DESIGN AND PERFORMANCE OF MIMO CHANNEL SIMULATORS DERIVED FROM THE TWO-RING SCATTERING MODEL

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Abstract – This paper deals with the design and performance analysis of a new class of space-time channel simulators based on a double-sum of complex harmonic exponentials. The starting point of the design procedure is a non-realizable stochastic (multiple-input multiple-output) MIMO reference model that has been developed for the well-known geometrical two-ring scattering model consisting of an infinite number of local scatterers laying on two separated rings — one around the transmitter and the other one around the receiver. The statistical properties of both the reference model and the simulation model are studied analytically, where the three-dimensional space-time cross-correlation function (CCF) is the focal point of our interest. By exploiting the fact that the space-time CCF can be expressed as the product of the so-called transmit and receive CCF, we present two methods enabling the computation of the model parameters of the MIMO channel simulator. The excellent performance of the proposed simulation model is demonstrated by comparing its transmit and receive correlation functions with those of the reference model.

Keywords – Mobile fading channels, MIMO channel modelling, stochastic channel modelling, multielement antenna systems, Rice's sum-of-sinusoids, MIMO channel capacity.

I. INTRODUCTION

Realistic models of the mobile radio channel are of great importance to the design, analysis, optimization, and test of mobile communication systems. To develop advanced mobile communication systems employing multiple antenna arrays with M_T transmit and M_R receive antennas, sophisticated MIMO channel models are required that accurately reflect all relevant features of the physical channel. An overview of channel models developed for MIMO systems can be found in [1]. Two of the most well-known narrowband MIMO channel models are the one-ring model [2] and the two-ring model [3]. The one-ring model assumes that the base station (BS) is evaluated and therefore not obstructed by local scatterers, while the mobile station (MS) is located in the center of a ring of surrounding scatterers. Such an assumption is appropriate for propagation environments in rural and sub-urban areas, where tall BS antennas are used. In the two-ring model, it is assumed that both the transmitter and receiver are surrounded by scatterers, which is generally the case in indoor environments.

This paper contributes to the two-ring model by focussing on the design and performance analysis of a new class of space-time channel simulators using a double-sum of complex harmonic exponentials. The starting point of the design procedure is a non-realizable stochastic MIMO reference model as it has been developed for the two-ring scattering model in [3]. Our procedure can be considered as an extension of Rice's sum-of-sinusoids to a double-sum of complex harmonic exponentials. For the computation of the parameters of the simulation model, two methods are proposed. The first method is an extension of the method of exact Doppler spread (MEDS) [4], which results in closed-form expressions for the model parameters. This method has especially been developed for propagation scenarios described by uniformly distributed angles of departure (AOD) and angles of arrival (AOA). The second method is a variant of the L_p-norm method (LPNM) [4]. Although this procedure is more complex than the first one, it has the advantage that the model parameters can be obtained for many types of distribution functions of the AOD and AOA, including the von Mises distribution [5], the Gaussian distribution [6], the truncated Laplacian distribution [7,8], and the truncated uniform distribution [9].

The rest of the paper is organized as follows. Section II reviews briefly the geometrical two-ring model and introduces the notation. Section III describes the stochastic reference model from which the stochastic and deterministic simulation models are derived in Section IV. Section V presents two parameter computation methods, and Section VI deals with the performance analysis of the resulting MIMO channel simulator. Finally, Section VII presents some concluding remarks.

II. THE GEOMETRICAL TWO-RING MODEL

In the following, we describe the geometrical two-ring model shown in Fig. 1 for a narrowband MIMO system with two omnidirectional antennas at both the transmitter and the receiver side, i.e., $M_T = M_R = 2$. This elementary 2×2 antenna configuration can be used to construct any other type of two-dimensional multielement antenna arrays, e.g., the uniform linear array, the hexagonal array, as well as the circular antenna array. For simplicity, it is assumed that there is no line-of-sight between the transmitter and the receiver, which are taken to be the BS and the MS, respectively. The two-ring model is an appropriate model for describing propagation scenarios, where the BS is not evaluated and, just like the MS, surrounded by a large number of local scatterers.

In the two-ring model, only local scattering is considered, since it is assumed that the contribution of remote scatterers to the total received power can be neglected due to high path loss. The local scatterers around the transmitter, denoted by $S_T^{(m)}$ $(m = 1, 2, \ldots)$, are located on a ring of radius R_T, while the local scatterers $S_R^{(n)}$ $(n = 1, 2, \ldots)$ around the receiver lie on a second ring of radius R_R. It is assumed that the radii R_T and R_R are small in comparison with the distance D between the transmitter and the receiver, i.e., $\max\{R_T, R_R\} \ll D$. The antenna spacings at the transmitter and the receiver are denoted by δ_T and δ_R, respectively. It is usually assumed that these quantities are small in comparison with the radii R_T and R_R, i.e., $\max\{\delta_T, \delta_R\} \ll \min\{R_T, R_R\}$. As can be seen from Fig. 1, the tilt angle between the x-axis and the orientation of the transmitter's antenna array is denoted by α_T. Analogously, the tilt angle α_R describes the orientation of the antenna array at the receiver. We assume that the transmitter is fixed, while the receiver moves with speed v in the direction determined by the angle of motion α_{V}. Furthermore, it is assumed that all waves reaching the receiver antenna array are equal in power.

