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Wide Angle SAR Imaging Based on LS-CS-Residual

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***Abstract*—Wide angle synthetic aperture radar (WASAR) receives data from a large angle, which brings the problem of aspect dependent scattering. The subaperture approach can accommodate for it. However, the scattering recovery is not so accurate. In this paper, we propose a novel WASAR imaging method based on least squares on compressed sensing residual (LS-CS-Residual). LS-CS-Residual is to replace compressed sensing (CS) on the observation by CS on the least squares (LS) residual computed using the previous estimate of the support. It can achieve accurate recovery compared with CS. The support sets of the subapertures are highly overlapped across the whole aperture. We first implement backprojection (BP) on the whole aperture data and estimate the support from the BP image. Then we perform LS-CS-Residual on the subapertures. The WASAR imaging based on LS-CS-Rresidual can recover the aspect dependent scattering better than CS and can recover the scene with less artifacts. Real data processing results validate the proposed method.**

***Index Terms*—wide angle, aspect dependent, LS-CS-Residual.**

I. INTRODUCTION

Wide angle synthetic aperture radar (WASAR) receives echoes from a large angle. The azimuth resolution can be increased and more scattering of the targets can be obtained. However, it brings the problem of aspect dependent scattering [1] [2]. Aspect dependent scattering is brought by especially the man-made targets [3]. The traditional imaging methods are based on the isotropic assumption which means that the scattering is isotropic in the synthetic aperture angle. It does not hold for WASAR.

To accommodate the aspect dependent scattering, there are mainly two approaches, the subaperture approach and full aperture approach. The subaperture approach [1] [2] divides the whole aperture into the subapertures and assumes that the scattering holds in the subaperture. Then the narrow angle imaging methods such as matched filtering, non-quadratic optimization can be adopted for the subaperture imaging. For the full aperture approach, they can be divided into two kinds. The first one assumes that the scattering during one subaperture is isotropic and construct imaging models with all subapertures included [4-6]. The imaging models are recovered jointly. The other is the parametric method [7-9]. It assumes that the scene includes some scattering targets and their scattering follows some functions. The scattering functions of the targets are fitting with the whole aperture data included. The subaperture approach often faces inaccurate aspect dependent scattering recovery. During some downsampled rate, there are also some artifacts remained in the recovered image.

In this paper, we proposed a novel subaperture imaging method based on least squares on compressed sensing residual (LS-CS residual) [10]. The proposed method firstly implements Backprojection (BP) on the whole aperture data. Then the coarse support set is estimated from the BP image. The least squares estimate on the support set is calculated. Then the observation residual is calculated. With the residual data, we can solve the residual observation model with L_1 regularization. The accurate supports of subaperture images are estimated from the L_1 regularization image. Finally, the LS estimate on the accurate supports is calculated. The proposed method can recover the aspect dependent scattering more accurately than the subaperture method. The result of LS-CS has less artifacts.

This paper is organised as follows. In Section II, we introduce the WASAR imaging model. Then LS-CS-Residual based WASAR imaging method is proposed. In Section IV, we use the simulations and experiments to validate the proposed method. Finally, the conclusion is provided in Section V.

II. WASAR SUBAPERTURE IMAGING MODEL

WASAR receives echoes from a large angle. The geometry of WASAR is shown in Fig. 1 .

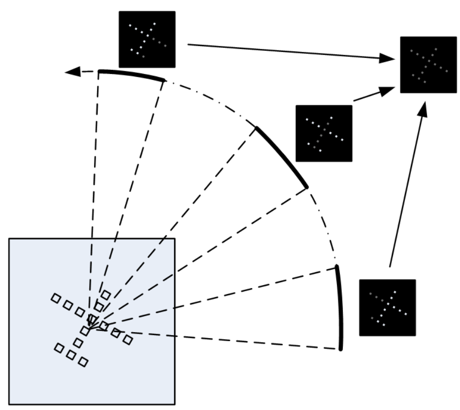

Fig. 1. The geometry of WASAR.

