A New Joint AOA/Delay Estimator for Wideband Spread Spectrum Systems

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***Abstract*—In this paper, we present a joint delay/AOA estimator for wideband spread spectrum (Wideband-SS) systems in a multiuser environment. The algorithm is applicable to space-time channel estimation for space-time processing systems and to location systems that employ hybrid AOA/TOA technology. The estimator is based on a novel formulation of the data model in which the delays and AOAs of each user can be estimated and automatically paired. The estimator avoids the computational burden of a matching, or pairing, procedure and does not require the use of a Fourier transform to aid the estimation of the delays. Further, the algorithm has lower computational complexity than other joint angle/delay estimators. The Cramér-Rao lower bound (CRLB) is derived. Simulations are presented to show the performance of the estimator.**

I. Introduction

Joint parameter estimation plays an important role in several application technologies. For example, some wireless location algorithms are based on hybrid AOA/TOA measurements [1]. In addition, channel parameters are required to enhance the signal reception of the transmitted signal in space-time processing systems, which is useful especially in multipath situations. By estimating the channel parameters, the received signals from different paths can be weighted and shifted according to those channel parameters to get a stronger signal, such as the algorithms in [2, 3], and in RAKE receivers [4].

Many papers present joint AOA/delay estimators based on the narrow band model, in which the AOA array response is in the form of exponentials. But this formulation is not applicable for a wide band system. In this paper we assume that the AOA will add to the delay in the received data at different antenna elements assuming that the chip time is much smaller in Wideband-SS system than the one in CDMA system.

When the AOAs and delays of each user are necessary, it is important to properly match the delays and AOAs of each user and do so as efficiently as possible. In systems in which it is desired to estimate the parameters of multiple users, the problem of matching the delays and AOAs for each user can be very complex. Rather than matching the measurements for each user through a complex pairing or matching procedure (such as [5]), it is of interest to estimate the channel parameters jointly in a procedure that also automatically pairs the parameters for each user. Therefore, it is of interest to find low complexity and high resolution joint AOA/delay estimators.

The AOA estimation for wideband signals are developed in [6–13]. In [6], the authors build on the IQML algorithm suggested by Bresler and Macoviski for the case of narrow band signals to extend it for wideband AOA estimation. In [7], the authors introduce an algorithm which takes a sub band of the signal and applies MUSIC to perform the AOA estimation. The authors in [8] develop the AOA estimation based on dividing the signal band into sub-bands, then a focusing autocorrelation matrix is calculated from all the sub-band autocorrelation matrices using neural networks. In [14], the authors develop a Unitary-ESPRIT algorithm to perform the AOA estimation. In [9], the authors apply MUSIC algorithm on chirp source signals to perform the AOA estimation. The AOA estimator in [10] uses an interpolation technique to generate virtual arrays each of different frequency having the same array manifold, the covariance matrices of these arrays are added up to produce a composite covariance matrix then applies MUSIC algorithm by eigendecomposition of the composite covariance matrix. The algorithm in [11] proposes a strategy that combines a robust near-optimal data-adaptive statistic, called the weighted average of weighted average of signal subspaces for robust wideband direction finding signal subspaces (WAVES), with an enhanced design of focusing matrices to ensure a statistically robust preprocessing of wideband data to make the estimation process using the focusing matrices more robust. In [12], the authors propose an STFD-based wideband root-MUSIC estimator to perform the AOA estimation. This technique employs an extended coherent signal subspace (CSS) principle involving coherent averaging over a preselected set of time-frequency points rather than the conventional frequency-only averaging procedure. The AOA estimator in [13] is based on the cyclic cross spectral density matrix of the fourier transform of the received signals. For the delay estimation, several algorithms were suggested for delay estimation in a CDMA system such as the ones in [15, 16]. Those delay estimators use subspace techniques to perform the delay estimation.

