RECOVERY OF IMAGES FROM A MIXTURE WITH MULTIPLICATIVE NOISE

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ABSTRACT

The use of independent component analysis (ICA) in coherent images needs to take into account the presence of the multiplicative noise that exits in this kind of images. In this paper, the recovery of original images from a mixture contaminated with this type of noise is studied using the ICA ideas. The mixing matrix is obtained using the fourth order multiplicative ICA method, which extracts the mixture before removing the noise. The result is a noisy version of the original images, where the effect of other images is reduced. The quality of the images is finally improved with the used of a multiplicative noise removal method. The proposed approach is compared with the direct use of ICA method over the noisy mixture or a denoise version of it, using simulated images.

Index Terms— Independent Component Analysis, Multiplicative Noise, Image Processing, Coherent Images.

1. INTRODUCTION

Independent component analysis (ICA) has been used extensively in the extraction of linear information from many types of signals. The ICA methods can be applied to the data recorded in some sensors in case where the later result from the mixing of independent signals. In its simplest and original form, ICA is designed to obtain the mixing matrix when the data are the instantaneous linear mixture of independent sources (with the same or smaller dimension than the data). Once the mixture is estimated, the original sources are computed by solving the *blind source separation* (BSS) problem. Since this first model was formulated in [1], the ICA methods have been extended to treat with non-linear mixture, additive noise, more sources than signals, convolutive mixture, partially dependent sources, etc., which has made possible their application to many fields.

One of the fields where ICA performance and applicablility has significantly improved since the initial attempt in [2], is image processing, where it has been applied to many kinds of images, as natural [3], video [4], multispectral [5] or hyperspectral [6]. The linear information obtained by the ICA methods has been used to solve different problems like texture classification, creation of thematic maps, restoration of images and unsupervised classification.

A type of images where ICA has been less applied, or even the application has been unsuccessful [7], is coherent images, such as ultrasound, laser, or synthetic aperture radar (SAR) images. The limitation of ICA in these images is mainly caused by the existence of multiplicative noise, which is due to the coherent formation process. The information in a pixel is the result of the sum of different backscatterings from the same region with fluctuating phase, which can be modeled as a noise free image multiplied by an independent noise of mean one [8].

In this paper, it is supposed that some original images are mixed and the data recorded in the sensor consist in this mixture of the original images but contaminated with multiplicative noise. If ICA is applied to these noisy mixture of images, even for a low level of noise, it does not estimate correctly the linear information, and, therefore, the original images can not be recovered using the estimated mixture. This behavior was shown for random sources in [9] [10].

Other option to recover the images would be to use a multiplicative noise removal method, in first place, and then use a standard ICA method to extract the mixture. The subsequent application of the inverse of the obtained mixture over the denoised data produces an estimation of the original images. If the method works perfectly, the results are the correct unknown mixture and the original images. However, the noise removal method is far from being perfect, so it will identify linear information as noise, destroying part of the structure of the mixture, which will produce a poor posterior estimation of it and, therefore, of the original images. The above scheme, although it might be proved efficient for low noise levels, it fails in the cases where the noise is high.

The approach in this paper is different, and consists in extracting the mixture *before* any noise removal algorithm. This can be done with the *fourth-order multiplicative ICA* (FMICA) method described in [10]. The algorithm obtain the

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inverse of the unknown mixture, and also some statistical information about the noise and the original images. Even if the actual mixture was known, the inverse mixture matrix application can not lead to the full recovery of the original images due to the presence of noise. However, the obtained images are going to be the original ones contaminated by multiplicative noise [11]. As a result, a multiplicative noise removal method can be used as a post processing stage in order to improve their quality.

The independence of the original images assumption it might be considered quite restrictive. However, the objective of this work is to apply the recovery method in situations where the ICA analysis is expected to obtain good results, which implicitly assumes the existence of independent images. Even though ICA methods rely on this restrictive condition of independence, it has been shown that they can be successfully used in more general image processing problems. We expect, that this is going to be the case for coherent images as well.

Many methods for multiplicative noise reduction or removal have been proposed for more than twenty years, mainly in the field of radar images, where this noise is called *speckle noise*. In [12] these methods are classified in two main groups. The first group are principally *minimum mean square error* (MMSE) and the second are *maximum a posteriori* (MAP) filters. In this paper, MMSE filters will be used as multiplicative noise removal methods.

The signal model is studied in Section 2. In Section 3, the FMICA and noise removal methods are briefly introduced, while in Section 4 the behaviour of the proposed approach is tested using simulations. The paper finalizes with the main conclusions and future work.