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The subaperture imaging method divides the whole aperture into subapertures. Subaperture data are processed individually. We consider the i-th subaperture at aspect angle θ_i. The phase history data is formulated as

$$r_i(f_p,\theta_q)=\sum_{m=1}^{M}\sum_{n=1}^{N}s_i(x_m,y_n)\cdot\exp\{-j\frac{4\pi f_p}{c}\cdot (x_m\cos(\theta_q)+y_n\sin(\theta_q))\}+z_i \tag{1}$$

where r is the phase history data, s is the scattering reflectivity located at (x_m,y_n), f_p $(p=1,2\cdots P)$ is the frequency, c is the light velocity, θ_q $(q=1,2\cdots Q)$ is the aspect angle, z_{θ_i} is the noise at aspect angle θ_i.

(1) can be expressed in a compact form

$$\boldsymbol{r}_i=\boldsymbol{\Phi}_i\cdot\boldsymbol{s}_i+\boldsymbol{z}_i \tag{2}$$

where $\boldsymbol{r}_i$ is the history data of θ_i in vector form, $\boldsymbol{s}_i s$ is the scattering of i, $\boldsymbol{\Phi}_i$ is the measurement matrix, z_i is the noise.

The subaperture imaging methods for WASAR imaging assume that the scattering of the scattering is not relevant to the aspect angle in a little angle range. Then traditional imaging method such as BP, feature enhanced method can be implemented for subaperture image focusing. When the scene is sparse and the matrix satisfies RIP [14], (2) can be solved via the following expression

$$\min_{\boldsymbol{s}_i}\|\boldsymbol{r}_i-\boldsymbol{\Phi}_i\cdot\boldsymbol{s}_i\|_2^2+\lambda\|\boldsymbol{s}_i\|_1. \tag{3}$$

III. WASAR IMAGING BASED ON LS-CS-RESIDUAL

In this section, we propose a novel WASAR imaging based on LS-CS-Residual. We first introduce LS-CS-Residual. Then the detailed algorithm is given. To avoid the high memory cost in the real scene imaging, the azimuth-range decouple scheme is adopted in LS-CS-Residual based WASAR imaging.

A. LS-CS-Residual

CS focuses on the simple sparsity. In real world, there exists other sparse pattern. For example, the support of the vector slowly changes with time. To solve the aforesaid problem, one can solve it with CS. However, it indicates that the prior information is not used in the recovery, which would need more measurement. LS-CS-Residual [10] has been proposed for dynamic CS problems. The key idea of LS-CS-Residual is to replace CS on the least squares residual computed using the previous support estimation. It is shown that it needs less samples and the bounded reconstruction error is less than the traditional CS. For Eq. (1), if the support set of $\boldsymbol{s}_i$ is known, we could simply compute the LS estimate on the support while setting all other values to zeros. The previous support can be estimated from the prior information. Suppose the estimated support is T, to compute and initial LS estimate

$$(\boldsymbol{s}_{i,init})_T=(\boldsymbol{\Phi_{i}}_T)^{\dagger}\boldsymbol{r}_i,(\boldsymbol{s}_{i,init})_{T^c}=0 \tag{4}$$

where $\boldsymbol{\Phi_i}_T^{\dagger}=\left(\boldsymbol{\Phi_i}_T^H\boldsymbol{\Phi_i}_T\right)^{-1}\boldsymbol{\Phi_i}_T^H$, T^C denotes the complement of T.

Then the LS residual is calculated as

$$\boldsymbol{r}_{i,res}=\boldsymbol{r}_i-\boldsymbol{\Phi}_i\boldsymbol{s}_{i,init}. \tag{5}$$

LS-CS-Residual includes the following steps.

Firstly, the LS estimation is calculated with (4). Then the LS residual can be calculated from (5). CS is implemented on the LS residual

$$\min\|\boldsymbol{r}_{i,res}-\boldsymbol{\Phi}_i\boldsymbol{\beta}_i\|_2^2+\lambda\|\boldsymbol{\beta}_i\|_1. \tag{6}$$