A few joint AOA/delay estimators for narrowband signals have been developed in the literature [5, 17–20], but not for the wideband signals. All of the estimators in [5, 17–20] apply for the multiuser, or multiple signal source, case. In this paper, we present a joint AOA/delay estimator for multiple users with multipath for each user for wideband-SS system. The estimator is based on a data model presented in [15]. Compared with the algorithms in [5, 17, 18], the new algorithm does not require the Fourier transformation of the received signal, nor does it require a pairing procedure to match the delays and their corresponding AOAs for each user. The algorithm requires less searching complexity than the methods in [19–21]. The algorithm is also applicable to wideband-SS systems

Section II introduces the system model that forms the foundation for our estimators. Sections III presents the joint AOA/delay estimator. Section IV presents the Cramér-Rao Lower Bound for the estimators. Section V provides the simulated performance of the joint AOA/delay estimators, while conclusions are drawn in Section VI.

II. System Model

We assume an M element antenna array at a base station (BS) in a system with K users and L multipaths for each user. A wideband-SS system is considered that employs BPSK modulation with data bit duration T and chip duration $T_c = T/Q$, where Q is an integer. The code waveforms are rectangular with unit amplitude and of period QT_c. The baseband signal of the kth user, $s_k(t)$, is formed by modulating the data stream, $d_k(i)$, with a period of the code waveform, $b_k(t)$, as

$$s_k(t) = \sum_{i=-\infty}^{\infty} d_k(i) b_k(t - iT) \quad (1)$$

where i indicates the data symbol and $b_k(t)$ is the spreading code wave form consisting of rectangular chip waveforms. The received signal at the mth antenna element is

$$\begin{aligned} r_m(t) &= n_m(t) + \sum_{k=1}^{K}\sum_{l=1}^{L} \beta_{k,l} s_k\left(t - \tau_{k,l}\right) \\ &\cdot \cos\left(2\pi f_c(t - \tau_{k,l,m}\right) \end{aligned}$$

where f_c is the carrier frequency, $n_m(t)$ is additive white Gaussian noise with power spectral density $N_o/2$, $\tau_{k,l,m}$ is the excess delay due to distance between the antenna elements and is given by $\tau_{k,l,m} = \tau_{k,l} + (m-1)d_s \sin(\theta_{k,l})/c$, d_s is the distance between the two consecutive antenna elements (which assumed constant over each consecutive elements), m is the index for the antenna element, and it has the values between $[1, \cdots, M]$, and $\beta_{k,l}$, $\theta_{k,l}$, and $\tau_{k,l}$ are the complex propagation factor ($\beta_{k,l} = \iota_{k,l} e^{j\phi_{k,l}}$), arrival angle and delay of the lth path of the kth user, respectively.

Following the data model in [15], after an I-Q stage followed by chip-rate sampling in the receiver, the received signal at the antenna array is given by

$$\mathbf{r} = \mathbf{A}_{TS}\mathbf{Z} + \mathbf{n} \quad (2)$$

where

$$\mathbf{A}_{\text{TS}} = \begin{bmatrix} \mathbf{A}_{1,1,1} & \cdots & \mathbf{A}_{k,l,1} & \cdots & \mathbf{A}_{K,L,1} \\ \vdots & & \vdots & & \vdots \\ \mathbf{A}_{1,1,M} & \cdots & \mathbf{A}_{k,l,M} & \cdots & \mathbf{A}_{K,L,M} \end{bmatrix} \quad (3)$$

$$\mathbf{Z} = \left[\beta_{1,1}\mathbf{z}_1^T \quad \cdots \quad \beta_{k,l}\mathbf{z}_k^T \quad \cdots \quad \beta_{K,L}\mathbf{z}_K^T \right]^T \quad (4)$$

and $\mathbf{z}_k$ contains the data symbol information, and $\mathbf{A}_{k,l,m}$ contains the delay information and the array response vector for lth path of the kth user and mth element, respectively. The definition of $\mathbf{z}_k$ is

$$\mathbf{z}_k = \left[z_{2k-1} \quad z_{2k} \right]^T \quad (5)$$

with

$$\begin{aligned} z_{2k-1}(n) &= 0.5\left[d_k(n) + d_k(n-1)\right] \\ z_{2k}(n) &= 0.5\left[d_k(n) - d_k(n-1)\right], \end{aligned}$$

