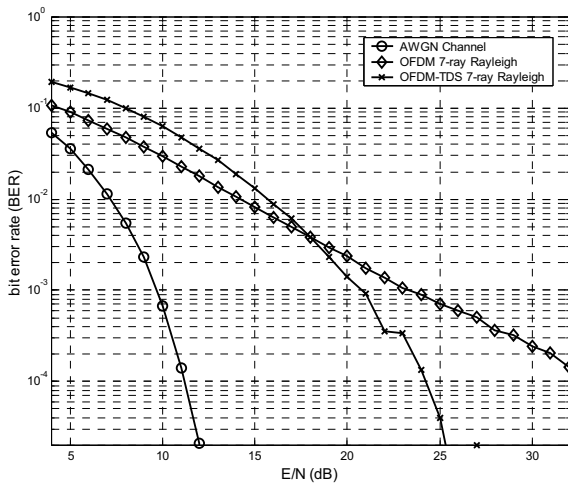


Fig. 7 BER for TDS-OFDM system with 64 subcarriers and 52 used subcarriers. The channel has 7-ray and  $f_d = 20\text{Hz}$

energy of original QAM symbol at deep fading subcarriers into eigenvalues of transform domain. In other words, the degradation of eigenvalues of transform domain will decrease the impact of multipath fading at decision stage.

## V CONCLUSION

A transform domain scrambling (TDS) approach has been proposed for reducing the PAPR in OFDM system. In this method, amplitude/phase of each subcarrier is updated by an orthogonal transform coding scheme. Unlike the conventional PTS and SLM method, the specified transform kernel is used to decrease correlation of original QAM signal only for PAPR reduction and no side information is needed to be transmitted. Simulation shows that the PAPR is typically reduced  $\sim 4.3\text{dB}$  of the maximum possible peak power at the probability

of  $10^{-3}$ . From the simulation results of the BER with mobile channel environment, it can be shown that TDS-OFDM can get better performance in high SNR region. The complexity and overhead of the proposed approach has only minor effect and can be easily implemented in various OFDM systems such as IEEE802.11a/g and DVB-T.

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