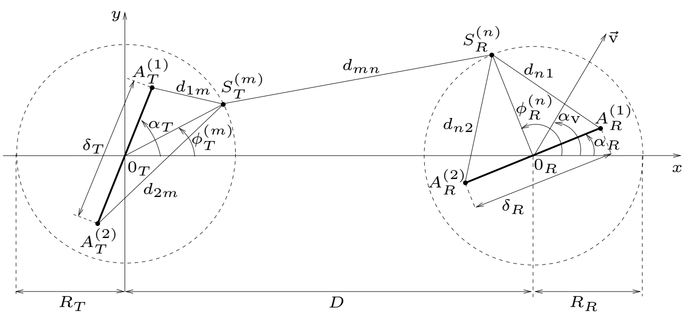

Figure 1: Geometrical model (two-ring model) for a 2×2 channel with local scatterers around the BS and the MS.

III. THE STOCHASTIC REFERENCE MODEL

A. Derivation of the Reference Model

In this subsection, we derive the reference model for the two-ring model. The starting point is the geometrical two-ring model shown in Fig. 1 from which we observe that the mth homogeneous plane wave emitted from the first antenna element $A_T^{(1)}$ of the transmitter travels over the local scatterers $S_T^{(m)}$ and $S_R^{(n)}$ and impinges finally on the first antenna element $A_R^{(1)}$ of the receiver. The reference model is based on the assumption that the number of local scatterers around the transmitter and receiver is infinite. Hence, the diffuse component at the receive antenna $A_R^{(1)}$ is composed of an infinite number of homogeneous plane waves, each of which carries power that is negligible compared to the total mean power of the diffuse component. Considering the geometrical model in Fig. 1 and using the results in [3], it follows that the diffuse component of the channel describing the link from $A_T^{(1)}$ to $A_R^{(1)}$ can be written as

$$h_{11}(t) = \lim_{\substack{M\to\infty \\ N\to\infty}} \frac{1}{\sqrt{MN}} \sum_{m=1}^{M} \sum_{n=1}^{N} g_{mn} e^{j(2\pi f_n t + \theta_{mn} + \theta_0)} \tag{1}$$

where

$$\begin{aligned}
g_{mn} &= a_m\, b_n\, c_{mn} && (2)\\
a_m &= e^{j\pi(\delta_T/\lambda)\cos(\phi_T^{(m)} - \alpha_T)} && (3)\\
b_n &= e^{j\pi(\delta_R/\lambda)\cos(\phi_R^{(n)} - \alpha_R)} && (4)\\
c_{mn} &= e^{j\frac{2\pi}{\lambda}(R_T \cos\phi_T^{(m)} - R_R \cos\phi_R^{(n)})} && (5)\\
f_n &= f_{\max}\cos(\phi_R^{(n)} - \alpha_{\mathrm{V}}) && (6)\\
\theta_0 &= -\frac{2\pi}{\lambda}(R_T + D + R_R)\,. && (7)
\end{aligned}$$

In (1), the phases θ_{mn} are independent, identically distributed (i.i.d) random variables, which are uniformly distributed over $[0, 2\pi)$. Without loss of generality, the constant phase θ_0 defined by (7) can be set to 0. From the statistical properties of the diffuse component in (1) it follows that the mean value and the power of $h_{11}(t)$ are equal to 0 and 1, respectively. Hence, the central limit theorem states that $h_{11}(t)$ is a zero-mean complex Gaussian process with unit variance. Consequently, the envelope $|h_{11}(t)|$ is a Rayleigh fading process.

One can show that the diffuse component $h_{22}(t)$ of the link from $A_T^{(2)}$ to $A_R^{(2)}$ can be obtained from (1) by replacing a_m and b_n by their respective complex conjugates a_m^* and b_n^*. Similarly, the diffuse components $h_{12}(t)$ and $h_{21}(t)$ can directly be obtained from (1) by performing the substitutions $a_m \to a_m^*$ and $b_n \to b_n^*$, respectively. The four diffuse components $h_{ij}(t)$ $(i, j = 1, 2)$ of the $A_T^{(j)} - A_R^{(i)}$ link can be combined to the so-called stochastic channel matrix

$$\mathbf{H}(t) = \begin{pmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{pmatrix} \tag{8}$$

which describes completely the reference model of the proposed two-ring MIMO frequency-nonselective Rayleigh fading channel. The capacity $C(t)$ of this channel in bits/s/Hz is defined as

$$C(t) := \log_2\left[\det\left(\mathbf{I}_2 + \frac{P_T}{M_T N_0}\mathbf{H}(t)\mathbf{H}^H(t)\right)\right] \tag{9}$$

where $\det(\cdot)$ denotes the determinant, $\mathbf{I}_2$ is the 2×2 identity matrix, P_T designates the total transmitted power allocated uniformly to all of the M_T antenna elements of the transmitter, N_0 is the noise power, and $(\cdot)^H$ denotes the complex conjugate (Hermitian) transpose operator.