Iterative shrinkage thresholding algorithm (ISTA) [18] can be adopted to solve (6). ISTA is used to solve (6). The iteration is formulated as

$$\hat{\boldsymbol{\beta}}_i^t=\boldsymbol{\beta}_i^t+\mu\left[\boldsymbol{\Phi}_i^H(\boldsymbol{r}_i-\boldsymbol{\Phi}_i\boldsymbol{\beta}_i^t)\right], \tag{7}$$

$$\boldsymbol{\beta}_i^{t+1}=f_{\lambda\mu}\left(\hat{\boldsymbol{\beta}}_i^t\right)=\begin{cases}\mathrm{sgn}(\hat{\boldsymbol{\beta}}_i^t)(|\hat{\boldsymbol{\beta}}_i^t|-\lambda\mu), & \text{if } |\hat{\boldsymbol{\beta}}_i^t|>\lambda\mu\\ 0, & \text{otherwise.}\end{cases} \tag{8}$$

where $\mu\in(0,\|\boldsymbol{\Phi_i}\|_2^{-2})$ is the step size controlling the convergence, λ is the regularization parameter, f is the iterative function of ISTA. In the iteration, the value of λ is

$$\lambda=|\hat{\boldsymbol{\beta}}_i^t|_{K+1}/\mu \tag{9}$$

where $|\hat{\boldsymbol{\beta}}_i^t|_{K+1}$ is the $(K+1)$-th largest element of $\hat{\boldsymbol{\beta}}_i^t$ and $K=\|\hat{\boldsymbol{\beta}}_i^t\|_0$.

The final estimation is

$$\hat{\boldsymbol{s}}_i=\boldsymbol{\beta}_i+\boldsymbol{s}_{i,\mathrm{init}}. \tag{10}$$

It is shown that $\boldsymbol{\beta}_i$ is obtained after L_1 regularization, the estimation will be biased to towards zeros. Thus a debiased step is needed

$$T'=\mathrm{supp}(\hat{\boldsymbol{s}}_i), \tag{11}$$

$$\boldsymbol{s}_{iT}=(\boldsymbol{\Phi}_{iT'})^{\dagger}\boldsymbol{r}_i,\boldsymbol{s}_{iT'^C}=0. \tag{12}$$

After the construction of the subaperture images, the generalized likelihood ratio test (GLRT) [2] can be implemented for the final composite image. GLRT is defined as

$$s_{x,y}=\max_i|s_{x,y}^i| \tag{13}$$

where $s_{x,y}^i$ is the scattering at pixel (x,y).

B. LS-CS-Residual Based WASAR Imaging

In WASAR, the scattering of the targets is aspect dependent. However, the support sets of the subaperture images are highly overlapped, which means that a fairly accurate support T can be estimated from the data. T is estimated via

$$T=supp(\boldsymbol{s}_0:|\boldsymbol{s_0}|>\alpha) \tag{14}$$

which is the support of the elements whos amplitudes are larger than α. In [13], the threshold α is determinded by the $b\%$-Energy support which means that T contains at least $b\%$ of the signal energy. The prior knowledge is usually not very accurate. However, [13] shows that T can contain some errors. In WASAR imaging, we set $b\%=90\%$.

Then with the known support, LS-CS-Residual can be implemented for WASAR subaperture imaging. Backprojection can serve to approximate an inverse operator by the adjoint operator. This adjoint operator is the matched filter for ideal point scattering. This adjoint operator is the matched filter for ideal point scattering.

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TABLE I
WIDE ANGLE SAR IMAGING BASED ON LS-CS-RESIDUAL

Input:	Subaperture echo data $\boldsymbol{r}_i$ ($i = 1 : I$ and I is the number of the subapertures), measurement matrix $\boldsymbol{\Phi}_i$, iterative parameter μ, error parameter $\varepsilon = 10^{-6}$, maxmum number of iterative steps $T_{\max}$.
Initialization:	Implement BP on the whole aperture data, estimate T from the BP image. $\boldsymbol{s}_i = 0$ $(i = 1 : I)$, $t = 0$.
Iteration:	**for** $i = 1 : I$ $(\boldsymbol{s}_{i,init})_T = (\boldsymbol{\Phi}_{iT})^{\dagger}\boldsymbol{r}_i, (\boldsymbol{x}_{i,init})_{T^c} = 0,$ $\boldsymbol{r}_{i,res} = \boldsymbol{r}_i - \boldsymbol{\Phi}_i\boldsymbol{s}_{i,init}$ $\boldsymbol{\beta}_i^0 = 0$ $Res = \varepsilon + 1;$ **While** $t < T_{\max}$ and $Res > \varepsilon$ $\boldsymbol{\beta}_i^{t+1} = f_{\lambda\mu}(\boldsymbol{\beta}_i^t + \mu\left[\boldsymbol{\Phi}_i^H(\boldsymbol{r}_i - \boldsymbol{\Phi}\boldsymbol{s}_{i,res}^t)\right])$ $Res = \Vert\boldsymbol{\beta}_i^{t+1} - \boldsymbol{\beta}_i^t\Vert_2;$ $t = t + 1;$ **end while**; $\hat{\boldsymbol{s}}_i = \boldsymbol{\beta}_i^{t+1} + \boldsymbol{x}_{i,\text{init}}$ $T' = \text{supp}(\hat{\boldsymbol{s}}_i)$ $s_i = {A_{T'}}^{\dagger}y_i$ **end for** .
GLRT:	$s_{x,y} = \max_i \vert s_{x,y}^i\vert$.