n is the time index, and $d_k(n)$ is the data bit for the kth in the nth symbol interval. The matrix $\mathbf{A}_{k,l,m} = \left[\mathbf{a}_{2k-1,l,m} \quad \mathbf{a}_{2k,l,m} \right]$ where

$$\mathbf{a}_{2k-1,l,m} = \left[\frac{\delta_{k,l,m}}{T_c}\mathbf{D}(p_{k,l,m}+1,1) + \left(1 - \frac{\delta_{k,l,m}}{T_c}\right)\mathbf{D}(p_{k,l,m},1) \right]\mathbf{c}_k \quad (6)$$

$$\mathbf{a}_{2k,l,m} = \left[\frac{\delta_{k,l,m}}{T_c}\mathbf{D}(p_{k,l,m}+1,-1) + \left(1 - \frac{\delta_{k,l,m}}{T_c}\right)\mathbf{D}(p_{k,l,m},-1) \right]\mathbf{c}_k, \quad (7)$$

the delay $\tau_{k,l,m}$ (to be estimated) is $\tau_{k,l,m} = p_{k,l,m}T_c + \delta_{k,l,m}$, where $p_{k,l,m}$ is an integer, and $\delta_{k,l,m}$ is the remainder after dividing $\tau_{k,l,m}$ by T_c. The vector $\mathbf{c}_k$ is a vector with its n^{th} component given by the n^{th} chip of the spreading sequence:

$$c_k^n = \frac{1}{T_c}\int_{(n-1)T_c}^{nT_c} b_k(t)dt. \quad (8)$$

The waveform $b_k(t)$ is the code sequence. The vector $\mathbf{c}_k$ is defined as

$$\mathbf{c}_k = \left[c_k^1 \quad \cdots \quad c_k^n \quad \cdots \quad c_k^Q \right].$$

The matrix

$$\mathbf{D}_{(p,s)} = \begin{bmatrix} 0 & \mathbf{I}_{Q-p} \\ s\mathbf{I}_p & 0 \end{bmatrix}$$

where Q is the size of the code sequence (number of chips per bit). $\mathbf{n}$ is the noise vector after the I-Q stage and it has a variance of $\sigma^2 = N_o/2$. We also define the vectors $(\mathbf{a}_{2k,l}, \mathbf{a}_{2k-1,l})$ as

$$\begin{aligned} \mathbf{a}_{2k,l} &= \left[\mathbf{a}_{2k,l,1}^T \quad \cdots \quad \mathbf{a}_{2k,l,M}^T \right]^T \\ \mathbf{a}_{2k-1,l} &= \left[\mathbf{a}_{2k-1,l,1}^T \quad \cdots \quad \mathbf{a}_{2k-1,l,M}^T \right]^T \end{aligned}$$

where $\mathbf{a}_{2k,l}$ and $\mathbf{a}_{2k-1,l}$ are $QM \times 1$ vectors. The vectors $\mathbf{a}_{2k,l,m}$ and $\mathbf{a}_{2k-1,l,m}$ are $Q \times 1$ and are shown as

$$\begin{aligned} \mathbf{a}_{2k,l,m} &= \left[a_{2k,l,m}^1 \quad \cdots \quad a_{2k,l,m}^n \quad \cdots \quad a_{2k,l,m}^Q \right]^T \\ \mathbf{a}_{2k-1,l,m} &= \left[a_{2k-1,l,m}^1 \quad \cdots \quad a_{2k-1,l,m}^n \quad \cdots \quad a_{2k-1,l,m}^Q \right]^T . \end{aligned}$$

From the formulation above, we find that the delay and AOA information for the different users are included in $\mathbf{A}_{TS}$.