B. The Space-Time CCF of the Reference Model

According to [5], the space-time CCF between the two links $A_T^{(1)}$ — $A_R^{(1)}$ and $A_T^{(2)}$—$A_R^{(2)}$ is defined as a correlation between the diffuse components $h_{11}(t)$ and $h_{22}^*(t)$, i.e.,

$$\rho_{11,22}(\delta_T,\delta_R,\tau) := E\{h_{11}(t)h_{22}^*(t+\tau)\}\,. \tag{10}$$

Here, the expectation operator applies on all random variables: phases $\{\theta_{m,n}\}$, AOD $\{\phi_T^{(m)}\}$, and AOA $\{\phi_R^{(n)}\}$. Starting from (1) and making use of the fact that $h_{22}(t)$ can be obtained from $h_{11}(t)$ by performing the above mentioned substitutions $a_m \to a_m^*$ and $b_n \to b_n^*$, we can express the space-time CCF as

$$\rho_{11,22}(\delta_T,\delta_R,\tau) := \lim_{\substack{M\to\infty\\ N\to\infty}} \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} E\{a_m^2 b_n^2 e^{-j2\pi f_n \tau}\}. \tag{11}$$

This result has been obtained by averaging only over the phases θ_{mn}. Note that a_m is a function of the angle of departure $\phi_T^{(m)}$, while b_n and f_n are functions of the angle of arrival $\phi_R^{(n)}$. If the number of local scatterers approaches infinity $(M, N \to \infty)$, then the discrete random variables $\phi_T^{(m)}$ and $\phi_R^{(n)}$ become continuous random variables ϕ_T and ϕ_R, each of which is characterized by a certain distribution, denoted by $p(\phi_T)$ and $p(\phi_R)$, respectively. The infinitesimal power of the diffuse components corresponding to the differential angles $d\phi_T$ and $d\phi_R$ is proportional to $p(\phi_T)p(\phi_R)\,d\phi_T\,d\phi_R$. As $M \to \infty$ and $N \to \infty$, this contribution must be equal to $1/(MN)$, i.e., $1/(MN) = p(\phi_T)p(\phi_R)d\phi_T d\phi_R$. Hence, it follows from (11) that the space-time CCF of the reference model can be expressed as

$$\rho_{11,22}(\delta_T,\delta_R,\tau) = \rho_T(\delta_T)\cdot\rho_R(\delta_R,\tau) \tag{12}$$

where

$$\rho_T(\delta_T) = \int_{-\pi}^{\pi} a^2(\delta_T,\phi_T)\,p(\phi_T)\,d\phi_T \tag{13}$$

and

$$\rho_R(\delta_R,\tau) = \int_{-\pi}^{\pi} b^2(\delta_R,\phi_R)e^{-j2\pi f(\phi_R)\tau}p(\phi_R)\,d\phi_R \tag{14}$$

are correlation functions related to the transmit side and the receive side, respectively, and

$$a(\delta_T,\phi_T) = e^{j\pi(\delta_T/\lambda)\cos(\phi_T-\alpha_T)} \tag{15}$$

$$b(\delta_R,\phi_R) = e^{j\pi(\delta_R/\lambda)\cos(\phi_R-\alpha_R)} \tag{16}$$

$$f(\phi_R) = f_{\max}\cos(\phi_R-\alpha_V)\,. \tag{17}$$

From (12), we observe that the space-time CCF $\rho_{11,22}(\delta_T,\delta_R,\tau)$ can be expressed as the product of the transmit correlation function $\rho_T(\delta_T)$ and the receive correlation function $\rho_R(\delta_R,\tau)$. This fact was first pointed out in [10]. For the one-ring model, however, this property does in general not hold [5,11]. The two-dimensional space CCF $\rho(\delta_T,\delta_R)$, defined as $\rho(\delta_T,\delta_R) := E\{h_{11}(t)h_{22}^*(t)\}$, equals the space-time CCF $\rho_{11,22}(\delta_T,\delta_R,\tau)$ at $\tau = 0$, i.e.,

$$\begin{aligned}\rho(\delta_T,\delta_R) &= \rho_{11,22}(\delta_T,\delta_R,0)\\ &= \rho_T(\delta_T)\cdot\rho_R(\delta_R)\end{aligned} \tag{18}$$

where we have used the notation $\rho_R(\delta_R) = \rho_R(\delta_R,0)$. The temporal ACF $r_{h_{ij}}(\tau)$ of the diffuse component $h_{ij}(\tau)$ of the link from $A_T^{(j)}$ to $A_R^{(i)}$ $(i,j=1,2)$ is defined by