2. SIGNAL MODEL

In the model we assume, the data are a linear mixture of independent images, contaminated with multiplicative noise. Hereafter, this model will be refered to as *multiplicative ICA* (MICA) model. The data consist in N images of $M_1 \times M_2$, and each one of them can be ordered as a row vector of length $M = M_1 M_2$. Then, the data are arranged in a $N \times M$ matrix $\mathbf{Z}$, where each row $[z_i(1), \ldots, z_i(M)]$ corresponds to one image (so $i = 1, \ldots, N$) and each column $\mathbf{z}(t)$ corresponds to the information in a pixel position for the N images, where the variable t characterizes the pixel position, so it goes from 1 to M. These images are supposed to be noisy version of the mixture of some independent images. The ith original image is notated as $s_i(t)$, with $t = 1, \ldots, M$, while i goes from 1 to the number of independent images, that is assumed to be N for simplicity, although the development would be the same for any other value, smaller than N. The original images can be collected in a $N \times M$ matrix $\mathbf{S}$, where its elements are $[\mathbf{S}]_{it} = s_i(t)$. These images are mixed with a full rank matrix $\mathbf{A}$, that is called the mixing matrix, such that the noise-free

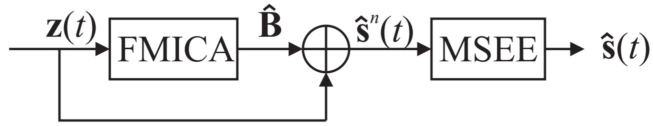


Fig. 1. FMICA recovery method

mixture of images is collected in a $N \times M$ matrix $\mathbf{X}$, with $\mathbf{X} = \mathbf{AS}$, where each row, $[x_i(1), \ldots x_i(M)]$, corresponds to a mixed image and each column, $\mathbf{x}(t)$, corresponds to the information in the t pixel for the different mixed images. Then, the recorded data follow the model[8]:

$$z_i(t) = v_i(t) x_i(t) \tag{1}$$

where the process $v_i(t)$ for $t = 1, \ldots M$ is the multiplicative noise that corrupts the ith image, having mean one and being independent both from image to image and from the vectors $x_j(t)$, for all $j = 1, \ldots, N$. As it has been done before with $\mathbf{s}(t)$ and $\mathbf{x}(t)$, the noise processes can be grouped in a matrix $\mathbf{V}$, where each row, $[v_i(1), \ldots v_i(M)]$, corresponds to the multiplicative noise present in the ith mixed image and each column, $\mathbf{v}(t)$, corresponds to the noise in the tth pixel of the different images. This is a standard model in SAR or other coherent images and is the base of many speckle reduction methods[8].

The approach proposed in this paper consist in two steps. First the FMICA algorithm in [10] is used to obtain an estimate of the inverse of the mixing matrix, $\mathbf{B}$, and and then the estimation of a noisy version of the original images can be achieved via equation $\hat{\mathbf{s}}^n(t) = \hat{\mathbf{B}}\mathbf{z}$. Even if the estimation of the mixing matrix was exact, (which means that $\mathbf{BA} = \mathbf{I}$)), the ith estimated source whould not consist of the desired one only but of the desired source plus a zero-mean, signal depend noise term, i.e., $\hat{s}_i^n(t) = s_i(t) + \sum_k \hat{B}_{ik}(v_{ik} - 1)s_k$. Due to the fact that such a type of noise can be effectively eliminated by MMSE filters [13] [12], the final estimate $\hat{\mathbf{s}}(t)$ of the original images result from the MMSE filtering of the estimated independent components $\hat{s}_i^n(t)$. The proposed method is shown in Figure 1, and will be called FMICA recovery method.

As it can be seen, the key step in FMICA recovery method is the FMICA algorithm, which is going to be introduced in the next section. In order to do so, it is necessary to study some of the statistical properties of the model.