In large scale SAR imaging problem, restoring the measurement matrix could cost huge memory, which is unbearable for most of the computers. Instead of calculating the LS estimation, we implement BP firstly. BP alone serves to approximate an inverse operator by the adjoint operator. This adjoint operator is the matched filter for ideal point scattering. So the result can be regarded as the least squares estimation, which would be adopted in LS-CS-Residual based WASAR imaging.

After the construction of the subaperture images, the generalized likelihood ratio test (GLRT) [1] can be implemented for the final composite image. GLRT is defined as

$$s_{x,y} = \max_i |s_{x,y}^i| \tag{15}$$

where $s_{x,y}^i$ is the scattering at pixel (x, y).

The algorithm is summarized in Tabel I.

C. Azimuth-Range Decouple Scheme

In real SAR data imaging, implementing CS on the data faces huge memory cost. Because the storage of the measurement matrix and its complex conjugate transpose cost huge memory, which is unbearable for most of the computer. We take scene size to be 1024×1024 and the raw data size is also 1024×1024. Every complex number occupies 16 bytes. The memory cost for the measurement matrix is more than 16 TB. To reduce the memory, [12] and [15] proposed an azimuth-range decouple scheme. The azimuth-range decouple method substites the measurement matrix and its complex conjugate transpose with the traditional matched filter method based operators.

In WASAR, the scattering of the subaperture can be regarded as isotropy. BP is implemented in subaperture imaging. In this paper, we take BP based operators in real WASAR subaperture imaging.

BP mainly include two operations, Fourier transform and azimuth coherent addition. Assume the BP based imaging process is $\mathcal{I}(\cdot)$ and data generation process is $\mathcal{G}(\cdot)$. $\mathcal{I}(\cdot)$ and $\mathcal{G}(\cdot)$ is formulated as

$$\mathcal{I}\{\cdot\} \cong \mathcal{R}^{-1}\{\mathcal{H}\{\mathcal{F}^{-1}\{\mathcal{R}\{\cdot\}\}\}\}, \tag{16}$$

$$\mathcal{G}\{\cdot\} \cong \mathcal{R}\{\mathcal{F}\{\mathcal{H}^{-1}\{\mathcal{R}^{-1}\{\cdot\}\}\}\} \tag{17}$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the the Fourier transform pairs, $\mathcal{H}$ is azimuth coherent addition operator and $(\mathcal{H})^{-1}$ is its inverse operation, $\mathcal{R}$ reshapes the vector into matrix and $\mathcal{R}^{-1}$ reshapes the matrix into vector.

We now consider the memory cost. Without loss of generality, PQ is supposed to be equal to MN. Since the proposed method only needs to store the input, output and