$$r_{h_{ij}}(\tau) := E\{h_{ij}(t)\,h_{ij}^*(t+\tau)\} \tag{19}$$

and can be expressed as

$$r_{h_{ij}}(\tau) = \int_{-\pi}^{\pi} e^{-j2\pi f_{\max}\cos(\phi_R-\alpha_V)\tau}p(\phi_R)\,d\phi_R \tag{20}$$

for all $i,j\in\{1,2\}$. Notice that $r_{h_{ij}}(\tau)$ equals the space-time CCF $\rho_{11,22}(\delta_T,\delta_R,\tau)$ at $\delta_T=\delta_R=0$, i.e., $r_{h_{ij}}(\tau) = \rho_{11,22}(0,0,\tau) = \rho_T(0)\rho_R(0,\tau) = \rho_R(0,\tau)$. This result shows that the temporal ACFs of all four diffuse components are identical. It has been mentioned in [11] that this statement holds also for the one-ring model. The reference model described above is basically a theoretical model, which grounds on the assumption that the number of scatterers (N, M) are infinite. Although such a model is unrealizable, it is of central importance for the derivation of a stochastic simulation model that has approximately the same statistical properties as the reference model. To show this will be the topic of the next two sections.

IV. THE SIMULATION MODEL

In this section, we derive first a stochastic simulation model from the reference model, and then we focus on the corresponding deterministic simulation model, which is obtained from the stochastic one by fixing all model parameters.

A. The Stochastic Simulation Model

From the reference model described in Section III, we can easily derive a stochastic simulation model by performing the following three steps: (a) using finite values for the numbers of scatterers (M, N), (b) setting θ_0 to zero, and (c) considering the AOD $\phi_T^{(m)}$ and AOA $\phi_R^{(n)}$ as constants. Under these conditions, the diffuse component $h_{11}(t)$ of the reference model [see (1)] leads to the diffuse component

$$\hat{h}_{11}(t) = \frac{1}{\sqrt{MN}} \sum_{m=1}^{M} \sum_{n=1}^{N} g_{mn} e^{j(2\pi f_n t + \theta_{mn})} \qquad (21)$$

of the stochastic simulation model. In the above equation, g_{mn} and f_n are given by (2) and (6), respectively, and θ_{mn} are still i.i.d. random variables, each having a uniform distribution over $(0, 2\pi]$. Both the AOD $\phi_T^{(m)}$ ($m = 1, 2, \ldots, M$) and the AOA $\phi_R^{(n)}$ ($n = 1, 2, \ldots, N$) of the simulation model are henceforth constants, which will be determined in Section V. Thus, the inphase and the quadrature component of $\hat{h}_{11}(t)$ represent stochastic processes, each of which can be identified as a double sum-of-sinusoids with constant gains $g_{mn}/\sqrt{MN}$, constant Doppler frequencies f_n, and i.i.d. random phases θ_{mn}.

By analogy to the reference model, the other diffuse components $\hat{h}_{12}(t), \hat{h}_{21}(t)$, and $\hat{h}_{22}(t)$ can be obtained from $\hat{h}_{11}(t)$ by performing the following substitutions:

(a) If $a_m \rightarrow a_m^*$, then $\hat{h}_{11}(t) \rightarrow \hat{h}_{12}(t)$. (22)

(b) If $b_n \rightarrow b_n^*$, then $\hat{h}_{11}(t) \rightarrow \hat{h}_{21}(t)$. (23)

(c) If $a_m \rightarrow a_m^*$ and $b_n \rightarrow b_n^*$, then $\hat{h}_{11}(t) \rightarrow \hat{h}_{22}(t)$. (24)

With these relationships, the corresponding channel matrix $\hat{\mathbf{H}}(t) := [\hat{h}_{ij}]$ of the stochastic simulation model is completely defined. Hence, the statistics of the channel capacity

$$\hat{C}(t) := \log_2 \left[\det \left(\mathbf{I}_2 + \frac{P_T}{M_T N_0} \hat{\mathbf{H}}(t) \hat{\mathbf{H}}^H(t) \right) \right] \text{(bits/s/Hz)} \qquad (25)$$

can be analyzed by simulation. The software or hardware realization of $\hat{C}(t)$ is called the *(stochastic) capacity simulator* for a MIMO frequency-nonselective Rayleigh fading channel. The usefulness of such a simulator as a tool for analyzing the capacity of the two-ring model is demonstrated in an accompanying paper [12].