The ICA methods use the statistics of the outputs of a linear transformation $\mathbf{u(t)} = \mathbf{Wx(t)}$ to find the inverse of the mixing matrix, called the *unmixing matrix*, since in the case $\mathbf{W} = \mathbf{A}^{-1}$ the components of the output $\mathbf{u}(t)$ are going to be independent. As in the ICA case, the FMICA algorithm uses the statistical properties of $\mathbf{y}(t) = \mathbf{Bz}(t)$, where $\mathbf{B} = \mathbf{A}^{-1}$, but in this case the unmixing process is not going to lead to independent components $\mathbf{y}(t)$ are not independent. Specifically, the covariance, third- and fourth-order cumulants of the components of $\mathbf{y}(t)$ depend on some statistical functions of the noise and the original sources, and on the mixing and the unmixing matrix, which can be collected in the following parameters:

$$\{\gamma_i^s, \kappa_i^s, \eta_i, \omega_{ij}, \rho_i, \phi_i, B_{ij}\} \quad (2)$$

where γ_i^s and κ_i^s are the skewness and kurtosis of $s_i(t)$ and the rest of parameters are

$$\eta_i = \sqrt{\sigma_i^v}\mu_i^x;\;\; \omega_{ij} = \sqrt{\sigma_i^v}A_{ij};\;\; \rho_i = \gamma_i^v/(\sigma_i^v)^{3/2};$$
$$\phi_i = \kappa_i^v(\kappa_{iiii}^x + 4\mu_i^x\gamma_{iii}^x + 6(\mu_i^x)^2\sigma_{ii}^x + 3(\sigma_{ii}^x)^2 + (\mu_i^x)^4) \quad (3)$$

with σ_i^v, γ_i^v and κ_i^v being the variance, skewness and kurtosis of v_i; and μ_i^x, σ_{ii}^x, γ_{iii}^x and κ_{iiii}^x being the mean, variance, skewness and kurtosis of x_i. The indices in the parameters go from 1 to N. With these parameters and the unmixing matrix $\mathbf{B}$, the theoretical structure of γ_{ij}^y, σ_{ijk}^y and κ_{ijkl}^y is completely defined. The specific structure can be found in [10].

It is important to point out that, as the problem is blind, the parameters $\{\eta_i, \omega_{ij}, \rho_i, \phi_i, \gamma_i^s, \kappa_i^s\}$ and the unmixing matrix $\mathbf{B}$ are unknown.

3. METHODS

In this section, first the FMICA algorithm and then the multiplicative noise removal is briefly discussed.

3.1. FMICA algorithm

ICA searches for the linear transformation which renders the components of its output as independent as possible. If the data satisfy the ICA model, the solution is the inverse of the mixing matrix. In the case of MICA model, it has been shown that the components of the output of the unmixing matrix are not independent, but they possess a specific statistical structure, and FMICA exploits it in order to find the solution. The structure is explicitly shown in [10], which is satisfied if the unmixing matrix $\mathbf{B}$ is the inverse of the mixing matrix, and the rest of the parameters in (2) take their theoretical values.

On the other hand, the covariance, third- and fourth-order cumulants of the output $\mathbf{y}(t)$ can be estimated from the noisy data, for any matrix $\mathbf{B}$. If these three estimated functions are noted as $\hat{\sigma}_{ij}^y$, $\hat{\gamma}_{ijk}^y$ and $\hat{\kappa}_{ijkl}^y$, they can be obtained from the covariance, third- and fourth-order cumulants of the noisy data $\mathbf{z}(t)$, which are noted as $\hat{\sigma}_{ij}^z$, $\hat{\gamma}_{ijk}^z$ and $\hat{\kappa}_{ijkl}^z$, respectively. The explicit relation is straightforward to obtain, taking into account the relation $\mathbf{y(t)} = \mathbf{Bz(t)}$:

$$\hat{\sigma}_{ij}^y = \sum_{mn} B_{im}B_{jn}\hat{\sigma}_{mn}^z\;;\; \hat{\gamma}_{ijk}^y = \sum_{mnp} B_{im}B_{jn}B_{kp}\hat{\gamma}_{mnp}^z$$
$$\hat{\kappa}_{ijkl}^y = \sum_{mnpq} B_{im}B_{jn}B_{kp}B_{lq}\hat{\kappa}_{mnpq}^z \quad (4)$$

It can be seen that the functions $\hat{\sigma}_{ij}^y$, $\hat{\gamma}_{ijk}^y$ and $\hat{\kappa}_{ijkl}^y$ depend only on the unmixing matrix $\mathbf{B}$, while the functions σ_{ij}^y, γ_{ijk}^y and κ_{ijkl}^y depend both on the unmixing matrix and the set of parameters in (2) which are unknown due to the fact that the problem is blind.