III. Joint Estimation

If we assume the received signal vector at the l^{th} time is $\mathbf{R}_x(l)$, then the autocorrelation matrix, $\mathbf{U}_T^{xx}$, is given by

$$\mathbf{U}_T^{xx} = E[\mathbf{R}_x^H(l)\mathbf{R}_x(l)] = \mathbf{A}_{\text{TS}}\mathbf{P}_s\mathbf{A}_{\text{TS}}^H + \sigma^2\mathbf{I} \quad (9)$$

If we look at the eigenvalues of $\mathbf{U}_T^{xx}$, and from the definition of the data matrix $\mathbf{Z}$, we see that $\mathbf{U}_T^{xx}$ has pairs of equal eigenvalues that correspond to each path of each user. Thus, for the

lth path of the kth user, we can express the eigenvalues of $\mathbf{U}_T^{xx}$, $(\lambda_{k,l,1}, \lambda_{k,l,2})$, and their corresponding eigenvectors, $(\mathbf{v}_1, \mathbf{v}_2)$, as a linear combination of the two vectors (the column vectors of the A_{TS}), $\mathbf{a}_{2k-1,l}$ and $\mathbf{a}_{2k,l}$:

$$\mathbf{v}_1 = q_1\mathbf{a}_{2k,l} + q_2\mathbf{a}_{2k-1,l}$$
$$\mathbf{v}_2 = q_3\mathbf{a}_{2k,l} + q_4\mathbf{a}_{2k-1,l}$$

where (q_1, q_2, q_3, q_4) are unknown constants while $\mathbf{a}_{2k}$ and $\mathbf{a}_{2k-1,l}$, and consequently $\mathbf{v}_i$, $i = 1, 2$, are functions of the unknown delay $\tau_{k,l}$.

Next, we need to evaluate the coefficients q_1, q_2, q_3, and q_4, but since the two eigenvectors provide redundant information, we only focus on evaluating q_1 and q_2. Since v_1 and v_2 are defined in terms of $a_{2k,l}$ and $a_{2k-1,l}$ which in turn are vectors composed of M subvectors $a_{2k-1,l,m}$ and $a_{2k,l,m}$, we define the mth subvector of v_i ($i = 1, 2$), as $v_{i,m}$. Furthermore, from the definitions of $a_{2k,l,m}$ and $a_{2k-1,l,m}$, we find that the jth element of the vector $v_{1,m}$ is $v_{1,m}^{(j)} = q_1 a_{2k,l,m}^{(j)} + q_2 a_{2k-1,l,m}^{(j)}$. We can notice from (6) and (7) that when $j < p_{k,l,m}$, we find $a_{2k-1,l,m}^{(j)} = a_{2k,l,m}^{(j)}$, and when $j > p_{k,l,m}$, we find $a_{2k-1,l,m}^{(j)} = -a_{2k,l,m}^{(j)}$. Given these, we define three conditions on which to evaluate q1 and q2.

$$\begin{aligned} &\text{if} \quad 0 < p_{k,m} < Q-1 \\ &\quad q_1 + q_2 = v_{1,m}^1 / a_{2k-1,l,m}^1 \\ &\quad q_1 - q_2 = v_{1,m}^Q / a_{2k-1,l,m}^Q. \end{aligned}$$

For the above case, we have two equations with two unknowns, and by adding the two equations and dividing by two we can get q_1, and by subtracting the two equations and dividing by two we can get q_2. For the second condition

$$\begin{aligned} &\text{if} \quad p_{k,l,m} = 0 \\ &\quad \text{let} \quad \kappa = \begin{bmatrix} v_{1,m}^1 / a_{2k-1,l,m}^1 \\ v_{1,m}^{Q-p_{k,l,m}} \end{bmatrix} \\ &\quad \mathbf{C} = \begin{bmatrix} 1 & 1 \\ a_{2k-1,l,m}^{Q-p_{k,l,m}} & a_{2k,l,m}^{Q-p_{k,l,m}} \end{bmatrix} \\ &\quad \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \mathbf{C}^{-1}\kappa. \end{aligned}$$

Similarly for the third condition

$$\begin{aligned} &\text{if} \quad p_{k,l,m} = Q-1 \\ &\quad \text{let} \quad \kappa = \begin{bmatrix} v_{1,m}^Q / \mathbf{a}_{2k-1,l,m}^Q \\ v_{1,m}^{Q-p_{k,l,m}} \end{bmatrix} \\ &\quad \mathbf{C} = \begin{bmatrix} 1 & -1 \\ a_{2k-1,l,m}^{Q-p_{k,l,m}} & a_{2k,m}^{Q-p_{k,l,m}} \end{bmatrix} \\ &\quad \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \mathbf{C}^{-1}\kappa. \end{aligned}$$