By analogy to (10), the space-time CCF between $\hat{h}_{11}(t)$ and $\hat{h}_{22}^*(t)$ is defined by

$$\hat{\rho}_{11,22}(\delta_T, \delta_R, \tau) := E\{\hat{h}_{11}(t) \hat{h}_{22}^*(t+\tau)\} \qquad (26)$$

where we have to take into account that the expectation operator applies now only on the random phases θ_{mn}, since all other parameters are constant. By using (21) and (24), it can be shown that (26) can be expressed in closed form as

$$\begin{aligned} \hat{\rho}_{11,22}(\delta_T, \delta_R, \tau) &= \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} a_m^2(\delta_T) b_n^2(\delta_R) e^{-j2\pi f_n \tau} \\ &= \hat{\rho}_T(\delta_T) \cdot \hat{\rho}_R(\delta_R, \tau) \qquad (27) \end{aligned}$$

where

$$\hat{\rho}_T(\delta_T) = \frac{1}{M} \sum_{m=1}^{M} a_m^2(\delta_T) \qquad (28)$$

and

$$\hat{\rho}_R(\delta_R, \tau) = \frac{1}{N} \sum_{n=1}^{N} b_n^2(\delta_R) e^{-j2\pi f_n \tau} \qquad (29)$$

are the transmit and receive correlation functions of the stochastic simulation model, respectively. Obviously, the property of separable transmit and receive antenna correlation functions holds for the stochastic simulation model as well. A further result is that the two-dimensional space CCF $\hat{\rho}(\delta_T, \delta_R)$ can be written as

$$\begin{aligned} \hat{\rho}(\delta_T, \delta_R) &= \hat{\rho}_{11,22}(\delta_T, \delta_R, 0) \\ &= \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} a_m^2(\delta_T) b_n^2(\delta_R) \\ &= \hat{\rho}_T(\delta_T) \cdot \hat{\rho}_R(\delta_R) \qquad (30) \end{aligned}$$

where $\hat{\rho}_R(\delta_R) = \hat{\rho}_R(\delta_R, 0)$. By analogy to (19), the temporal ACF of $\hat{h}_{ij}(t)$ can be expressed in closed form as

$$\begin{aligned} \hat{r}_{h_{ij}}(\tau) &:= E\{\hat{h}_{ij}(t) \hat{h}_{ij}^*(t+\tau)\} \\ &= \frac{1}{N} \sum_{n=1}^{N} e^{-j2\pi f_n \tau}, \ \forall\, i, j = 1, 2. \qquad (31) \end{aligned}$$

Notice that the temporal ACF $\hat{r}_{h_{ij}}(\tau)$ and the correlation functions introduced in (27)–(29) are related through $\hat{r}_{h_{ij}}(\tau) = \hat{\rho}_{11,22}(0, 0, \tau) = \hat{\rho}_T(0) \cdot \hat{\rho}_R(0, \tau) = \hat{\rho}_R(0, \tau)$.

B. The Deterministic Simulation Model

A stochastic process can be interpreted as a family of sample functions [13]. In our case, the stochastic process $\hat{h}_{ij}(t)$ is a family of time functions depending on the parameters θ_{mn}. A sample function of the stochastic process $\hat{h}_{ij}(t)$ is obtained if all phases θ_{mn} are fixed. A set of fixed phases, $\{\theta_{mn}\}$, can be obtained, e.g., by generating $M \cdot N$ outcomes of a random generator with a uniform distribution over $[0, 2\pi)$. To distinguish between stochastic processes $\hat{h}_{ij}(t)$ and sample functions, we denote the latter by $\tilde{h}_{ij}(t)$ and the corresponding channel matrix by $\tilde{\mathbf{H}}(t) := [\tilde{h}_{ij}(t)]$. Since $\tilde{h}_{ij}(t)$ is a deterministic function of t, it follows that the channel matrix $\tilde{\mathbf{H}}(t)$ is also time variant and deterministic. Hence, $\tilde{\mathbf{H}}(t)$ defines a deterministic simulation model for a MIMO frequency-nonselective Rayleigh fading channel. The correlation properties of deterministic space-time MIMO channel models have to be analyzed by using time averages rather than statistical averages. For this reason, the space-time CCF between $\tilde{h}_{11}(t)$ and $\tilde{h}_{22}(t)$ has to be computed by using

$$\tilde{\rho}_{11,22}(\delta_T, \delta_R, \tau) := < \tilde{h}_{11}(t)\tilde{h}_{22}^*(t+\tau) > \quad (32)$$

where $< \cdot >$ denotes the time average operator. Without proof, we state that the space-time CCF of the deterministic simulation model can be expressed in closed form as

$$\tilde{\rho}_{11,22}(\delta_T, \delta_R, \tau) = \hat{\rho}_{11,22}(\delta_T, \delta_R, \tau) + \Delta\tilde{\rho}_{11,22}(\delta_T, \delta_R, \tau) \quad (33)$$

where $\hat{\rho}_{11,22}(\delta_T, \delta_R, \tau)$ is given by (27) and

$$\Delta\tilde{\rho}_{11,22}(\delta_T, \delta_R, \tau) = \frac{1}{MN} \sum_{m=1}^{M} \sum_{\substack{m'=1 \\ m' \neq m}}^{M} \sum_{n=1}^{N} a_m a_{m'} b_n^2 c_{mn} c_{m'n}^* \cdot e^{-j(2\pi f_n \tau + \theta_{mn} - \theta_{m'n})}. \quad (34)$$

From the result in (34), we must conclude that the second term in (33) is generally different from zero. Therefore, it follows $\tilde{\rho}_{11,22}(\delta_T, \delta_R, \tau) \neq \hat{\rho}_{11,22}(\delta_T, \delta_R, \tau)$, stating that the proposed stochastic MIMO channel simulator is non-ergodic with respect to the space-time CCF. This result is in contrast to the ergodic MIMO channel simulator derived from the geometrical one-ring scattering model in [11].