Hence, the estimated functions (4) will be equal to the theoretical ones when $\mathbf{B} = \mathbf{A}^{-1}$ and the rest of the parameters in (2), take their theoretical values. The latter, will be called the *correct solution*. To measure how well the structure is reproduced for a specific matrix $\mathbf{B}$ and a specific set of parameters (2), a cost function $J = J(B_{ij}, \eta_i, \omega_{ij}, \rho_i, \phi_i, \gamma_i^s, \kappa_i^s)$ can be built as:

$$J = \sum_{ij}(\mu_i^x \sum_k \omega_{ik}B_{kj} - \eta_i\delta_{ij})^2 + \sum_{i\geq j}\left(\sigma_{ij}^y - \hat{\sigma}_{ij}^y\right)^2$$
$$+ \sum_{i\geq j\geq k}\left(\gamma_{ijk}^y - \hat{\gamma}_{ijk}^y\right)^2 + \sum_{i\geq j\geq k\geq l}\left(\kappa_{ijkl}^y - \hat{\kappa}_{ijkl}^y\right)^2 \quad (5)$$

The first term in the cost function has been included in order to take into account the theoretical relation between the parameters B_{ij}, ω_{ij} and η_i. The cost function , which is formed by $N_2 = N(N+1)/2((N+2)/3(1+(N+3)/4)) + N^2$ terms, is function of $N_1 = N(2N+5)$ parameters, and will be zero at the correct solution.

Thus, the problem is reduced to find the value of the parameters $\{\phi_i, \eta_i, \omega_{ij}, \rho_i, \gamma_i^s, \kappa_i^s, B_{ij}\}_{i,j=1,\ldots,N}$ that minimizes the cost function (5), which means a problem of non-linear minimization of J. Although the non-linear minimization method mostly used in the ICA literature is the steepest descend using the natural gradient, it is necessary to resort to another minimization method here. The natural gradient of the cost function is not easy to establish, since the set of parameters is not a multiplicative group, and the standard steepest descendant method is too slow. In the FMICA algorithm, the minimization is accomplished using the quasi-Newton method called BFGS (Broyden- Fletcher- Goldfard-Shanno). In this method the set of parameters, which are grouped in a $N_1 \times 1$ vector $\mathbf{b}$, is updated in the step k as:

$$\mathbf{b}(k+1) = \mathbf{b}(k) - \mu(k)\mathbf{H}(k)\nabla_k J \quad (6)$$

where $\mu(k)$ is the learning rate in the step k, $\nabla_k J$ is the gradient of J in the step k, and the matrix $\mathbf{H}(k)$ is an estimate of the inverse of the Hessian in the step k, which is forced to be positive definite and symmetrical, and is obtained using the value of the parameters and the gradient of J in the steps k and $k-1$. For more details about BFGS, readers are referred to literature on non-linear optimization, for example [14].

Only the gradient and the starting point are necessary for the FMICA algorithm to be completed. With respect to the gradient, its derivation can be found in the appendix of [10]. With respect to the starting point, as in most non-linear optimization methods, adequate starting values are necessary for the method to converge. In our case, appropriate initial values for all parameters but the unmixing matrix, $\mathbf{B}(0)$ can be determined by assuming a not too high noise level. With respect to $\mathbf{B}(0)$, it is necessary to resort to a standard ICA method, the FastICA in this paper. In other words, the solution of the FastICA is set as initial for $\mathbf{B}$ in the FMICA algorithm. Summarizing, the initialization of the FMICA algorithm is as follows:

$$\begin{aligned} \mathbf{B}(0) =& \text{solution of FastICA method;} \\ \phi_i(0) =& \rho_i(0) = 0; \;\; \gamma_i^s(0) = \hat{\gamma}_{iii}^{\tilde{y}}; \;\; \kappa_i^s(0) = \hat{\kappa}_{iiii}^{\tilde{y}}; \\ \eta_i(0) =& \sqrt{\hat{\sigma}_i^v}\hat{\mu}_i^z \;\text{ and }\; \omega_{ij}(0) = \sqrt{\hat{\sigma}_i^v}[\mathbf{B}(0)^{-1}]_{ij} \end{aligned} \tag{7}$$

where $\hat{\sigma}_i^v$ is obtained as $\hat{\sigma}_i^v = \frac{\hat{\sigma}_{ii}^z - \sum_k [\mathbf{B}(0)^{-1}]_{ik}}{\sum_k [\mathbf{B}(0)^{-1}]_{ik}^2 + (\hat{\mu}_i^z)^2}$ and $\tilde{\mathbf{y}}(t) = \mathbf{B}(0)\mathbf{z}(t)$. These last estimatated values are inexact, due to the errors in $\mathbf{B}(0)$, but this is not a problem since the value of $\hat{\sigma}_i^v$ is inside other parameters (η_i and ω_{ij}), and is updated in the minimization process. In fact, a initial value of zero for all the $\hat{\sigma}_i^v$ provides, in most of the cases, a good initialization, such as the method converges, but the convergence is slower. The FMICA algorithm consists in the minimization of (5) with the update formula (6), the gradient of J and the initialization (7).