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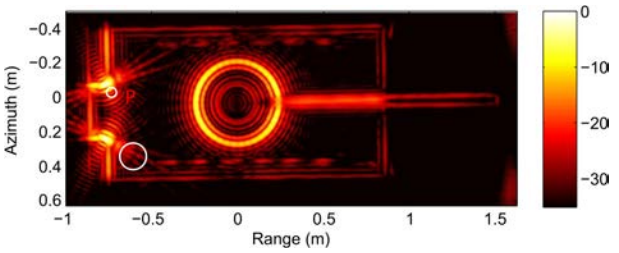
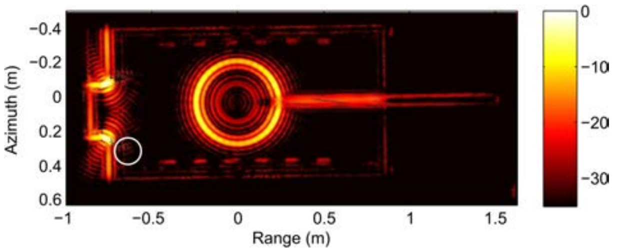
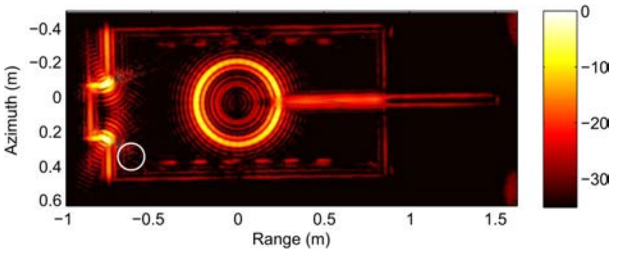


(a)



(b)



(c)

Fig. 2. Results of the three models. (a) GLRT result of CS. (b) GLRT result of LS-CS. (c) GLRT result of MCS.

several matrices, its memory cost is $\mathcal{O}(PQ)$ bytes. In comparison, without adoption of azimuth-range decouple scheme, the memory cost of the measurement matrix and its conjugate transpose is $\mathcal{O}((PQ)^2)$ bytes. The proposed method reduces the memory cost dramatically and makes the large scale real data processing possible.

IV. Experimental Results

In this section, we use real data collected by the Institute of Electronics, Chinese Academy of Sciences to show the effectiveness of the proposed method. The real data of a metal tank model are measured in an anechoic chamber on a turntable, which is in uniform circular motion. The radar is a stepped frequency type and has a center frequency of 15 GHz and bandwidth 6 GHz. The 360° whole aperture is divided into 36 subapertures. The pixel size of the SAR image is 0.25cm×0.38cm. We reconstruct the subaperture images with CS and LS-CS. In the experiment, we also implement another model called Modified compressed sensing (MCS) [13] [17], which also adopts the partial known information in the subaperture reconstruction. Fig. 2 (a), (b) and (c) show that the three models can reconstruct the targets. However, for LS-CS and MCS, the support is estimated from the BP image which is generated with all subaperture data. The resolution of BP is higher than the subaperture images, which means fairly accurate partial known support estimation. Thus, less sidelobes

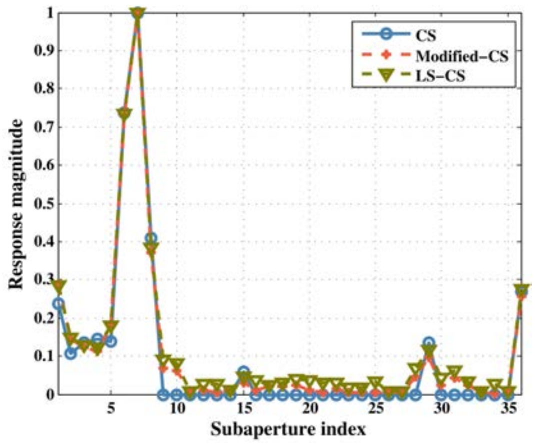


Fig. 3. Aspect dependent scattering curve of P.

remain in the GLRT image of LS-CS and MCS than those in the GLRT image of CS.

Fig. 3 is the aspect dependent scattering of the selected target. The scattering is normalized. It is shown that the three can reconstruct the main scattering of the targets. However, CS may fail to reconstruct the weak scattering. Since the prior information are adopted in LS-CS and MCS, the supports of weak scattering targets are preserved in the subapertures. So LS-CS and MCS can recover the aspect dependent scattering more accurately than CS.

V. Conclusion

This paper has proposed a novel WASAR subaperture imaging algorithm based on LS-CS-Residual. Backprojection is first implemented with whole aperture data to estimate the partial known support. Then the subaperture images are reconstructed individually with LS-CS-Residual. Backprojection based azimuth-range decouple operators are implemented in the reconstructions of subaperture images. Compared with the traditional compressive sensing, the proposed method can reconstruct the scattering more accurately. The proposed method can also reconstruct the scene with less undesirable artifacts.

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