V. PARAMETER COMPUTATION METHODS

In this section, we present two methods for the computation of the parameters determining the statistics of the simulation model. The first method is proposed for the case of isotropic scattering around the transmitter and receiver, where $p(\phi_T) = p(\phi_R) = 1/(2\pi)$ holds, while the other method can be applied to any given distributions of ϕ_T and ϕ_R. The number of discrete scatterers used in the simulation model is determined by the pair (M, N). For these parameters, the user must choose proper values by finding a compromise between complexity and precision. As a guideline for our methods, it is recommended to choose a value between 20 and 25 for M, while N should be in the range from 40 to 50. Hence, for any given pair (M, N) the only model parameters to be determined are the discrete AOD $\phi_T^{(m)}$ $(m = 1, 2, \ldots, M)$ and the discrete AOA $\phi_R^{(n)}$ $(n = 1, 2, \ldots, N)$. All other parameters of the stochastic simulation model are identical to the corresponding parameters of the reference model.

A. Extended Method of Exact Doppler Spread (MEDS)

The MEDS has originally been proposed in [4] as a high-performance parameter computation method for sum-of-sinusoids-based simulation models developed for single-input single-output Rayleigh fading channels in isotropic scattering environments. In case that the two-ring model is considered under isotropic scattering conditions, where $p(\phi_T) = p(\phi_R) = 1/(2\pi)$ holds, the original MEDS needs to be extended, which results in the following closed-form solutions:

$$\phi_T^{(m)} = \frac{\pi}{M}\left(m - \frac{1}{2}\right) + \alpha_T, \quad m = 1, 2, \ldots, M \quad (35)$$

$$\phi_R^{(n)} = \frac{2\pi}{N}\left(n - \frac{1}{2}\right) + \alpha_R, \quad n = 1, 2, \ldots, N. \quad (36)$$

Note that for large values of M and N, the AODs $\phi_T^{(m)}$ of the simulation model are restricted to the interval $(\alpha_T, \alpha_T + \pi)$, while the AOAs $\phi_R^{(n)}$ are within the interval $(0, 2\pi)$. The performance of this method is demonstrated in Section VI.

B. L_p-Norm Method (LPNM)

In case of non-isotropic scattering, we recommend using the LPNM, which is described in detail in [14]. This method is very general and can be applied to determine the sets $\{\phi_T^{(m)}\}_{m=1}^M$ and $\{\phi_R^{(n)}\}_{n=1}^N$ for any given distributions $p(\phi_T)$ and $p(\phi_R)$ proposed in the literature, including the von Mises distribution [5], the Gaussian distribution [6], the truncated Laplacian distribution [7, 8], and the truncated uniform distribution [9].

Using the LPNM to compute the model parameters $\phi_T^{(m)}$ and $\phi_R^{(n)}$ requires in case of the two-ring model the minimization of the following two error norms:

$$E_1^{(p)} := \left\{ w_1 \int_0^{\delta_{T,\max}} |\rho_T(\delta_T) - \hat{\rho}_T(\delta_T)|^p d\delta_T \right\}^{1/p} \quad (37)$$

$$E_2^{(p)} := \left\{ w_2 \int_0^{\delta_{R,\max}} \int_0^{\tau_{\max}} |\rho_R(\delta_R, \tau) - \hat{\rho}_R(\delta_R, \tau)|^p d\delta_R d\tau \right\}^{1/p} \quad (38)$$

where $w_1 = 1/\delta_{T,\max}$, $w_2 = 1/(\delta_{R,\max}\tau_{\max})$, and $p = 1, 2, \ldots$ In (37), the quantity $\delta_{T,\max}$ denotes the maximum transmit antenna spacing defining the range within the approximation $\rho_T(\delta_T) \approx \hat{\rho}_T(\delta_T)$ is of interest. Analogously, the quantities $\delta_{R,\max}$ and $\tau_{\max}$ in (38) define the domain within the fitting of $\hat{\rho}_R(\delta_R, \tau)$ to $\rho_R(\delta_R, \tau)$ is of importance. The minimization of the error norms $E_1^{(p)}$ and $E_2^{(p)}$ must be performed numerically, e.g., by using the Fletcher-Powell optimization algorithm [15]. Note that the parameters $\phi_T^{(m)}$ and $\phi_R^{(n)}$ can be optimized independently. This follows from the fact that $E_1^{(p)}$ and $E_2^{(p)}$ are independent of $\phi_R^{(n)}$ and $\phi_T^{(m)}$, respectively. We mention that another variant of the LPNM has also successfully been applied to the design of the MIMO channel simulator in [11], which has been proposed for the one-ring scattering model.