Since the delay and the excess delay (due to the AOAs) are unknowns, we search over different values of $p_{k,l,m}$ for $\delta_{k,l,m}$ which will minimize the function $h(\tau)$, i.e.,

$$\hat{\tau}_{k,l,m} = \arg\,(\min_{\tau}\{h(\tau)\}). \tag{10}$$

where

$$\begin{aligned} h(\tau) \;=\; & (\mathbf{A}_{k,l,m}\mathbf{Q}_1 - \mathbf{v}_{1,m})^T(\mathbf{A}_{k,l,m}\mathbf{Q}_1 - \mathbf{v}_{1,m}) \\ & + (\mathbf{A}_{k,l,m}\mathbf{Q}_2 - \mathbf{v}_{2,m})^T(\mathbf{A}_{k,l,m}\mathbf{Q}_2 - \mathbf{v}_{2,m}) \end{aligned} \tag{11}$$

where

$$\mathbf{Q}_1 = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{12}$$

$$\mathbf{Q}_2 = \begin{bmatrix} q_3 \\ q_4 \end{bmatrix}. \tag{13}$$

The reason for choosing to minimize the function $h(\tau)$ is because it is a measure of the vector $\mathbf{A}_{k,l,m}$ error power. Since we are considering L multipaths for each user, we consider the lowest L different minima for $h(\tau)$ and their corresponding $p_{k,l,m}$ and $\delta_{k,l,m}$ for each user to evaluate the delay and AOA for each path for that user.

From the above we see that the delay can be evaluated by evaluating $\tau_{k,l,1}$, and the AOA can be evaluated by evaluating $\tau_{k,l,m}$ for different values of m, then

$$\theta_{k,l} = \sin^{-1}\left(\frac{1}{M-1}\sum_{m=2}^{M}\frac{(\tau_{k,l,m} - \tau_{k,l,m-1})}{(m-1)d_s/c}\right)$$

So we can see that each pair of eigenvector contains the delay and the AOA for each path of each user. We also notice that for a certain path, l, of a certain user, k, by evaluating $\hat{\tau}_{k,l,m}$ in (10) over different values of m we will be evaluating the delay and excess delay which are contained in the same eigenvector. Thus, we can find the delay and AOA for each path of each user with automatic matching.

IV. Cramér-Rao Lower Bound

The Cramér-Rao Lower Bound (CRLB) is given by the inverse of the Fisher matrix (FIM). To compute the FIM we first define the parameter vector $\mathbf{\Upsilon} = [\sigma, \boldsymbol{\iota}, \mathbf{\Phi}, \boldsymbol{\tau}, \mathbf{\Theta}]$, where

$$\begin{aligned} \boldsymbol{\iota} &= [\,\iota_{,11} \ \cdots \ \iota_{k,l} \ \cdots \ \iota_{K,L}\,]^T \\ \mathbf{\Phi} &= [\,\phi_{1,1} \ \cdots \ \phi_{k,l} \ \cdots \ \phi_{K,L}\,]^T \\ \boldsymbol{\tau} &= [\,\tau_{1,1} \ \cdots \ \tau_{k,l} \ \cdots \ \tau_{K,L}\,]^T \\ \mathbf{\Theta} &= [\,\theta_{1,1} \ \cdots \ \theta_{k,l} \ \cdots \ \theta_{K,L}\,]^T \end{aligned}$$

Since the noise is AWGN, then the probability of error for the received signal is expressed as

$$H = \prod_{n=1}^{W} P(n)$$

where $P(n)$ is the probability of the received signal error at the L elements at the the nth time instant, and is expressed as

$$P(n) = \frac{1}{(2\pi\sigma^2)^{LQ}}\exp\left(-\frac{||\mathbf{r}(n) - \mathbf{A}_{\mathrm{TS}}\mathbf{z}(n)||^2}{\sigma^2}\right)$$