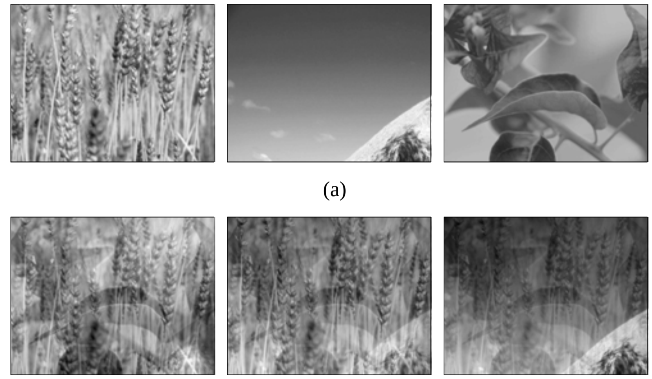
3.2. Multiplicative noise removal methods

The literature on multiplicative noise removal methods is very large and an extensive but not exhaustive list of them can be found in [12], where they are classified in two main types: MMSE and MAP methods. The MMSE filters assume different multiplicative models in the images and perform the filtering using non-stationarity estimators of the noise free scene. Examples of this methods are the Lee [15], the Kuan *et al.*[16] and the Frost *et al.* [17] filters. The MAP filters use the *power density function* (PDF) of the noise (that is known for a specific application) and assume some PDF for the noise free scene. This statistical information is used to calculate the most likely image. Both PDFs, the noise's and the noise-free image's, are based on theoretical and practical studies of images. For the first PDF, this corresponds to a study about the coherent formation process, and for the second PDF, to a study about the reflective properties of typical scenes. examples of this type of methods are the Bayesian Gaussian [18] and the Gamma [19].

The statistical properties of noise are well known, at least in fully developed speckle SAR images, where it is circular Rayleigh in amplitude and negative exponential in intensity, but the PDF of the original images depend of the nature of the scene and it is not known in a general. For this reason, in this paper a MSSE method is used, specifically the Lee filter [15].

4. RESULTS

In this section, the behavior of the proposed recovery method is studied via simulations. As said before, working with simulated data gives us the possibility to establish a direct comparison between the recovered images and the initial ones used to generated the data. This is not possible working with real data, where the performance of the method would be judged as a function of another task such as classification or detection.



(a)

(b)

Fig. 2. (a) Original images; (b) Mixture of images.

In our simulation study, three original images, shown in Figure 2(a), are used to generate the data. The variance of these images is normalized to the unity, to avoid the scale indetermination that appears in the ICA model [1]. The images are not completely independent, but the results will show that the ICA principles can still be applied.

These images are mixed with a 3×3 matrix, whose elements have been randomly generated. The resulted mixture of images appears in Figure 2(b). On other hand, the noise $\mathbf{V}$ is randomly generated following a Gaussian PDF, changing the variance to study how the behavior of the method changes with the level of noise. None of the involved methods depend theoretically on the PDF of the noise, so Gaussian has been chosen for simplicity.

The performance of the FMICA recovery method is compared with the standard recovery method that incorporates the FastICA which is shown in Figure 3. As the standard ICA method can not be directly applied to the noisy mixture, the FastICA recovery method first uses the MSEE filter to obtain an estimation of the noise-free mixed images $\hat{\mathbf{x}}'(t)$. If the multiplicative noise removal method works perfectly, the process will produce the original mixed images, but if not, it will destroy part of the linear information and will worsen the posterior separation of the original images. After the denoise step, the standard FastICA method [20] is used, and it produces an estimate of the unmixing matrix $\hat{\mathbf{B}}'$. With this matrix, the estimation of the original images is obtained as $\hat{\mathbf{s}}'(t) = \hat{\mathbf{B}}'\hat{\mathbf{x}}'(t)$.