VI. PERFORMANCE EVALUATION

In this section, we assess the accuracy of the proposed stochastic simulation model by comparing its statistical properties with those of the reference model. From the results in Section III, it is clear that the transmit and receive correlation functions are proper statistical quantities enabling a fair performance analysis. The following results are based on the isotropic scattering assumption, i.e., $p(\phi_T) = p(\phi_R) = 1/(2\pi)$. The tilt angles α_T and α_R are defined as $\alpha_T = \alpha_R = \pi/2$, and the angle of motion $\alpha_{\rm v}$ is set to π. The ring radii R_T and R_R were equal to 10 m. A maximum Doppler frequency of $f_{\max} = 1$ Hz has been assumed, and the wave length λ was set to $\lambda = 0.15$ m. The parameters $\phi_T^{(m)}$ and $\phi_R^{(n)}$ of the stochastic simulation model have been computed by using the proposed extended MEDS with $M = 20$ and $N = 40$.

Under isotropic scattering conditions, the transmit and receive correlation functions of the reference model can be expressed in closed form. By using the integrals [16, eq. (3.338–4)], [16, eq. (3.339)], and the relationship [17, eq. (9.6.3)], we obtain

$$\rho_T(\delta_T) = J_0(2\pi\delta_T/\lambda) \quad (39)$$

$$\rho_R(\delta_R,\tau) = J_0\left(2\pi\sqrt{\left(\frac{\delta_R}{\lambda}\right)^2 + (f_{\max}\tau)^2 - 2g(\delta_R,\tau)}\right) \quad (40)$$

where $g(\delta_R,\tau) = (\delta_R/\lambda) f_{\max}\tau\cos(\alpha_R - \alpha_{\rm v})$. In the above equations, $J_0(\cdot)$ denotes the Bessel function of the first kind of order zero. From (39), (40), and (18), we may conclude that the two-dimensional space CCF $\rho(\delta_T,\delta_R)$ can be expressed as the product of two Bessel functions according to $\rho(\delta_T,\delta_R) = J_0(2\pi\delta_T/\lambda)\cdot J_0(2\pi\delta_R/\lambda)$. Finally, we notice that the temporal ACF $r_{h_{ij}}(\tau)$ of the reference model results in $r_{h_{ij}}(\tau) = \rho_R(0,\tau) = J_0(2\pi f_{\max}\tau)$ for all $i,j = 1, 2$.

The transmit correlation function $\rho_T(\delta_T)$ [see (39)] of the reference model and the corresponding correlation function $\hat{\rho}_T(\delta_T)$ [see (28)] of the stochastic simulation model are presented in Fig. 2. This figure also shows the simulation results for $\hat{\rho}_T(\delta_T)$ obtained by averaging over 150 trials. Notice that the presented results not only confirm the theory but they visualize as well that the approximation $\rho_T(\delta_T) \approx \hat{\rho}_T(\delta_T)$ is excellent in the range of 0 to $\delta_{T,\max}/\lambda = M/4 = 5$. Figure 3 illustrates the shape of reference model's receive correlation function $\rho_R(\delta_R,\tau)$ introduced in (40). For comparison, the received correlation function $\hat{\rho}_R(\delta_R,\tau)$ of the stochastic simulation is shown in Fig. 4. The results presented in this figure have been obtained by evaluating (29) in combination with (36) and the parameters as defined above. A comparison of the plots in Figs. 3 and 4 demonstrates that the approximation $\rho_R(\delta_R,\tau) \approx \hat{\rho}_R(\delta_R,\tau)$ is excellent in the region of interest, which is in the present case determined by a circle with radius $N/8$, i.e., $[(\delta_R/\lambda)^2 + (f_{\max}\tau)^2]^{1/2} \le N/8 = 5$. From the results shown in the Figs. 2—4, we may conclude that the stochastic space-time MIMO channel simulator has nearly the same temporal and spatial correlation properties as the reference model.

A performance investigation of the LPNM has revealed that this method outperforms the extended MEDS only sightly in case of isotropic scattering.

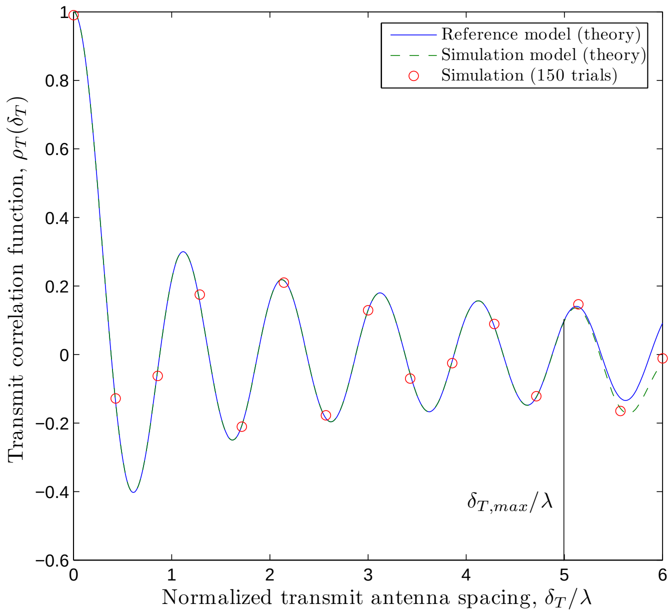

Figure 2: The transmit correlation functions $\rho_T(\delta_T)$ (reference model) and $\hat{\rho}_T(\delta_T)$ (stochastic simulation model) designed with the extended MEDS (M=20).