The log-likelihood function is given by

$$\ln\mathrm{H} = \mathrm{constant} - \mathrm{LQW}\ln\sigma^2 - \frac{1}{\sigma^2}\sum_{n=1}^{W} ||\mathbf{r}(n) - \mathbf{A}_{\mathrm{TS}}\mathbf{z}(n)||^2$$

The Cramér-Rao lower bound is given

$$\mathbf{CRLB}(\boldsymbol{\Upsilon}) = \mathbf{J}^{-1}$$

where $\mathbf{J}$ is the FIM and is given by

$$\mathbf{J} = E\left[\left(\frac{\partial \ln\mathrm{H}}{\partial \boldsymbol{\Upsilon}}\right)\ \left(\frac{\partial \ln\mathrm{H}}{\partial \boldsymbol{\Upsilon}}\right)^T\right] \quad (14)$$

The elements of the fisher matrix, $\mathbf{J}$, are shown in the appendix. Thus, the Cramér-Rao lower bounds for θ and τ can be simplified and expressed as

$$\mathbf{CRLB}(\boldsymbol{\theta}) = \left(\mathbf{J}_{\boldsymbol{\theta\theta}} - \mathbf{J}^T_{\boldsymbol{\phi\theta}}\mathbf{J}^{-1}_{\boldsymbol{\phi\phi}}\mathbf{J}_{\boldsymbol{\phi\theta}}\right)^{-1}$$

$$\mathbf{CRLB}(\boldsymbol{\tau}) = \left(\mathbf{J}_{\boldsymbol{\tau\tau}} - \mathbf{J}^T_{\boldsymbol{\iota\tau}}\mathbf{J}^{-1}_{\boldsymbol{\iota\iota}}\mathbf{J}_{\boldsymbol{\iota\tau}}\right)^{-1} \quad .$$

V. Simulation Results

Simulations of the joint AOA/delay estimator for WDCDMA were completed to assess its performance. For the simulations, the chip time, T_c, and the carrier frequency were set to 0.05μ sec and 30MHz, respectively. The reason for considering this setup is that we consider that wideband signals take more time to cross the antenna array than the chip time. The propagation constants, β_k, were obtained from a Rayleigh time varying fading channel, which is simulated using modified Jakes model (the model is described in [22]). The spreading codes were generated from Gold-sequences. Each code contains 31 chips. The spacing between the elements were set to half the wavelength.

Figs. 1 and 2 show the first user's root-mean-square (RMS) error for the AOA and delay, respectively, for varying numbers of users with $M = 6$ and SNR=10 dB for all users. For those two figures, to see the effect of multiuser only, we assume no multipath. The AOA for the first, second and third users were $15°$, $45°$ and $65°$, respectively, while their delays were $5.1T_c$, $7.6T_c$ and $9.26T_c$, respectively. The results show that as we increase the number of users the performance is degraded.

Figs. 3 and 4 show the first user's RMS error of the AOA and delay estimates, respectively, in a two users system with two multipath for each user over various numbers of antenna elements with SNR=10 dB for both users. For those two figures, we assume the AOA for the first and second path for the first and second users were $15°$, $45°$, $55°$ and $70°$, respectively, while their delays were set to $5.1T_c$, $7.6T_c$, $9.26T_c$ and $18.3T_c$, respectively. The results show that the performance of the AOA and delay estimation are generally constant as we increase the number of antenna elements.

VI. Conclusions

We presented a novel joint AOA/delay estimator that is capable of estimating the delays and arrival angles for each path for each user in a wideband-SS system. The estimator does an automatic pairing between the AOAs and delays from the structure of the eigenvectors, that flow from the structure of the underlying data model.