When the standard deviation of the noise is 0.06, the noisy mixture of images appears in Figure 4(a). Both method (the FMICA recovery and the FastICA recovery methods) are ap-

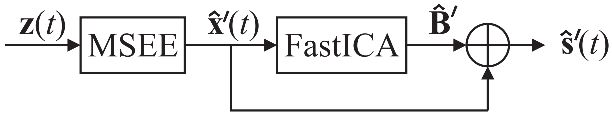
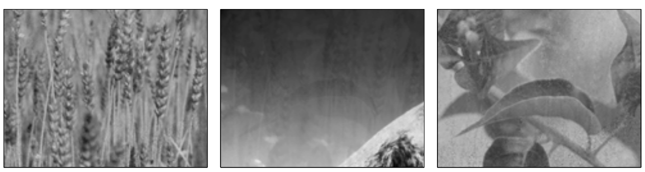


Fig. 3. Standard method

(a)

(b)



(c)

Fig. 4. (a) Mixture of images multiplied by Gaussian noise of 0.06 standard deviation (b) Recovered images with the FastICA recovery method; (c) Recovered images with the FMICA recovery method

plied to this noisy mixture, and the results are presented in Figure 4(b) and 4(c), where corrections in the order and sign of the recovered images have been applied when needed. These corrections are needed since, although the scale indetermination has been fixed with the normalization over the original images, the permutation and sign indeterminations of the ICA model still remain [1].

As it can be seen in these images, both methods obtain reasonably good estimations of the original images. Even so, it can be seen that the separation obtained by the proposed method is better than the one obtained by the FastICA. This is due to the fact that the MSEE denoising method is not perfect and it affects the ability of the FastICA method to estimate the correct unmixing matrix. On the other hand, the FMICA algorithm obtain the mixture before the denosing process, so the demixing of the image is better with the proposed method. The different separation capacity of both methods is clearer in the second image, where traces of the other two images (specially the first) are clearly visible in the results of the FastICA recovery method, while this behaviour is much less evident

(a)

(b)

Fig. 5. For noise of standard deviation equal to 0.1 (a) Recovered images with the FastICA recovery method; (b) Recovered images with the FMICA recovery method.

in the results of the one. This visual observation can be measured by the Frobenious norm of the difference between the original and the estimated images from the FastICA recovery method ($\hat{s}_2^1(t)$) and the FMICA recovery method ($\hat{s}_2^2(t)$) yields to:

$$||\hat{s}_2^1(t) - s_2(t)||_{\text{Fro}} = 94.9\ ;\ ||\hat{s}_2^2(t) - s_2(t)||_{\text{Fro}} = 42.5$$

The behavior observed in Figure 4 is clearer when the level of noise is increased. If the standard deviation of the noise is 0.1, the recovered images are shown in Figure 5. In this case, the distances from the original to the estimated images from the FastICA recovery method and the FMICA recovery method yields to:

$$||\hat{s}_2^1(t) - s_2(t)||_{\text{Fro}} = 102.1,\ ||\hat{s}_2^2(t) - s_2(t)||_{\text{Fro}} = 45.1$$

In Figure 6, the distance between the recovered and the original images is shown as a function of the noise level. The distance is computed as the mean for the three images of the norm of the difference between estimated and original images. It can be seen how, except for very small level of noise, the proposed method is always better than the FastICA recovery method.

5. CONCLUSIONS AND FUTURE WORKS

In this paper, a method for recovering images from a mixture contaminated with multiplicative noise is proposed. The approach try to extend the use of ICA ideas to coherent images, a field where the ICA application has been very limited. The proposed method used the FMICA algorithm to recovered the mixture before any denoise process. This approach is more efficient that trying to demix a denoise version of the data, since the denoise process destroy part of the linear information. In future work, the statistical information about the

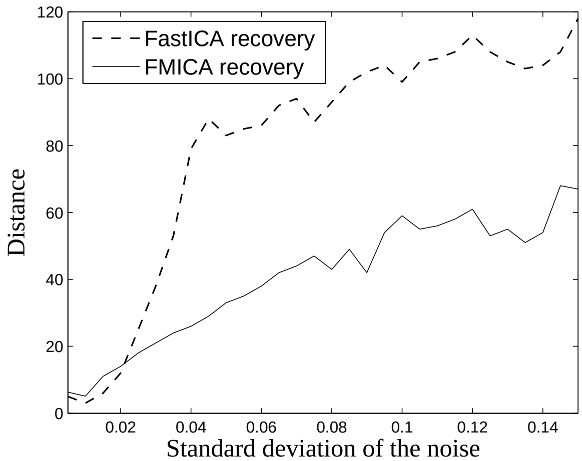


Fig. 6. Distance between recovered and original images with FastICA recovery method and FMICA recovery method.

original images and the noise that the FMICA algorithm also provides, will be incorporated to newly designed methods in order to enhance the overall recovering performance.

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