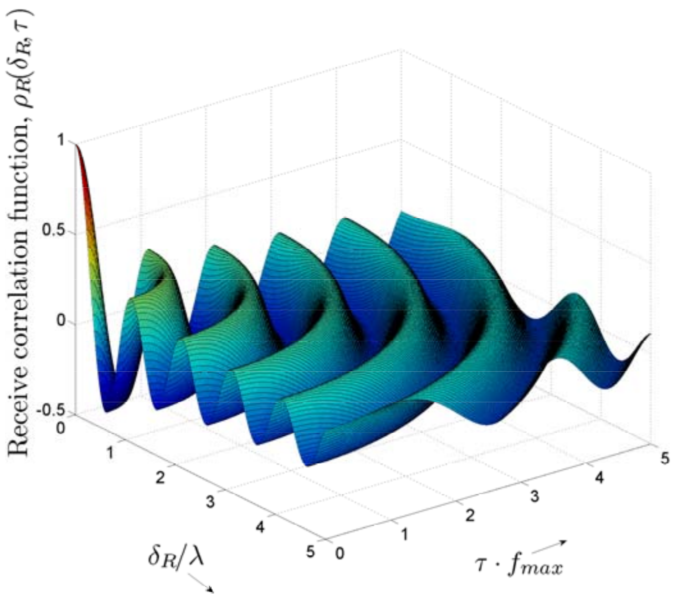

Figure 3: The receive correlation function $\rho_R(\delta_R,\tau)$ of the reference model.

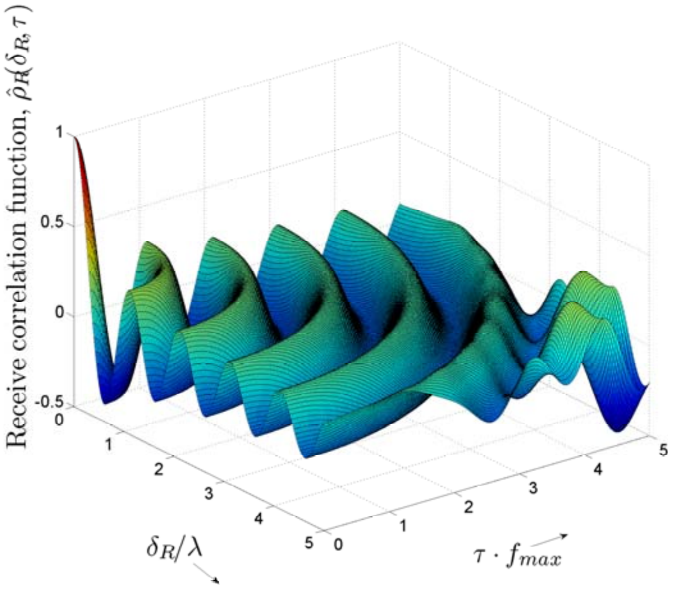

Figure 4: The receive correlation function $\hat{\rho}_R(\delta_R, \tau)$ of the stochastic simulation model (extended MEDS with $N = 40$).

VII. CONCLUSION

A general procedure has been proposed for the design of stochastic and deterministic simulation models for frequency-nonselective MIMO Rayleigh fading channels based on the geometrical two-ring scattering model. Starting from the geometrical scattering model via a non-realizable stochastic reference model, we have shown that a simulation model can easily be derived by extending the sum-of-sinusoid principle with respect to MIMO fading channels. The resulting MIMO channel simulator takes not only the spatial correlation properties at the transmit side and the receive side into account, but models also accurately the Doppler effect. Two methods have been proposed for the computation of the parameters of the simulation model — the extended MEDS and a variant of the L_p-norm method. The application of the former method is highly recommended for isotropic scattering scenarios, while the latter displays its full capabilities when the AOD and AOA are non-uniformly distributed, as it is generally the case in real-world environments. The performance study has shown that the space-time CCF of the stochastic simulation model approximates very well that of the reference model within an area that increases linearly with the number of discrete scatterers N and M. Due to its simplicity, acceptable complexity, and high precision, we believe that the proposed channel simulator is quite useful for MIMO systems whenever such systems need to be designed, tested, analyzed, and\or optimized. Moreover, the proposed MIMO channel simulator enables an exact analysis of the channel capacity by simulation.

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