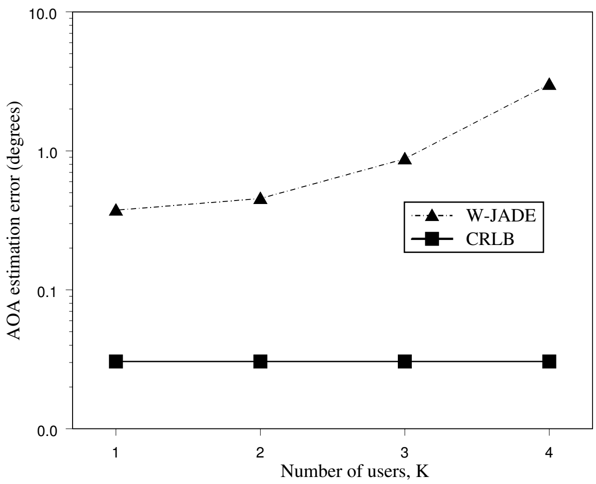

Fig. 1. RMS error for AOA estimation for different numbers of antenna elements with SNR=10 dB.

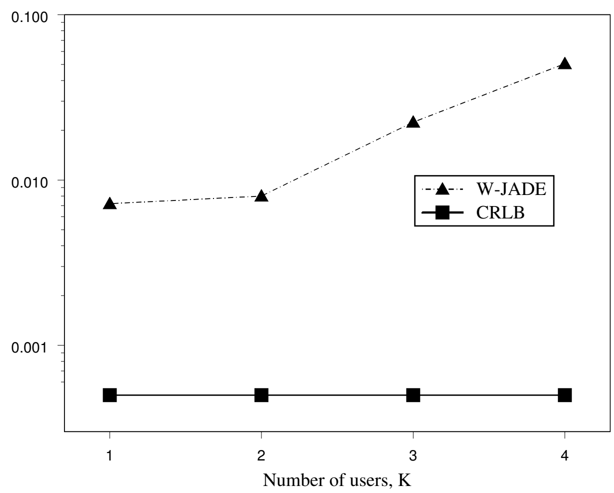

Fig. 2. RMS error for delay estimation for different numbers of antenna elements with SNR=10 dB.

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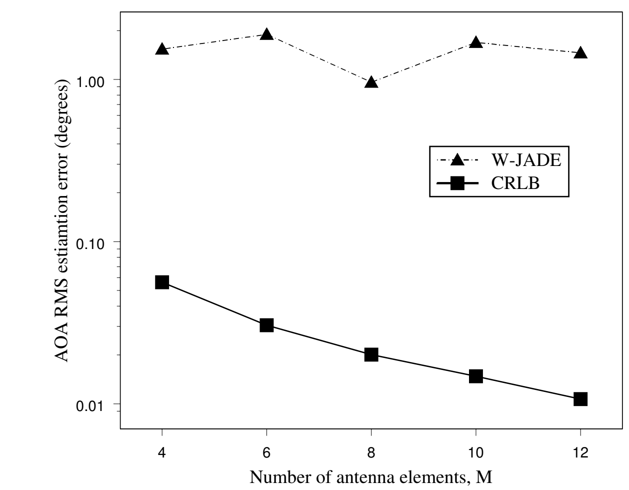

Fig. 3. RMS error for AOA estimation for different numbers of antenna elements with SNR=10 dB.

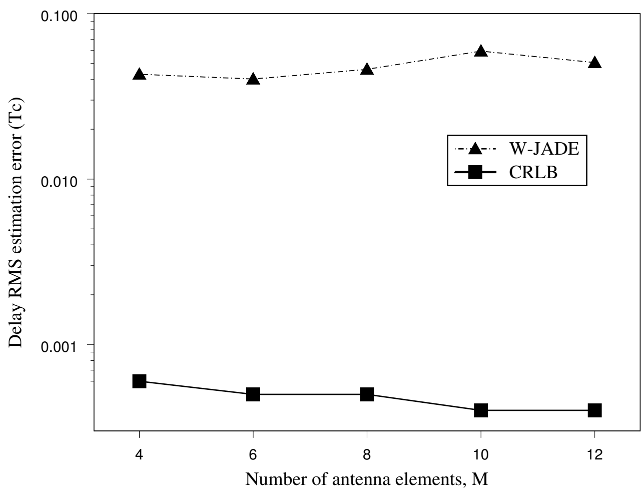

Fig. 4. RMS error for delay estimation for different numbers of antenna elements with SNR=10 dB.

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